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ABSTRACT

This guide represents the final experimental version of a pilot project conducted in the United States between 1973 and 1976. The ideas and manner of presentation are based on the works of Georges and Frederique Papy. They are recognized as having introduced colored arrow drawings ("papygrams") and models of our numeration system (the Papy "minicomputer") into the teaching of mathematics at the elementary and secondary level in Belgium. This program follows the "spiral approach." This guide includes the General Introduction and "The World of Numbers." The program description includes suggestions for teaching the materials and using the work pages. Suggestions for organizing the program are also given. Worksheets for the students accompany the guide. (Author/SA)

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omprehensive School Mathematics Program
CEMREL, Inc.
Final Experimental Version
2-33201

CSMP Mathematics for the Upper Primary Grades Part I

Teacher's Guide

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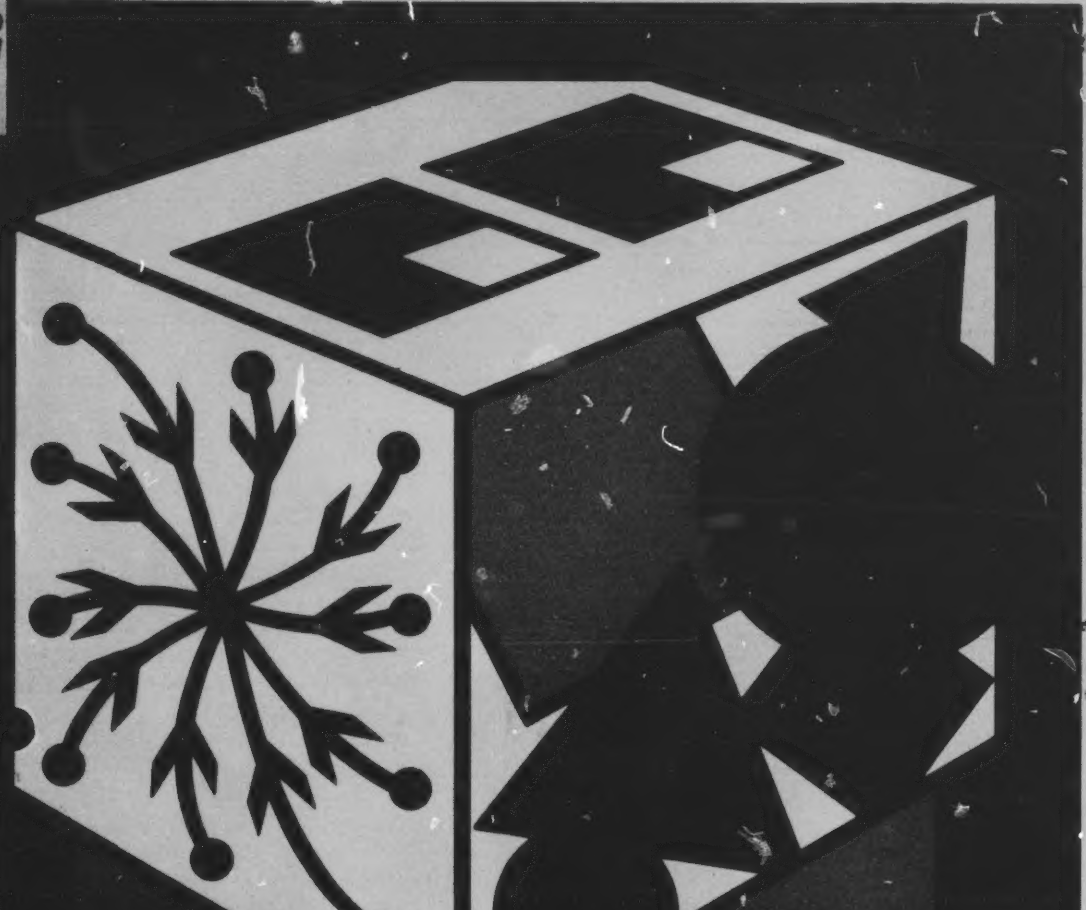
General Introduction
The World of Numbers



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Comprehensive School Mathematics Program

CSMP Mathematics for the Upper Primary Grades

Part I

Teacher's Guide

CEMREL, Inc.

PREFACE

CSMP Mathematics for the Upper Primary Grades (UPG) owes its existence to the coming together under the leadership of Burt Kaufman (CSMP Director) of a special group of people who have been concerned not only about the kind of mathematics that is being taught to children but also about the manner in which it is taught. This is even more important in the crucial early years of a child's life than in later years.

The CSMP elementary school curriculum is in large part the outgrowth of a great deal of active involvement with children in ordinary classroom settings, and its present form was strongly influenced by information gathered from classes participating in an Extended Pilot Test conducted nationwide during 1974-77. But its manner of presentation and its main pedagogical ideas and tools, which stand out from all other curricula, are based on the pioneering works of Georges and Frédérique Papy, which we acknowledge here with deep gratitude.

Use of colored arrow diagrams* -

Georges Papy, Die Erste Elemente der Moderne Mathematik: Shriftenreihe zur Mathematik, Hefte 10-11 (Frankfurt and Hamburg: Otto Salle Verlag, 1962-63)

Mathématique Moderne, Vols. 1, 2, 3, 5, 6 (Brussels and Paris: Didier, 1964-1967)

Modern Mathematics 1, 2 (London: Collier-Macmillan Ltd., 1968)

Frédérique and Georges Papy, L'Enfant et les Graphes (Brussels, Montreal and Paris: Didier, 1968)

Graphs and the Child (Montreal: Algonquin, 1970)

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* In 1959, Georges Papy introduced colored arrow diagrams into the teaching of mathematics at the elementary and secondary school level, and for this reason, they are sometimes referred to as "papygrams" in his honor.

Les Enfants de 4 Ans et le Langage des Graphes (Paris: Classiques Hachette, 1973)

Origin of the Papy Minicomputer-

Georges Papy, Minicomputer (Brussels: IVAC, 1968) in collaboration with Frédérique Papy

Use of graphs and the Papy Minicomputer with young children-

Frédérique Papy, Les Enfants et la Mathématique[*], Vols. 1, 2, 3, 4 (Brussels, Montreal and Paris: Didier, 1970-1976)

Mathematics and the Child 1, (Montreal: Algonquin, 1971)

By making this wealth of experience available to CSMP, Frédérique and Georges Papy have made an invaluable contribution to the production of this program. It must also be acknowledged that the fundamental idea of presenting mathematics to children via "pedagogical situations" is due to the mathematician and psychologist Caleb Gattegno.

Various prototype versions of the UPG Program dating back to 1973 were due to the efforts of former CSMP staff members José Agusti, Phyllis Klein, Edward Martin, and Christiane Vandeputte, and staff members Burt Kaufman and Frédérique Papy.

The large amount of information gleaned, thanks to the interest and concern of many teachers, in the Extended Pilot Tests since 1974 led to the revision of the program that culminated in this final version. During the 1976-77 school year, this final version of Part I was tried out by CSMP staff members Clare Heidema and Janis Schweitzer in Jackson Park Elementary School, University City, Missouri. In this effort they received valued collaboration and support of second grade teacher Marylou Long and the motivating responses of her students.

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[*] A series of books reporting on an experiment in teaching mathematics in grades 1 to 6 in Brussels, using the languages of strings, arrows and the Papy Minicomputer. The reports of the first four years have been published by Didier; the reports of the last two years exist only in manuscript.

NOTE ON AUTHORSHIP

This printed copy of the final experimental version of CSMP Mathematics for the Upper Primary Grades, Part I is the result of the work of the following people:

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Comprehensive School Mathematics Program

CSMP Mathematics for the Upper Primary Grades

Part I

General Introduction

CEMREL, Inc.

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SECTION ONE

NOTES TO THE TEACHER

WELCOME

You are about to teach the first semester (Part I) of CSMP Mathematics for the Upper Primary Grades (grades 2 and 3), perhaps for the first time. We wish you success in your teaching and hope that both you and your students will enjoy the activities suggested in this teacher's guide. It is the purpose of these Notes to the Teacher to help you use this book effectively; to describe the materials and give suggestions for organizing the program; and to answer some frequently asked questions which may also be your questions about CSMP.

You will find, as you and your students take part in the activities during the year, that there is a definite CSMP viewpoint about elementary school mathematics, a viewpoint that we hope you will come to understand and share. We believe that learning takes place when children react to interesting situations (sometimes called a "pedagogy of situations"[*]), whether real life or fantasy, such as in stories or games. These situations involve children personally and allow arithmetic to take the form of "adventures in the world of numbers".

We believe that every child can and does learn something from each situation encountered in the program, sometimes suddenly and dramatically, sometimes latently. For this reason, CSMP teachers do not stick to one topic until it is "mastered". Indeed our experience indicates that true learning often stops when the purpose of a lesson is merely the mastery of a skill. Instead, we believe that learning is more a "spiral" process than a "straight line" process, that intuitive leaps play as big a role as acquiring small successive pieces of information, and that a variety of situations can provide the interrelated experiences through which we learn.

The Question and Answer section of these Notes to the Teacher gives more explanation of what we mean by a "pedagogy of situations" and a "spiral development", but you will probably learn their meanings only by experiencing them yourself as you teach the curriculum. Meanwhile we assure you that a large number of CSMP teachers across the country have come to understand and embrace the CSMP style of teaching and learning and are greatly pleased with the results.[*]

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[*] This phrase is due to the mathematician and psychologist Caleb Gattegno.

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[*] See evaluation Report 2-A-1 in the CSMP Evaluation Report Series, CEMREL, Inc., St. Louis, Missouri.

PROGRAM DESCRIPTION

CSMP Mathematics for the Upper Primary Grades (UPG) is designed as a program for students in second and third grades. There are four parts to the curriculum, each corresponding to one semester's work. This is Part I of the four. It follows the CSMP First Grade Program.

Since the learning process occurs through many interrelated experiences, we have developed this curriculum so that no single lesson is an end in itself. It is neither intended nor expected that every student will meet the full challenge of a particular situation in any given lesson. Furthermore, we have found that it is most effective to vary the situations from day to day rather than to continue one type of situation or topic until so-called mastery has occurred. This spiral development (see also Question 1 of Section Two) is reflected in the materials and scheduling of various topics in the curriculum. It is consistent with CSMP's pedagogy of situations since with such an approach, a child experiences each of several ideas a little at a time and then proceeds through increasing levels of sophistication as the situations become more challenging. We hope you will keep in mind the spiral approach as you teach the CSMP curriculum.

1. Description of Materials

a) Teacher's Guide

These two volumes (five books) which are directed specifically to you, the teacher, is known as the "Teacher's Guide to CSMP Mathematics for the Upper Primary Grades, Part I". (By way of abbreviation, in what follows we shall refer to this collection simply as the "Teacher's Guide".) With the exception of this book, "General Introduction", each book of the Teacher's Guide addresses itself to a single area of the curriculum, and each of these areas is known as a "strand". The four strands with abbreviations given in parentheses are:

- World of Numbers - (N)

- Languages of Strings and Arrows - (L)
- Geometry and Measurement - (G)
- Workbooks - (W)

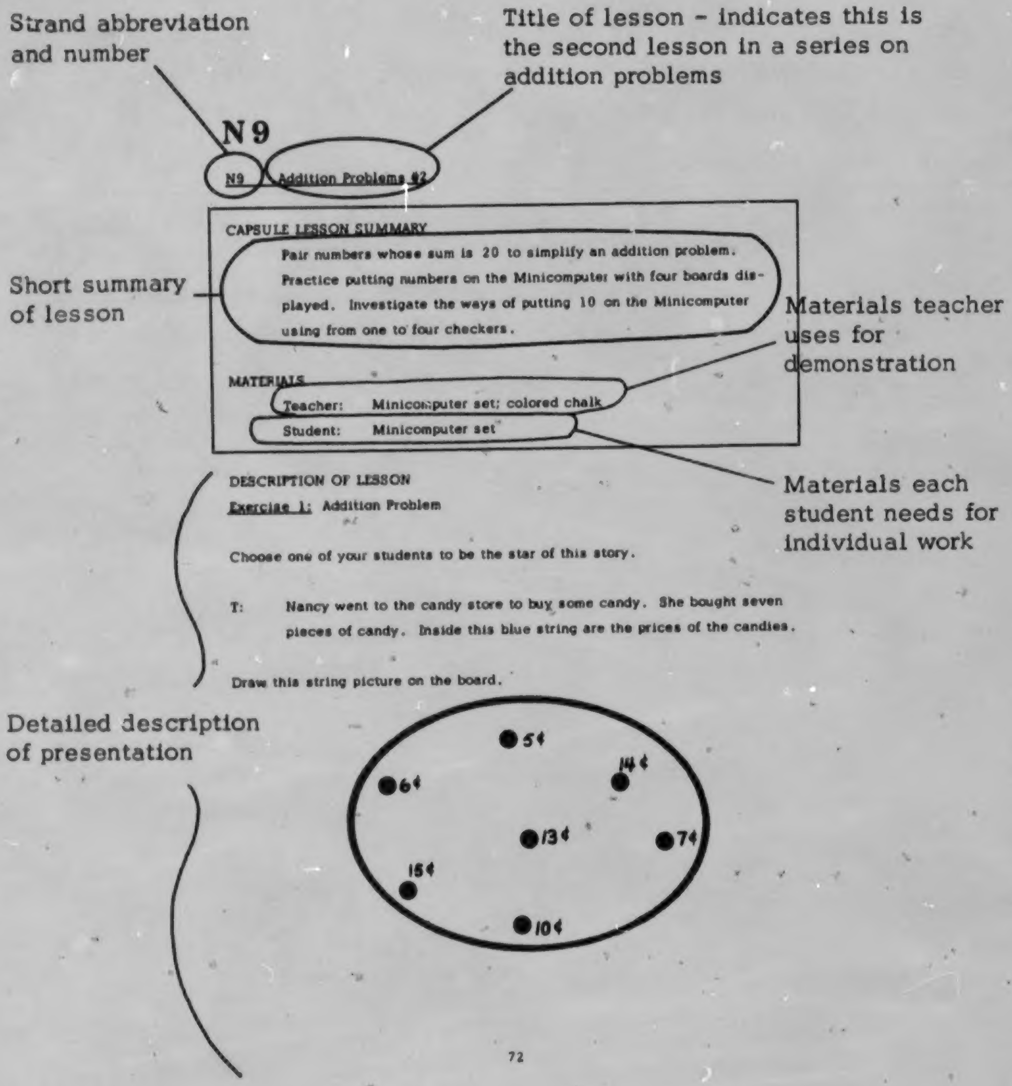
The five books comprising the Teacher's Guide are bound into two volumes, with a separation page inserted between the various books of a given volume. This volume contains both the General Introduction and the World of Numbers strand. The other volume contains the Languages of Strings and Arrows strand, the Geometry and Measurement strand, and the Workbook strand. Each of these five books has its own table of contents and is paginated individually.

The strands have the following arrangement of material.

- A table of contents. This enables you to locate whatever you need in that particular strand.
- An introduction to the strand. This provides you with information explaining why certain materials finds its place in that strand, what are the overall goals of study in that strand, and how this material fits in with that which is described in other strands.
- A series of lesson plans. These plans include Capsule Lesson Summaries, lists of needed materials for both teacher and students, and lesson descriptions which suggest possible classroom dialogue and indicate the use of worksheets and manipulative materials. The lesson descriptions also provide illustrations of diagrams the teacher puts on the board or otherwise displays for the class and answer keys for worksheets or other independent work during the lessons.

In each of the strand books, N, L, G, and W, the lessons are consecutively numbered. Each lesson is designated to be used during one full class period. For example, Lesson N8 is the eighth lesson in the N (World of Numbers) strand. Since this lesson is interspersed among lessons from other strands, it might be taught on, say, Wednesday of the fourth week of the term and the next N lesson (N9) might be taught on Monday of the fifth week.

Each lesson is presented in a form that includes its strand and number, title, summary, materials, and description. For example, here is the first page from the ninth lesson in the N (World of Numbers) strand.



Strand abbreviation and number

Title of lesson - indicates this is the second lesson in a series on addition problems

N9

N9 Addition Problems #2

CAPSULE LESSON SUMMARY

Pair numbers whose sum is 20 to simplify an addition problem. Practice putting numbers on the Minicomputer with four boards displayed. Investigate the ways of putting 10 on the Minicomputer using from one to four checkers.

Short summary of lesson

MATERIALS

Teacher:	Minicomputer set; colored chalk
Student:	Minicomputer set

Materials teacher uses for demonstration

Materials each student needs for individual work

DESCRIPTION OF LESSON

Exercise 1: Addition Problem

Choose one of your students to be the star of this story.

T: Nancy went to the candy store to buy some candy. She bought seven pieces of candy. Inside this blue string are the prices of the candies.

Draw this string picture on the board.

Detailed description of presentation

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Unlike the day-to-day presentation of lessons in the teacher's guide for the CSMP First Grade Program, the organization of lessons in this teacher's guide for UPG-I is a strand-to-strand presentation. By this we mean that on one day of the week a lesson is taught from a certain strand; and then on the next day of the week a lesson is taught from another strand; and so on. In this way we achieve the same spiraling effect as in the first grade program, but we also gain an extra advantage, namely, the benefit of organization of topics according to strands. For example, suppose you are interested in the way a topic in geometry is developed in the second grade. With a day-to-day format you would need to search through the entire guide for geometry lessons, but with a strand-to-strand format all these lessons are found, in their sequential order, in one part of the guide, the G-strand. You can then read the lessons in their natural order to gain an overview of the Geometry strand in second grade (first semester). At the same time, the strand-to-strand arrangement allows you to anticipate what is coming on the next "spoke" of the spiral development of any topic simply by reading ahead in the strand book in which that topic is developed.

b) Teacher's Materials

In addition to the teacher's guide, there are some demonstration manipulative materials you will need. The following materials are available from CSMP and should be in your classroom set of materials.

- Demonstration Minicomputer Set
  (4 magnetic Minicomputer boards; 1 set of magnetic checkers)
- A-Block String Game Kit
  (2 sets of A-Blocks; 2 attribute posters; 2 sets of string game cards; magnetic material)
- 0-109 Numeral Chart
- Number Line
- UPG-I Geometry Poster Set (four posters)
- Robot Walk Grid

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Other materials needed for individual lessons are listed under MATERIALS at the beginning of each lesson. Usually such materials (those not in your classroom set) are readily available or can be easily prepared from usual classroom supplies.

c) Student's Materials

There are some individual materials that each student (each pair of students in the case of storybooks) will need. The following materials are available from CSMP and should be in your classroom set of materials.

- Worksheet Booklet for UPG-I (with perforated pages which can be torn off sheet by sheet as needed)
- Workbooks
  (five titles: "7-12", "A Catalog of Problems #1", "A Catalog of Problems #2", "8?-15?", "Fishing for Numbers (Part I)")
- Tangram Booklet
- Addition Problems Booklet
- Storybooks from the STORIES BY FREDERIQUE Collection
  ("Where's My Nose" and "The Happy Puppet")
- Individual Minicomputer Set
  (2 sheets of Minicomputer boards; 1 sheet of punch-out checkers)
- Tangram Sheet
- Centimeter Tape Measure

Other materials needed for a lesson are listed under MATERIALS at the beginning of the lesson. Materials not available from CSMP are readily available or can be easily prepared from usual classroom supplies.

Some teachers find it convenient to provide each student with a sturdy cardboard box for storing manipulative materials. Others find it more convenient to store all the materials of one kind together in a container of appropriate size and to distribute these materials to the students as they are needed. In any case, you should probably keep the worksheet booklets, workbooks, and storybooks in your storage area and distribute them as called for in the lessons.

Some storybooks in the STORIES BY FREDERIQUE collection are provided in your classroom set of materials although they are not used in any lessons of the UPG-I Program. There are two copies of each such storybook; we suggest that these be available in a reading corner for students to read whenever they wish.

2. Role and Use of Worksheets, Workbooks, and Storybooks

a) Worksheets

With some lessons there are worksheets for independent work. Worksheets are designated with numbers corresponding to the lessons with which they are used and with stars indicating the order in which they are to be done by the students. The stars also suggest an increasing level of difficulty and, thereby, provide for various abilities of students. IMPORTANT: It is imperative that you do not insist on every student finishing every worksheet before going on to the next lesson. The worksheets are designed to accommodate differences in students interests and abilities. Some students will finish more worksheets than others during the allotted time. But the same ideas will emerge in new settings in later lessons, so that students will have other opportunities to work independently on the same topics.

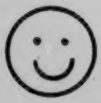
b) Workbooks

Part I of the CSMP Upper Primary Grades Program (UPG-I) includes five workbooks titled "Catalog of Problems #1, #2", "7-12", "8?-15?", and "Fishing for Numbers (Part I)". Each is a 32 page booklet that reviews problems which have been encountered up to that point in the curriculum. Two lessons are scheduled for each workbook, and we suggest that all students be directed to start at the beginning of the booklet and work independently through as many pages as they can during the two scheduled periods.

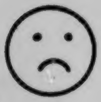
Ordinarily the first ten pages of a workbook are easy enough to guarantee some success for all the students; the next ten to twelve pages are average level difficulty; and the remaining pages are more difficult, often challenging even your best students. We would estimate that about two-thirds of your students should correctly finish the first ten pages during the two scheduled lessons, one-third should finish the first twenty pages, and a few might finish all or most of the pages. Of course, these proportions will vary from class to class.

Two full class periods are devoted to each workbook. At the end of the first period, you should collect the workbooks and check some or all the pages completed. Then when you return the workbooks at the beginning of the next workbook period, the students can correct their mistakes before going on with new problems.

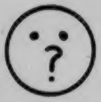
Notice that the cover page of each workbook includes spaces for marking your evaluation of the work on each page. One way to do this is to use these symbols:



meaning "well done" or "correct".

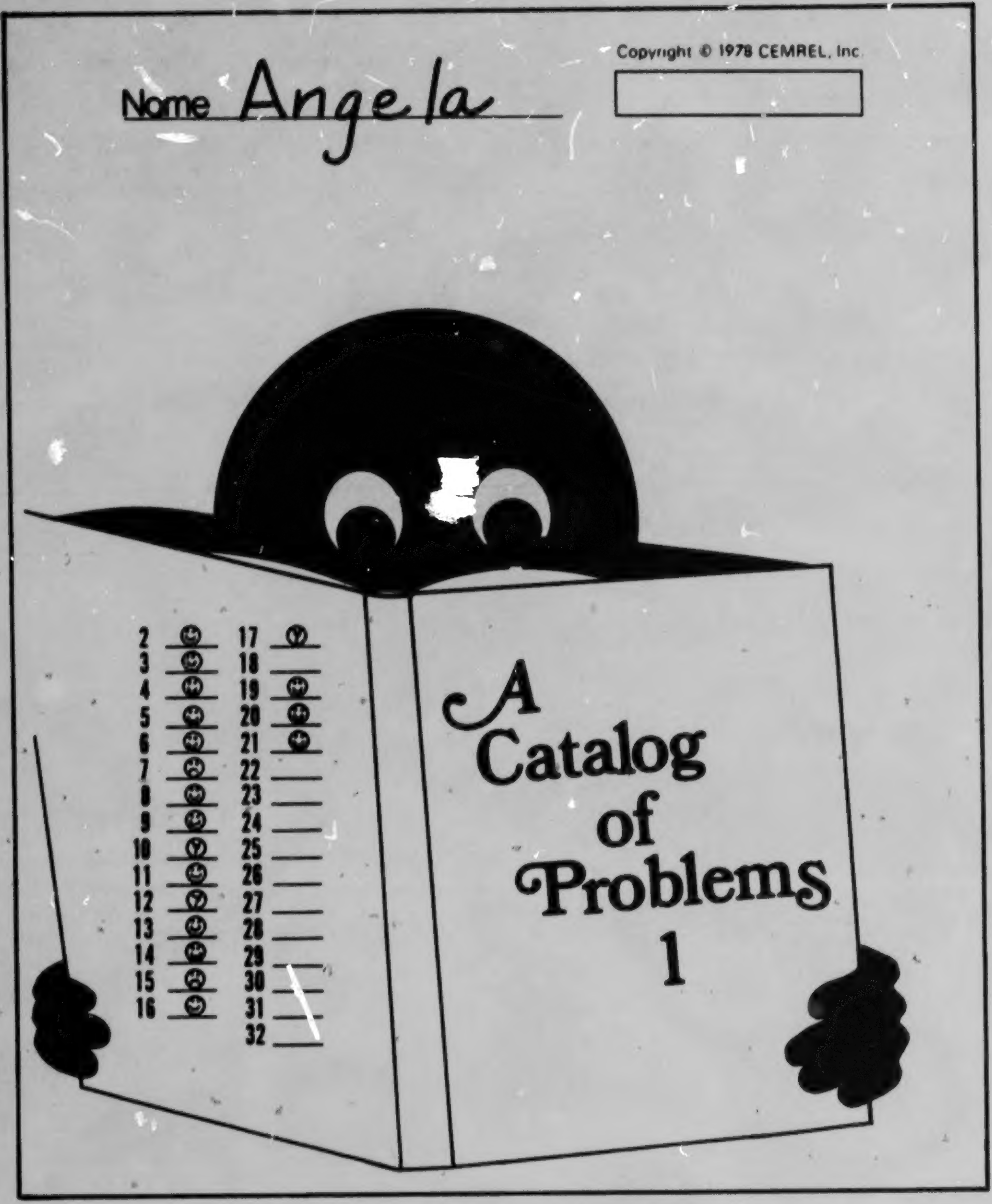


meaning "something wrong, try again"



meaning "did you forget this problem?"
or "do you need a hint?"

The next page shows the front cover of one student's A Catalog of Problems #1 workbook after it has been checked following the first lesson devoted to this workbook.



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Name Angela

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11	☺	26	
12	☮	27	
13	☺	28	
14	☺	29	
15	☹	30	
16	☺	31	
		32	

A Catalog of Problems 1

NOTE: This is the way Angela's workbook was returned to her at the beginning of the second lesson devoted to this workbook. The blanks on lines 18, 22-32 indicate that she has not yet worked on these pages. Before Angela begins on these pages during the second lesson, she should correct her mistakes on pages 7 and 15 and finish her work on pages 10, 12 and 17.

IMPORTANT: Again we ask you not to require or even encourage every student to finish every problem in a workbook. If, at the end of the two lessons allotted for a workbook, some students have not completed as much of the workbook as you expect they are capable of, you might suggest they spend some extra time on it during the next adjustment day class. It would not be useful to demand completion or "mastery" before permitting work to begin on the next workbook series.

After the second workbook period, and after you have checked the results, you might want to send the workbooks home with students. This could be one way to keep parents informed of their children's progress.

c) Storybooks

Presently, there are twenty-four storybooks for young readers in the collection STORIES BY FREDERIQUE. Each story is presented in a colorful newsprint booklet and can be used for individual or collective reading at school or at home. The stories are independent of each other and for the most part need no teacher explanations. The titles of the books in the collection and the intended age range for each story are:

Ages 5 to 8

THE PLAYFUL NUMBERS
81 ROSES
I AM A VERY HAPPY BOY
ONE OUT OF SEVEN
THE HAPPY PUPPET
TWO BY TWO
THE LITTLE DREAMER
THE LITTLE DONKEY
THE MAGIC BOX
THE BABY IS BORN
THE WEIRD STORY OF 24
THE OLD SHOEMAKER

Ages 8 to 12

WHERE'S MY NOSE?
SINGING FRIENDS
DANCING FRIENDS
THE LIVING LINES
I AM NOT MY NAME
NABU WINS AN AWARD
THE SQUARE TRAP
SUMMER SCHOOL IN THE OLD DAYS

Ages 10 to 14

THE HIDDEN TREASURE
A VERY STRANGE NEIGHBORHOOD
ELECTION IN THE NUMBER WORLD
A VALENTINE MYSTERY

Among the twelve storybooks intended for children ages 5 to 8, two are used as the basis of lessons in the UPG-I Program. Young people can become personally involved in these stories; there are questions to answer, pictures to relate to the story, information to be found in the pictures, and new adventures to invent for the characters in the stories. They find that in the world of numbers there are emotions to be expressed and aesthetic senses to be developed just as in the world of people. Your students will probably want to reread (or just look at) the storybooks many times after they have used them for lessons. Other storybooks (additional titles from the STORIES BY FREDERIQUE collection have been included in your classroom set of materials although they are not used with lessons) could be made available for individual reading during free time.

3. Role and Use of Hand-Calculators

The electronic hand-calculator, if used creatively, can have a remarkably beneficial effect on the students' interest in and understanding of arithmetic. Its use can be a contributing factor in the improvement of their capabilities in that area. The reasons for this are many and varied, but among them are certainly the following:

- By removing the burden of mechanical computation from the children's shoulders, the calculator frees them to concentrate on the thinking that is a necessary part of the solution to any meaningful problem.

- The instant feedback provided by the calculator enables the children to indulge in making and testing conjectures, an area into which they would not otherwise venture because the quantity of computation involved would be overwhelming.

This pedagogical tool is particularly useful in the introduction of new numerical notions such as negative integers, decimal numbers, multiples of a given whole number, and so on, because through suitably chosen, rich situations these concepts can be made to arise in a natural way.

Two battery powered hand-calculators are recommended for UPG-I[*] and many different kinds are available from CSMP as a distributor for National Semi-Conductor, Inc. and Texas Instruments, Inc. Any of the products of these two companies can be obtained at discount prices.

While two hand-calculators are adequate for the lessons in UPG, the ideal situation would be for each student to have one. Encourage the students to bring calculators from home. The hand-calculators that are available from CSMP have algebraic logic and function keys which are permanently in a constant mode. You should check to be sure that any calculators brought in by students also have these characteristics.

- Algebraic logic. This means that the calculator responds to instructions given in the order in which they are usually written. To check that this is the case, press $\boxed{2}$ $\boxed{+}$ $\boxed{3}$ $\boxed{=}$. If you have "5" on the display, then all is well.

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[*] A few UPG-I lessons require two hand-calculators in the class. During the second semester of the second grade (UPG-II) more lessons will require hand-calculators. If you already have calculators, be sure to check them for algebraic logic and constant mode (as explained on the following page).

- **Constant mode.** Some calculators have a switch so that the calculator can be set either in a constant mode or in a chain mode. If your calculator lacks a switch, it is quite likely that the calculator has an automatic constant. For calculators with an automatic constant, the first number entered during multiplication is usually the constant, while the second number entered during the other three operations (addition, subtraction, division) is usually the constant.

  To check your calculator for the constant mode operation of $\boxed{+}$ is a simple procedure. Press $\boxed{0}$ $\boxed{+}$ $\boxed{2}$ $\boxed{=}$ $\boxed{=}$ $\boxed{=}$ and so on. If the number on the display is increasing by two at each depression of $\boxed{=}$, then $\boxed{+}$ is operating in a constant mode. Check the constant mode operation of $\boxed{-}$ in a similar fashion.

  To check the constant mode operation of $\boxed{\times}$ (where the first number entered will be the constant) press $\boxed{2}$ $\boxed{\times}$ $\boxed{1}$ $\boxed{=}$ $\boxed{=}$ $\boxed{=}$ and so on. If the number on the display doubles at each depression of $\boxed{=}$, then $\boxed{\times}$ is operating in a constant mode.

While it would be possible to use calculators without these characteristics in the lessons, the adjustments that would be required in the instructions make the calculators unsuitable.

In several lessons, the hand-calculator is used to explore patterns in a sequence of numbers which appear on the display. For example, put 3 on the display and press $\boxed{+}$ $\boxed{5}$ $\boxed{=}$ $\boxed{=}$ $\boxed{=}$ and so on. The sequence 3, 8, 13, 18, 23, 28, ... should appear. Another example in which the positive multiples of some number are generated is to put 4 on the display and press $\boxed{+}$ $\boxed{=}$ $\boxed{=}$ $\boxed{=}$ and so on. The sequence of positive multiples of 4 should appear; that is, 4, 8, 12, 16, 20, 24, 28, ... Even if your calculators have algebraic logic, it may be necessary for you to check that they operate in the way described in the lessons. If you find that your calculators do not operate in this manner, then try replacing (for example)

$\boxed{+}\ \boxed{5}\ \boxed{=}\ \boxed{=}\ \boxed{=}$ . . . with $\boxed{+}\ \boxed{5}\ \boxed{=}\ \boxed{+}\ \boxed{5}\ \boxed{=}\ \boxed{+}\ \boxed{5}\ \boxed{=}$ . . .

and

$\boxed{4}\ \boxed{+}\ \boxed{=}\ \boxed{=}\ \boxed{=}$ . . . with $\boxed{4}\ \boxed{+}\ \boxed{4}\ \boxed{+}\ \boxed{4}\ \boxed{+}\ \boxed{4}\ \boxed{+}\ \boxed{4}$ . . .

or

$\boxed{4}\ \boxed{+}\ \boxed{4}\ \boxed{=}\ \boxed{+}\ \boxed{4}\ \boxed{=}\ \boxed{+}\ \boxed{4}\ \boxed{=}$ . . .

SUGGESTIONS FOR ORGANIZING THE PROGRAM

1. Schedule of Lessons

The design and schedule of the lessons in this guide should be regarded as suggestions; they reflect the experiences of the CSMP staff in developing the curriculum. It is difficult for curriculum developers to predict which topics will be easy for your students and which lessons will require more or less time for your particular class. Only you will be able to judge that as you teach the lessons. Thus the program is not meant to be a rigid sequence of lessons to be followed exactly, but only as a suggested guide.

The master schedule shown on pages 22 to 25, was developed as a one semester course for a hypothetical second grade class of average ability. It illustrates some important ideas about organizing the program. You will notice that several different strands are scheduled each week and that even within a strand the topics are interspersed. This was done to carry out the idea of a spiral curriculum. If you organized your own schedule, it is best to follow a similar pattern rather than to spend two or three full weeks, for example, "covering" one particular topic such as composition of functions or negative numbers. In a usual five day week, two days should be devoted to the World of Numbers strand, one day to the Languages of Strings and Arrows strand, one day to the Geometry and Measurement strand, and one day to the Workbook strand.

In the master schedule, a block represents one full period of approximately 45-50 minutes duration. It may very well be, however, that you and your class will spend more than one period or, possibly, less than one period on some lessons. Exercise your own professional judgement in deciding when to stop one particular lesson and when or whether to take it up again. Just be sure you do not insist that every single child understands every single aspect of a lesson before you go on. It is better to stop a lesson at a point when student interest is high and a problem remains unsolved than to teach so methodically that everyone in the class understands everything, but is completely bored.

Inside each block of the master schedule, the lesson of the day is designated in several ways. First, there is listed a number which indicates its suggested chronological order among the lessons in that strand. Hence, "N5" indicates that the lesson in question is suggested as the fifth lesson in the N (World of Numbers) strand. This identifying number also appears in the upper outside corner of each page in the description of that lesson, thus enabling a lesson to be located simply by flipping through the pages of the strand book. Second, you are told on which page of the strand book the description of the lesson in question begins. Third, you are given the title of the lesson.

Please bear in mind that the model schedule was designed for a hypothetical class with the usual range of abilities. In order to be successful with the program, you should adjust the schedule to the ability, maturity, and interests of your class. For example, you may wish to extend a lesson or a workbook for an additional period beyond those scheduled.

2. Adjustment Days

From time to time an adjustment day appears in the schedule. Since we have intended that UPG-I would be the mathematics curriculum for the first semester of the second grade and a usual school semester is 18 weeks, the schedule has been designed to indicate 18 weeks of lessons. However, it is also usual for there to be days when no mathematics lesson is taught; e.g., parent-teacher conference days, field trip days, snow days, and so on. These adjustment days should account for such non-class days, and should also give you opportunities to do many kinds of things. You could use an adjustment day to review a lesson you feel needs review, or to complete an unfinished lesson, or to extend some interesting ideas from completed lessons. You could also use an adjustment day to give your students time to become more familiar with the manipulative materials in the program. The students may especially enjoy playing games such as the Number Line Game or the String Game, and an adjustment day could give them another opportunity to play these games. You may wish to use an adjustment day to do some mental arithmetic exercises, or to teach a topic in mathematics you think would be especially interesting or useful to your class. For suggestions on ways to engage in mental arithmetic with your class, see the next section and Appendix B: MENTAL ARITHMETIC in the World of Numbers strand.

3. Supplementary Activities and Mental Arithmetic

Throughout the lessons there are many activities that children enjoy doing over and over in a variety of forms. For example, the String Game and the Number Line Game (to mention only two such activities) can be played whenever an extra time period is available. Of particular importance and enjoyment are the activities dealing with mental arithmetic; fully one-third of the lessons involve such activities. Other general suggestions about mental arithmetic are given in Appendix B to the World of Numbers strand. Teachers can invent clever variations of these mental arithmetic activities to be used often during a school day as adjuncts to the natural school routines. Some of the most obvious are: checking class attendance (How many present? How many should be here? How many absent?); the opening exercises (What day of the month is it? How many days in this month? How many days left?); distributing materials (How many students? Two pencils for each. How many pencils?).

Some teachers might want to do some supplementary drill work on writing numerals or on basic number facts. We would suggest that the drill be kept to a minimum so as not to suggest to children that arithmetic is a chore.

SUGGESTED SCHEDULE OF LESSONS

MASTER SUGGESTED SCHEDULE OF LESSONS FOR

WEEK	The World of Numbers	The Languages of Strings and Arrows
1	N1 Minicomputer Introduction #1 N p. 1	L1 Sending Letters L p. 1
2	N3 Minicomputer Introduction #2 N p. 27	L2 Where's My Nose? L p. 6
3	N5 Minicomputer Introduction #3 N p. 42	ADJUSTMENT DAY
4	N7 Addition Problems #1 N p. 61	L3 Set Membership L p. 14
5	N9 Addition Problems #2 N p. 72	L4 Introduction to A - Blocks L p. 23
6	N11 Multiplication and Subtraction #1 N p. 82	L5 Exchanging Names L p. 33
7	N13 Multiplication and Subtraction #2 N p. 91	L6 Detective Story with Coins #1 L p. 39
8	N15 Addition Problems #3 N p. 105	L7 String Pictures with A - Blocks L p. 46
9	N17 Addition Problems #4 N p. 116	L8 Messages L p. 51

CSMP MATHEMATICS FOR THE UPPER PRIMARY GRADES, PART I

The World of Numbers	Workbooks	Geometry and Measurement
N2 The Functions +3 and -3 N p. 21	W1 The Number Line Game W p. 1	ADJUSTMENT DAY
N4 The Functions +5 and -5 N p. 37	W2 Detective Story #1/ Introduction to Eli's Magic Peanuts W p. 12	G1 Introduction to Area G p. 1
N6 Twice As Many N p. 53	W3 7-12 (Lesson 1) W p. 22	G2 Taxi-Geometry #1 G p. 7
N8 The Functions +5, -5, +2, -2 N p. 66	W4 7-12 (Lesson 2) W p. 26	G3 Booker's Bakery #1 G p. 12
N10 Composition of Functions #1 N p. 77	W5 Eli the Elephant/ Detective Story #2 W p. 39	ADJUSTMENT DAY
N12 Introduction to Frames N p. 86	W6 A Catalog of Problems #1 (Lesson 1) W p. 49	G4 Measuring with Shoes and Pencils G p. 19
N14 Roads #1 N p. 100	W7 A Catalog of Problems #1 (Lesson 2) W p. 50	G5 Area with Tangram Triangles and Squares G p. 24
N16 The Function 2x N p. 110	W8 Detective Story #3/ Eli the Elephant Stories W p. 63	G6 Taxi-Geometry #2* G p. 31
N18 Becoming Acquainted With 37 N p. 126	W9 8? - 15? (Lesson 1) W p. 68	ADJUSTMENT DAY

* One exercise in this lesson relates to Halloween. If possible, schedule the lesson near the end of October.

MASTER SUGGESTED SCHEDULE OF LESSONS FOR

WEEK	The World of Numbers	The Languages of Strings and Arrows
10	N19 Multiplication and Subtraction #3 N p. 133	L9 String Game with A-Blocks #1 L p. 57
11	N21 Multiplication and Subtraction #4 N p. 143	L10 Composition #1 L p. 62
12	N23 Addition Problems #5 N p. 157	L11 Detective Story with Coins #2 L p. 73
13	N25 Addition Problems #6 N p. 170	L12 String Game with A-Blocks #2 L p. 82
14	N27 Multiplication and Subtraction #5 N p. 186	L13 String Pictures About Our School L p. 86
15	N29 Multiplication and Subtraction #6 N p. 201	L14 Changing Seats L p. 92
16	N31 Addition Algorithm #1 N p. 212	L15 Project: Making A Temperature Bar Graph L p. 101
17	N33 Addition Algorithm #2 N p. 240	L16 Composition #2 L p. 104
18	N35 Multiplying by Ten N p. 254	L17 The Happy Puppet L p. 110

The World of Numbers	Workbooks	Geometry and Measurement
N20 Frames of the Same Shape N p. 138	W10 8? - 15? (Lesson 2) W p. 71	G7 Millions of Dots/ Tangram Puzzles G p. 37
N22 Composition of Functions #2 N p. 150	W11 Detective Story #4/ Eli's Peanut Collection W p. 81	G8 Measuring with Centimeter Rods G p. 49
N24 Adding and Subtracting Three N p. 163	W12 A Catalog of Problems #2 (Lesson 1) W p. 89	G9 Comparison of Area G p. 50
N26 The Functions 2 x and 1/2 x N p. 179	W13 A Catalog of Problems #2 (Lesson 2) W p. 90	ADJUSTMENT DAY
N28 The Functions* +10 and -10 N p. 193	W14 Detective Story #5/ Eli Visits His Grandmother W p. 103	G10 Measuring with a Tape Measure G p. 60
N30 Roads #2 N p. 207	W15 Fishing for Numbers (Lesson 1) W p. 112	G11 Taxi-Geometry #3 G p. 66
N32 Patterns Generated by the Functions +5 and -5 N p. 232	W16 Fishing for Numbers (Lesson 2) W p. 114	G12 Higgledy Piggledy Dots G p. 72
N34 Less Than - More Than N p. 250	W17 Detective Story #6/ Eli Tricked by Clarence the Crocodile W p. 125	ADJUSTMENT DAY
N36 Composition of Functions #3 N p. 261	ADJUSTMENT DAY	G13 Booker's Bakery #2 G p. 80

* The worksheet for this lesson has a Christmas theme. If possible, schedule the lesson to be taught before but close to the Christmas holiday.

SECTION TWO

QUESTIONS AND ANSWERS ABOUT CSMP

As you teach the CSMP curriculum or even as you prepare to teach it, there will be some basic questions that come to your mind. Very likely they are the same kinds of questions that parents and supervisors will ask about a curriculum that appears to be different from the traditional program. Perhaps some of the questions that concern you are answered here.

1. Q: WHAT IS THE SPIRAL APPROACH? HOW DOES IT WORK?

A: We learn through many interrelated experiences. But no experience, particularly a second grade math lesson, is an end in itself. With this in mind, CSMP introduces an idea in one lesson, then another in the next lesson, then another. Later the first idea reappears in a slightly new setting or in a slightly expanded version; then a second topic reappears; and so on. Each of the many times that a particular idea or area of study reappears, it is treated in a different manner and at a slightly higher level than the last time. This is called a spiral development.

Children learn at different times and at different rates. Only part of the learning of a given topic actually takes place during a lesson on the topic. In between the times when the topic occurs on the learning "spiral", the child is mentally digesting the idea, sometimes consciously, usually unconsciously. In this way the spiral development gives each student a new chance to "catch on" at each stage.

We find it highly effective to follow this kind of development rather than continuing on and on with a topic until so called "mastery" has occurred.

Sticking with the spiral idea requires an act of faith on your part, particularly if you have been used to teaching a given topic until you are satisfied that all (or at least most) of the class knows it "cold". Belief in the spiral approach implies that some lessons may be stopped before some (or even a majority of) children appear to have caught on, or in a lesson involving worksheets or workbooks, before everyone has successfully completed all the pages. It involves knowing that the topic will reappear again and again, and that varying degrees of understanding will come at varying rates and times. It entails a belief that an idea planted now may not sprout until much later, and that it is best not to force its development. All our experience to date (including five years of extended pilot testing of the curriculum) indicates that this approach really works.

2.Q: I UNDERSTAND HOW THE SPIRAL DEVELOPMENT WORKS, BUT HOW CAN I ASSESS MY STUDENTS' PROGRESS UNDER SUCH AN APPROACH?

A: We don't believe it is desirable or reasonable to try to evaluate student mastery of skills from day to day or even from week to week. On the other hand, teachers, parents, and the children themselves benefit from some monitoring of their progress and their involvement in the activities. Most teachers have their own methods for doing this with which they are comfortable, and we suggest that you continue to use your own monitoring methods. You will find that group interaction in the lessons and the individual worksheets and workbooks in the materials provide some valuable information. The spiral development in fact may help rather than hinder your long term assessment of students' progress.

3.Q: FOR WHAT SORT OF STUDENT IS THE CSMP ELEMENTARY CURRICULUM DESIGNED?

A: It is designed for all children in the schools. Each learning situation is designed to involve all students at their own levels in such a way that the slowest can succeed in dealing with some aspects of the situation that challenge them. In fact, it has been found that such a "pedagogy of situations" (see Question 4) is particularly effective with the so-called "slow" learners, and recent work suggests it might be effective also with educable mentally handicapped children.

4. Q: BRIEFLY, HOW DOES CSMP DIFFER FROM A TRADITIONAL MATHEMATICS CURRICULUM?

A: The core content of CSMP is not revolutionary. The program aims to inculcate ideas and facts that have always been considered important in mathematics. The major differences are in method. Traditional teaching takes pride in disciplining students to be submitted to the usual paper and pencil algorithms. CSMP students are involved as well in many other real mathematical activities, with understanding at their level. For that purpose, CSMP employs new teaching aids such as non-verbal languages (see Question 7), which are themselves part of mathematics.

A pedagogy of situations is used to permit children to participate in the learning process. The students are faced with situations and are led by their reactions to important ideas in mathematics. Thereby, students begin to mathematize situations, a process which is of fundamental importance whenever mathematics is applied.

CSMP also makes use of a spiral approach rather than the mastery approach found in most mathematics curricula. (See Question 1)

5. Q: ISN'T THIS THE "NEW MATH"?

A: There is a current popular belief that all school mathematics today is either "new math" or "old math". Neither of these labels is well defined. "New math" usually denotes the programs that heavily emphasize the teaching of sets; introduce technical mathematical language for concepts such as "commutativity", associativity", distributivity", and so on; and that teach "concepts" before "skills". The "old math" refers to programs of the "good old days": memorization of the multiplication tables and complete emphasis on paper and pencil drill in calculations.

The CSMP curriculum is definitely not the "new math" nor the "old math" by any definition. It uses no technical language, yet achieves the understandings sought by the "new math". It uses no repetitive drills, yet elicits the ability to calculate stressed by the "old math". But these achievements are just by-products of a more pervasive goal: to help children use their curiosity, their intuition, their powers of reasoning to formulate and think about interesting problems and situations.

6. Q: ARE ALL THE TOPICS IN THE CSMP CURRICULUM REALLY MATHEMATICS?

A: Yes. Mathematics is much more than calculation with numbers, and its methods include more than paper and pencil algorithms. The CSMP curriculum is filled with experiences that directly and indirectly relate to the wide world of numbers, their interrelationships and the kinds of logical reasoning about these relationships that make mathematics an activity of interest and concern.

7. Q: WHY USE THE LANGUAGES OF STRINGS, ARROWS, AND THE PAPY MINICOMPUTER?

A: It is surprising that the language of strings was not used earlier and more extensively in the teaching of mathematics. Mathematicians have used strings for a very long time as they support classification notions so fundamental and natural that they go back to Aristotle. The scope of the string diagrams is broader than mathematics, the string diagram being a marvelous instrument for logical thinking. String diagrams permit the understanding of a situation at a glance; they generate math concepts and ideas through child-related discussion.

Arrow diagrams are probably the most important teaching aid in CSMP. They are used everywhere in our daily lives; signals, advertisements, directions, and so on. Why not use them in teaching mathematics? Arrows are a pictorial way to represent relations. Relations have always been important in mathematics, but in the past seemed to be something rather abstract, elusive, and mysterious. With arrows they become vivid, concrete, and easy to understand.

The Minicomputer allows very young children to become familiar with numbers and operations on them (especially large numbers and the more difficult calculations) before they are able to work with them on paper. Later, and more importantly, the Minicomputer is used as a vehicle for posing situations involving mental arithmetic and for examining the workings of the arithmetic operations. It provides the experience necessary so that, rather than memorizing an algorithm and applying it to situations, the situations give rise to the algorithm.

8. Q: IF THE STUDENTS USE THE MINICOMPUTER, WILL THEY BE ABLE TO CALCULATE WITHOUT IT?

A: This question suggests that the only purpose of the Minicomputer in the CSMP curriculum is to perform calculations. To be sure, the Minicomputer does enable very young children to deal with "large" numbers and perform relatively difficult calculations long before they know the paper and pencil algorithms for the calculations. But very soon these experiences with the Minicomputer suggest how the paper and pencil algorithms come about, and from that moment the Minicomputer is no longer regarded as a calculating device. Later, for the remainder of the curriculum, the Minicomputer assumes its primary roles: as a source of very interesting problems and games, as an effective vehicle for stimulating mental arithmetic, as a teaching device for introducing more advanced topics such as the negative numbers and the decimal numbers, and, in general, as an alternative to the usual representation of numbers.

9.Q: WHAT IS THE CSMP POSITION ON HAND-CALCULATORS AND WHAT ROLE DO THEY PLAY IN THE PROGRAM?

A: Hand-calculators should be a part of any mathematics curriculum. As more and more children have hand-calculators, these tools will become incorporated in the elementary and secondary school curricula in a natural way with uses very similar to those of the Minicomputer. In some cases, they may be used as a tool for mechanical computation where the difficulty of the calculation might turn an otherwise interesting problem into an exercise in drudgery. In other cases, they may be used as the context for posing situations leading to the discovery of important mathematical properties or providing opportunities for interesting exercises in mental arithmetic.

Hand-calculators are of particular benefit to "slow" learners. Many slow learners have mechanical difficulties rather than problems in thinking about the situation. The hand-calculator alleviates this problem.

Some lessons in the CSMP curriculum require only two calculators, while some lessons suggest that each student have one. Most often, however, lessons of this latter sort also include suggestions on how they may be modified since not every student may have a hand-calculator. In either situation, students with access to hand-calculators should always be encouraged to bring them to class.

10: Q: ARE THERE GOALS FOR EACH LESSON?

A: At the beginning of each lesson plan there is a CAPSULE LESSON SUMMARY, which describes briefly the activities and experiences included in the lesson. Indirectly these summaries suggest goals but, of course, the goals of the curriculum spread over whole series of lessons and cannot be broken into specific goals for individual lessons. If you read the introduction to the strand in which a lesson appears as well as the Capsule Lesson Summary for that lesson, we expect you will easily see the direction and intent of the lesson. We decline to predict for any one lesson what will happen in terms of student learning.

11.Q: SHOULD I SUPPLEMENT THE PROGRAM WITH MORE PRACTICE IN NUMBER FACTS?

A: There is a surprising amount of such practice throughout the program. It is implicit in the activities and is not in the form of practice for its own sake. If you still feel it is necessary to give supplementary practice, we suggest that you keep repetitive drill to a minimum so as not to endanger the students' interest in numerical situations.

12.Q: WHY IS THERE A LACK OF DRILL AND LITTLE EMPHASIS ON "SKILLS"?

A: CSMP believes that training for rote skill should never precede understanding. The avowed purpose of drill in arithmetic is to acquire the "skills", that is, the skills of calculation. Certain numerical skills are essential but, unfortunately for school children, the usual drill techniques for acquiring them are dull, stultifying and sometimes even counterproductive. The CSMP curriculum appears to place little emphasis on "skills" because it does not resort to drill. But as a matter of fact, the essential skills of both mental and written calculation are learned almost inadvertantly in response to the many situations where the ability to calculate is needed in order to respond to the situation: to win the number game, to identify the secret number of the detective story, or to build a number road. Calculation is always considered as a tool for doing interesting things, rather than a chore done for its own sake.

13.Q: HOW DO STUDENTS LEARN TO USE MATHEMATICS IN PRACTICAL SITUATIONS?

A: For students of elementary school age, a situation that intrigues them and catches their imagination or their curiosity will certainly motivate them to learn whatever is necessary in order to deal with the situation. But situations that adults consider "practical" very often are outside the realm of interest of school children and do not provide motivation for learning. CSMP has found that students will indeed learn to use mathematics in situations they consider worthy of their attention. As they approach adulthood they will soon consider "practical" situations to be most worthy of their attention, but in the meanwhile, they are more interested in games, fantasy, puzzles, stories, and so on.

14.Q: HOW ARE TELLING TIME, MONEY, AND THE METRIC SYSTEM TREATED?

A: Telling time is not included in the CSMP program. It is our belief that telling time is more naturally a part of a language program than of a mathematics program, although it is frequently included in the mathematics curriculum. Since there are many excellent materials available to help you teach telling time, we urge you to teach this topic using those methods and materials you particularly like or that have proved successful to you in the past.

The topic of money is first introduced in the CSMP curriculum in situations involving combinations of coins. It is treated again in the Upper Primary Grades and the Intermediate Grades as a support for the teaching of the decimal numbers. Although money is not treated as a separate topic, these situations provide experience with our monetary system. Should the students need more exposure to this topic, teachers should feel free to develop supplementary materials in order to provide extra practice.

Metric units are used whenever there is cause to measure in standard units. Activities involving the metric system are begun in the First

Grade Program and continue throughout the Geometry and Measurement strand of the Upper Primary Grades Program. We recommend that you do not ask the students to convert from the English to the metric system or vice versa (inches to centimeters, yards to meters, Celsius to Fahrenheit, grams to ounces, and so on). It would be confusing for children to deal with two systems simultaneously, especially at this early stage. Since the United States has not yet "gone metric", your students will still need to be familiar with the English system. We would suggest that you use whatever materials and methods you have found successful in the past for teaching this topic, but we repeat our recommendation that you not ask the students to convert from one system to the other.

15.Q: WHAT IS THE ROLE OF STORYBOOKS IN THE CURRICULUM?

A: The storybooks in the collection STORIES BY FREDERIQUE include 24 stories for young children, presented in colorful newsprint booklets. Two stories from this collection are used for lessons in UPG-I, with 15 copies of each storybook provided in a classroom set of materials. (If your school can afford it, we recommend that you order enough extra copies to provide one for each child.) Seven other storybooks, two copies of each, are provided for supplementary reading. Each of the stories is portrayed by the mathematical language of dots, arrows, and strings which reinforce the ideas of the stories. Young readers can become personally involved in the stories; there are questions to answer, pictures to relate to the story, and new adventures to invent for the characters of the stories. They find that emotions and aesthetic senses are involved in the world of mathematics just as in the world of people.

16.Q: WHAT PROVISION IS MADE FOR INDIVIDUALIZATION IN THE CSMP PROGRAM?

A: In many of the lessons there are worksheets to be done individually by the students. Sometimes the worksheets are done at the beginning of a lesson, followed by a full group discussion of them. But ordinarily a topic is first developed in a full group setting, followed by individual student work on related worksheets. If "individualization" means a procedure for allowing students to work alone at their individual rates and levels, then the worksheets (and also the workbooks, discussed below) do provide for (partial) individualization. A series of worksheets for a given lesson offers a whole spectrum of levels from very simple to quite challenging. Students all start with the first worksheet (one-star) and proceed at their own pace and levels as far as they are capable. The spiral approach accommodates this kind of individual work in the same manner as the full group work; that is, no worksheet is a prerequisite for the content of any succeeding lesson.

Ten lessons during the first semester of the second grade are devoted to the five UPG-I workbooks. Each workbook is also designed to move through a spectrum of levels of difficulty. We believe that the worksheets and workbooks realize the best sense of "individualization".

17.Q: WHAT PROVISIONS ARE MADE FOR THE VARIETY OF ABILITIES IN A USUAL CLASS?

A: Every lesson includes something for all students; there are opportunities for the slowest students to participate in a meaningful way, and there are challenges for even the very brightest of students. This means that you should not exclude any students from a full group lesson. At the same time, you should not expect every student to understand everything during the course of the lesson. Furthermore, with the spiral development many situations are open-ended and allow students to experience them at levels as high as they are capable.

Evaluation has shown that students traditionally labeled "slow" are doing better in CSMP classes than in non-CSMP classes. We believe this is due to the spiral development, to full group work on mathematical activities, and to the presence in the CSMP curriculum of certain nonnumerical activities in which many supposedly "slow" students can excel.

Additionally, CSMP provides for a variety of ability levels in the student materials. Worksheets and workbooks offer problems in a range from very easy to quite challenging. Indeed, some of the "higher starred" worksheets and pages toward the ends of workbooks may be accessible only to your very best students.

In an extreme case, you might seriously consider the possibility of arranging for exceptionally bright students to take their math with a higher grade level CSMP class.

18.Q: I PREFER TEACHING SMALL GROUPS, BUT MOST OF THE LESSONS SEEM TO BE AIMED AT THE WHOLE GROUP. WHAT DO YOU SUGGEST?

A: If your school routine allows you to teach each lesson twice, once to each half of your class, that would be an alternative way to proceed. But if you do this, it might be wise to vary the composition of the groups from day to day so that you are not tempted to group your students according to your perception of their ability. Otherwise you would guarantee that half your students would not have as rich an experience as the other half, through no fault of their own. The purpose of the full group experiences is to allow interaction among all the children in every situation posed in the lessons. Students learn quite a bit from each others' reactions. You will be surprised to find that in some situations the key responses are made by students you had labeled "slow". We feel the spiral approach has maximum potential in full group teaching.

The above response might give the impression that CSMP teaching takes place only in full group settings. This is not the case. There is much time devoted to individual student work on worksheets and workbooks, and there are various games to be played. We also encourage you to arrange small group sessions whenever you have time, particularly with your slower students. Small group activities could supplement the full group lessons, but should not replace them. Extra small group work should not be simply for drill and rehashing ideas, but rather should provide additional experiences.

19. Q: WHY IS A "DIALOGUE" STYLE USED AS A FORMAT FOR THE LESSON PLANS? ARE TEACHERS EXPECTED TO FOLLOW THIS DIALOGUE EXACTLY?

A: Most of the lessons are written in considerable detail; step by step descriptions are frequently given, together with suggestions for the exact words a teacher might use (in the form of a possible dialogue between teacher and students). These details are given only for the teachers who feel the need for them. You should never feel obligated to follow any lesson word for word as it is written in the guide. By the same token, you should never insist that your students give the same responses as those given in the imaginary dialogues. Use your own creativity in drawing out student responses as the situations suggest. Read the lesson plans very carefully beforehand to get an idea how the lessons are expected to proceed.

20. Q: IS IT NECESSARY TO FOLLOW THE SUGGESTED SCHEDULE EXACTLY?

A: This program is not meant to be a static sequence of exercises to be followed rigidly. The lessons in this guide are only suggestions based on our own experience teaching the materials. If this will be the first time you are teaching the program and you would like specific suggestions for organizing the lessons, we offer some guidelines and a suggested schedule of lessons. You may wish to follow it at the beginning of the school year and then deviate from it as the needs of your students may dictate. It is probably best to follow a one-day, one-strand pattern when you organize your own schedule, rather than to spend two or three full weeks "covering" one particular topic. In a usual five day week, two days should be devoted to the World of Numbers strand, one day to the Languages of Strings and Arrows strand, one day to the Geometry and Measurement strand, and one day to the Workbook strand.

Exercise your own professional judgment in deciding when to stop a particular lesson. It is better to stop a lesson when student interest is high than to teach so methodically that everyone in the class understands everything, but is completely bored. In order to be successful with this curriculum, you should adjust the schedule to the ability, maturity, and interests of your class.

21. Q: HOW DO I HANDLE STUDENTS WHO HAVE AN EXTENDED ABSENCE OR STUDENTS (NEW TO CSMP) WHO TRANSFER INTO MY CLASS DURING THE YEAR?

A: The spiral approach allows a student to miss one or two weeks of school without much "catch up" work needed. When the student returns, either you or classmates can help with any material that is deemed necessary.

If several months of classes have been missed, you might need to treat the student as you would a transfer student new to CSMP. Give the student a little extra attention during a lesson especially when the questions are review for most of the class.

In most cases, the differences between new and veteran students tend to fade rather quickly. One exception might be in lessons involving the Minicomputer. Veteran students who are unusually adept with the Minicomputer very often are good tutors for the transfer students in the use of this abacus.

It is probably best not to send worksheets and/or workbooks home to a child; without participating in the collective lessons that introduce the worksheets and workbooks, it would be very difficult for the student to proceed.

22. Q: HOW DO I HANDLE WHOLE CLASSES WHO ARE NEW TO CSMP?

A: CSMP does not assume that Kindergarten and first grade students have any prior experience with CSMP. The Kindergarten and First Grade Programs will need no adaptation for classes who are new to CSMP.

Second grade classes that are new to CSMP will initially progress more slowly than veteran classes. The lessons at the beginning of the second grade program are designed to serve either an an introduction to CSMP languages for new classes or as a quick review for veteran classes.

Third grade classes which are new to CSMP will need to become fluent in the CSMP languages before beginning the lessons in UPG Part III. A special schedule has been prepared for these classes which consists of prerequisite lessons and workbooks from UPG Parts I and II followed by a modified UPG Parts III and IV program.

23. Q: WHAT SHOULD I DO WITH MY CLASS IF THEY DID NOT FINISH THE PREVIOUS YEAR'S SCHEDULE?

A: No special measures are required. Each level of the CSMP curriculum begins with introductory lessons. In addition, the spiral approach in the lessons permits students to participate at their own level of understanding, even if that level is not quite as high as it might have been if the previous year's schedule had been completed.

24. Q: HOW DO CSMP STUDENTS DO ON STANDARDIZED TESTS?

A: While CSMP disagrees with the philosophy underlying standardized testing, test results are available to us from several sources. The results indicate that CSMP classes generally do as well or better on standardized tests than do non-CSMP classes. It should be pointed out that standardized testing and traditional programs are correlated, and that CSMP students have been exposed to a great deal of mathematics that is not taught in more traditional programs or that is not considered in the construction of the tests. There are also many unmeasurable or unmeasured areas such as enjoyment of mathematics, strategic thinking, organizational ability, and depth of understanding of the methods and uses of mathematics where we feel CSMP students have a definite advantage. Evaluation Report 2-A-3 in the CSMP Evaluation Report Series gives more detailed answers to this question. Copies are available upon request.

25. Q: WHAT PROBLEMS WOULD CSMP STUDENTS HAVE IF THEY RETURN TO A MORE TRADITIONAL PROGRAM IN THE NEXT SCHOOL YEAR?

A: There should be no problems for CSMP students who return to a traditional program. Test results indicate that CSMP students are at or above the level of non-CSMP students in material covered in traditional courses. In addition, CSMP students have encountered many situations that are not introduced in traditional programs nor tested for on standardized tests (see Question 24). In particular, their reasoning ability should be much sharper than their non-CSMP counterparts.

There are, of course, differences in timing as to when different programs introduce the standard algorithms and often CSMP's introduction will occur later than in some of the other programs. The reason for this is our belief that it is essential that the creative freedom of the student not be impaired by a premature emphasis on the standard algorithms for adding, subtracting, multiplying, and dividing. We believe that many of the present computational difficulties experienced by elementary and junior high school students were caused by their spending an inordinate amount of time trying to master the algorithms too early in their mathematical education. This shuts the students off from other experiences necessary for their mathematical growth. Students need plenty of time to explore the world of numbers. Mastering an algorithm should be the last step of this exploration, a sort of synthesis of previous experience. Furthermore, as electronic hand-calculators become available to nearly everyone, algorithmic mastery as a prerequisite to other numerical experiences should become less important.

CSMP students can usually solve a problem before having learned the standard algorithm. If they enter a program that expects an algorithm they have not learned, they are still more than adequately prepared for the algorithm and should have no trouble quickly learning to use it.

CSMP introduces the usual addition algorithm in the second grade, one of the two standard subtraction algorithms in the third grade, the multiplication algorithm in the fourth grade, and the division algorithm in the fifth grade.

26. Q: WHAT ARE SOME WAYS TO ORGANIZE MATERIALS AND MANAGE MY CLASS?

A: It is very important to organize your CSMP materials before school begins. Decide where you will store the materials once they are organized. Filing drawers, shelves, boxes, or closets are all satisfactory as long as you can easily get to the materials. Take them all out of their shipping boxes and spread them on several tables and/or desk tops in your classroom so they can be organized into the order you will be using them.

Workbooks: The teacher's guide is needed to find the order in which the workbooks are used. Locate the Workbook strand column in the Master Suggested Schedule of Lessons and collect all the workbooks in the order in which they are listed. After organizing the workbooks in order, place them in an upright position in your file drawer, box, shelf, etc., so that the first workbook to be used is at the front of the stack.

Worksheets: There is a separate worksheet booklet for each student and you should decide whether you will store these booklets and pass them out when a particular lesson calls for a worksheet, or simply let the students keep their own worksheet booklets in their desk.

Each worksheet can be removed from the booklet or left in depending on how you wish to manage worksheets. However, once a worksheet is removed from the booklet it cannot be replaced, so make sure students are careful about tearing off the correct sheet.

Manipulatives: Storage of each manipulative in a separate container, properly marked, is one way to make everything accessible. Another way is to provide a sturdy box (for example, a shoe box) for each student and to put individual manipulative materials in the box, properly labeled with the student's name.

Additional Materials: Be sure that you have plenty of high quality colored chalk, large sheets of unlined white paper, and that your students have colored pencils, pens, or crayons. Plan well ahead to be sure you have any needed materials that are not supplied by CSMP. Keep all teacher materials where they are easily accessible. You may find the guides easier to use if they are put in looseleaf binders; the books are perferated so that pages can be removed easily. The demonstration Minicomputer boards, the number line, and the 0-109 numeral chart should always be on display in the room.

27. Q: SOME OF THE CSMP MATERIAL IS NEW TO ME. HOW DO I LEARN ENOUGH TO TEACH A CLASS?

A: Every CSMP teacher should have been trained by his or her coordinator. Kindergarten and first grade teachers receive a minimum of 18 hours training, teachers of the Upper Primary Grades Program receive up to 32 hours training, and teachers of the Intermediate Grades Program receive up to 56 hours training. During the training sessions the CSMP content is taught to new teachers and suggestions are given for organization of their classes.

If the suggested training schedule has been followed, many lessons taken from the CSMP curriculum, or very similar lessons, have been presented. The training schedule also makes use of the spiral approach so that training sessions aim not only to teach CSMP mathematics but to familiarize teachers with the spirit and pedagogy of the CSMP curriculum.

28. Q: WHAT SHOULD BE DONE WHEN THERE IS A SUBSTITUTE TEACHER?

A: Some school systems have included people who frequently substitute in their CSMP training program. When this is not the case, it would be very difficult for the untrained substitute to teach from CSMP materials. One solution is to arrange for another CSMP teacher to teach math to your class while the substitute teacher teaches another subject to that teacher's class. If that can't be arranged and you will only miss one or two days, you may find certain CSMP material, uncompleted workbooks for example, that the substitute can manage. Otherwise, use those days to cover material not included in the CSMP curriculum, such as telling time. If your absence will be an extended one, arrangements should be made with your coordinator to train a substitute, rearrange schedules to put a CSMP teacher in your class, or otherwise provide for your students.

29. Q: THE PROGRAM MAKES USE OF THE STUDENT'S CREATIVE FREEDOM. HOW MUCH ROOM IS THERE FOR THE TEACHER'S INGENUITY?

A: CSMP not only allows for your ingenuity but depends on it. While the lessons may appear to be very rigid by presenting dialogue for teacher and student, it should be remembered that these are suggested questions, answers, and comments based on our experience with development classes. Even if you were to follow your part of the "script" your students are likely to offer responses which differ from those we have listed. It is the student's response that will determine the content and direction of your next question or comment.

In a traditional program your creativity is called upon the first time you develop a particular lesson, and to a lesser extent, as you revise lessons in succeeding years. There is little room for spur-of-the-moment creativity once a lesson begins. The CSMP curriculum, by creating a free and open situation emphasizing the student's creative freedom, continually presents you with unanticipated student response and calls on your ingenuity at all times.

30. Q: HOW CAN I EXPLAIN CSMP TO PARENTS?

A: The best way would be for your coordinator to organize a workshop for interested parents. The parents can then see the languages of the program in use and have some of the same experiences their children have had. Workshops can be of almost any length but anything short of two hours does not allow for much familiarity with the material. In such a workshop we would suggest that the language of the Minicomputer be explained with particular emphasis on its use in presenting situations where mental arithmetic is called upon and in the development of algorithms (for example, physically carrying a checker from the ones' to the tens' board in the "8 + 2 = 10" play, paralleling "carrying" in the paper and pencil operation). A lesson using the storybook "Where's My Nose?" provides a good example of the power of the language of strings. A nonnumerical example, such as the brother-sister graph and a numerical example, such as building a road from 2 to 23 using +5 and -2 arrows, will provide an introduction to the language of arrows. The lessons that are the favorites of the children would be excellent for a presentation to parents as well.

You might also extend an invitation to parents to visit your class during the math period. Many times such an invitation leads eventually to a closer parent-teacher relationship, with the parent offering voluntary services as an aide.

You will find a letter to parents at the end of this section which may be reproduced and sent home with the students.

TO THE PARENTS OF CSMP STUDENTS

Your child is participating in the Comprehensive School Mathematics Program's (CSMP) elementary school curriculum, developed under a contract with the National Institute of Education. While the program cannot be fully explained in a brief note, we hope that this letter will give you some feeling for the substance of your child's mathematics education.

The goals of the CSMP K - 6 curriculum include a development of the standard number systems and their operations (including an early introduction to the integers, rational numbers and decimal numbers) interwoven with a development of geometry and measurement. On an equal footing with the above, the CSMP curriculum includes a strong component of probability, statistics, and combinatorics which extends throughout grades 2-6. Powerful pedagogical devices are used, such as several nonverbal languages. These make the unifying ideas of the program accessible to very young children.

These languages include:

1) The Language of Strings This "language" mimics the basic methods of collecting and classifying data applying to the problems at hand. That is, it deals with the fundamentally useful and important mathematical notion of set.

2) The Language of Arrows This graphical language models the process of comparing and analyzing sets and operations on them. That is, it deals with the mathematical notions of relations and functions including, but not restricted to, ordering relations and the numerical processes of adding to, subtracting from, multiplying by, and sharing equally among.

3) The Language of the Papy Minicomputer The Papy Minicomputer, an extremely effective paper abacus invented by Georges Papy, models the positional structure of our system of numeration and lends itself to a multitude of numerical algorithms, including the standard algorithms for the basic numerical operations.

Among other things, these languages allow for the development of the basic operations on numbers in a natural way based on previous experiences with the languages. They also provide a context in which situations arise that call for the repeated use of arithmetic operations without resorting to drill exercises, which most children find boring.

The content is learned in an atmosphere of constant applications utilizing a "pedagogy of situations".[*] These vary from simple story situations in the early primary grades to challenging applications and nontrivial simulations of real world problems in the upper grades.

One of the tenets of CSMP's development philosophy is that mathematics is a unified whole and should be learned as such. As a result, along with CSMP's concern with the variation in ability levels in the typical heterogeneous class, the curriculum is structured in a spiral format throughout the K-6 program. The content is completely sequenced in spiral form so that each student is brought into contact with each area of content continuously throughout the program. This consciously avoids atomizing the content and mastering each bit before continuing to the next. Rather, each student spirals through repeated exposures to the content, building interlocking experiences of increasing sophistication.

Another tenet of CSMP's development philosophy, based on years of experimentation in the classroom, is that no single method of classroom management suffices to meet the needs of every student. Hence, the program is constructed to allow numerous opportunities for whole class participation, small group participation, and individual independent experiences. Individualization throughout the K-6 program is achieved through the independent work on workbooks and worksheets of varying levels of difficulty. Due to the spiral nature of the curriculum, each student can engage in a workbook without losing touch with the progress of the class while at the same time working to the limits of his or her ability.

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[*] Inspired by the work of the mathematician and psychologist, Caleb Gattegno.

A feature unique to the CSMP program is a collection of storybooks written by Frederique Papy (STORIES BY FREDERIQUE) and a collection of story-workbooks (THE CSMP LIBRARY). These probe the affective domain of learning and bring children into contact with mathematics at levels until now unexplored.

CSMP does not have its own system of student assessment and progress reporting. While standardized testing indicates that CSMP students develop the "basic skills" as early or earlier than do non-CSMP students, we hope you will not be obsessed with your child's development of these "skills". Each child is an individual and should not be judged against norms but on what he or she can and does accomplish. Many extremely important areas -- development of self-confidence, ability to think and reason, etc. -- are very difficult to measure and grade. If you have any questions regarding the progress of your child, his or her teacher is in the best position to answer those questions.

THE CSMP STAFF

THE
WORLD
OF
NUMBERS

Comprehensive School Mathematics Program

CSMP Mathematics for the Upper Primary Grades
Part I

The World of Numbers

CEMREL, Inc.

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THE WORLD OF NUMBERS

INTRODUCTION

Numbers are among the most important things that mathematics (at all levels) is about. Mathematicians are interested in numbers just as astronomers are interested in stars, botanists in plants, and sociologists in the organization and dynamics of human societies. Surely everyone agrees that one of the primary goals of any elementary school mathematics curriculum should be to introduce students to the world of numbers - to give them the opportunity to become familiar with numbers, their properties, and the relations between them. The ability to calculate with numbers is an important part — but not the only part — of being familiar and comfortable with numbers. The World of Numbers strand of the CSMP Upper Primary Grades Program is designed to provide the students with a wide variety of challenging experiences with numbers so that gradually they will become not only familiar, but comfortable with numbers; they will, so to speak, get to know numbers on a "first name basis".

The Minicomputer and the World of Numbers

The CSMP Upper Primary Grades program uses the Papy Minicomputer[*] as a support for the positional system of numeration; for calculations and estimation; for number patterns and mental arithmetic; and for modeling the basic operations involving whole numbers, integers, and decimal numbers. Although the Minicomputer can be seen as a tool for calculation and as a device to help students learn routine methods for calculations, its more exciting use is as a vehicle for posing interesting problems which challenge a child's intellectual curiosity about numbers and for presenting situations which dually encourage strategic thinking and reinforce numerical skills. The lessons which make use of the Minicomputer are often intended to be explorations into the world of numbers.

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[*] See also Questions 7 and 8 of Section Two, Questions and Answers About CSMP of the General Introduction and APPENDIX A: STANDARD CONFIGURATION AND STANDARD MINICOMPUTER MOVES.

Standard Algorithms of Arithmetic

CSMP seeks to develop basic numerical skills as well as an understanding of the underlying mathematical ideas. We are fully in agreement with the thesis that, along with the growth of understanding of the world of numbers, there must be a concommitant growth of familiarity and facility with numbers and operations on them. But facility should not be confused with understanding; they are partners in the growth of mathematical maturity. A balanced growth of each must be maintained, neither being sacrificed for the other.

Students must eventually learn mechanical algorithms for the basic operations (addition, subtraction, multiplication, division). However, premature presentation of these algorithms may actually stunt a student's ability to develop alternative algorithms, to do mental arithmetic, or to estimate.

Consider the problem of calculating "385 + 99". A second grader may have difficulty performing the standard (paper and pencil) addition algorithm. An easier and more efficient way to proceed is to add 100 to 385 and then to subtract 1 (385 + 100 = 485 and 485 - 1 = 484). To insist on a mechanistic response to such a problem would be to encourage inefficiency and might also inhibit the development of the flexibility necessary for problem solving. On the other hand, a rich array of situations in which students interact with numbers provides them with opportunities to gain the necessary facility with standard algorithmic procedures while retaining the openness required to respond creatively to new situations in the world of numbers.

Numerical Relations

One of the main aims of the World of Numbers strand is to familiarize students with numbers by studying explicitly and in a variety of contexts relations between numbers. (For more general comments about mathematical relations, see the introduction to the Languages of Strings and Arrows strand.) Arrow diagrams represent relations in a simple, suggestive, and pictorial way -- usually more conveniently than the same information could be given in words or other symbols.

Students are brought into contact with an assortment of challenging situations, many of which would be totally inaccessible to them were it not for the arrow diagrams. The problems and activities of this strand include many examples: solving linear equations presented in terms of arrows; studying iterated processes and patterns in sequences of numbers; tackling problems which may have many solutions or no solution; estimating or testing that a solution is reasonable; and exploring properties of operations on numbers.

In summary, what is most important in the study of numbers is to confront the students with a variety of problems and situations which capture their interests, challenge their abilities to reason, and stimulate their curiosities about numbers -- and, at the same time, provide them with tools for coping with these situations and problems.

N1 Minicomputer Introduction #1

CAPSULE LESSON SUMMARY

Review (or introduce) the value of the squares on the Minicomputer. Use the Minicomputer to represent numbers smaller than or equal to 100 .

MATERIALS

Teacher:	Minicomputer set[*]
Student:	Pencil; Worksheets N1*, **, ***, and ****

NOTE: Two different initial exercises have been included in this lesson. If all or most of the students in your class have had no previous experience with the Minicomputer, you should start with Exercise 1 - A and proceed to Exercise 2 . If most of the students in your class have been introduced to the Minicomputer, begin with Exercise 1 - B and proceed to Exercise 2 .

There are many exercises in this first lesson. Entry classes may be unable to cover all of them in one 40 - 50 minute session. Any exercise which you omit today or which is difficult for your class should be used, but not for more than five to ten minutes, before beginning Lesson N2 The Functions +3 and -3 or W1 The Number Line Game. In any case, your students should have an opportunity to work on the worksheets today.

---

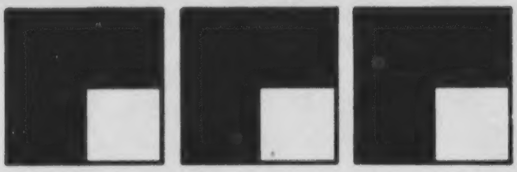
[*] A teacher's Minicomputer set consists of four demonstration Minicomputer boards and a sufficient number of demonstration Minicomputer checkers.

N1

DESCRIPTION OF LESSON

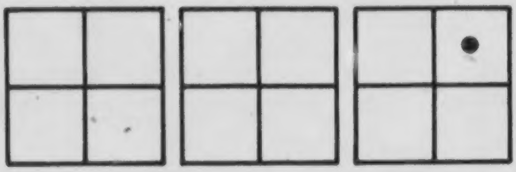
Exercise 1-A (for entry students)

Display three Minicomputer boards.



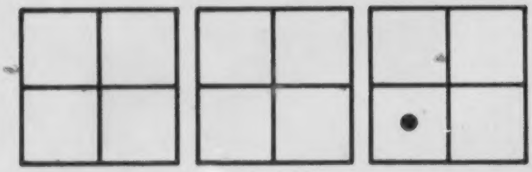
T: Show me four fingers. Trace a "4" on your desk. That's one way to make "4". Here's another way.

Put one checker on the purple square of the ones' board.



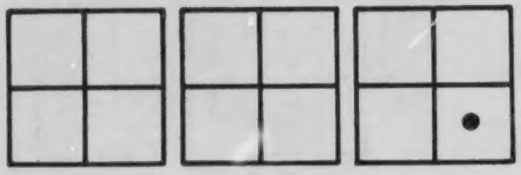
T: This is called the "Minicomputer". With this checker, I put 4 on the Minicomputer.

Move the checker from the purple square to the red square of the ones' board.



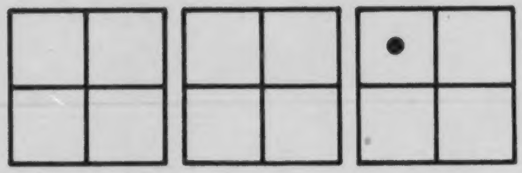
T: This is 2 on the Minicomputer.

Move the checker from the red square to the white square of the ones' board.



T: This is 1 on the Minicomputer.

Move the checker from the white square to the brown square of the ones' board.



T: Which number do you think this is? (Very often children will guess 3.) No, it's not 3. This is 8.

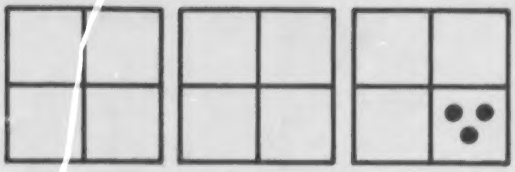
Review very briefly the placement of the checker for 1, 2, 4, and 8. Remove the checker from the Minicomputer.

T: Who can put 3 on the Minicomputer?

Ask volunteers to show with their fingers how many checkers they will need. (Two and three are both correct answers.) Let a student put 3 on the Minicomputer. If an answer is not correct, say so and ask for another volunteer.

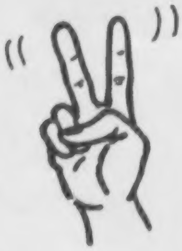
A student might put three checkers on the white square of the ones' board.

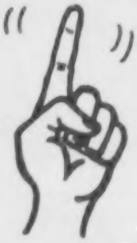
N1



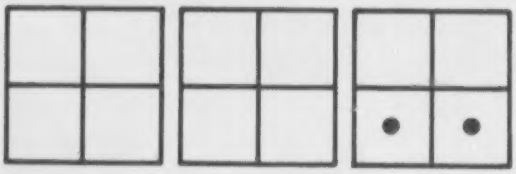
If a student places the checkers this way, lift the checkers one by one and say "1 + 1 + 1 = 3." Remove the checkers from the Minicomputer. Ask if someone can put 3 on the Minicomputer in another way.

If your students need a hint, alternately wave two fingers on one hand and one finger on the other hand saying, "3 = 2 + 1 ."





By now someone should be able to put this configuration for 3 on the Minicomputer.

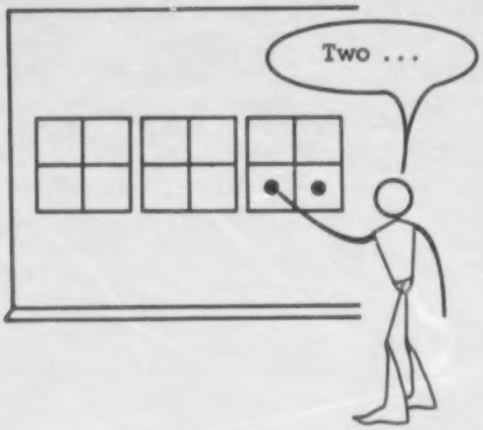


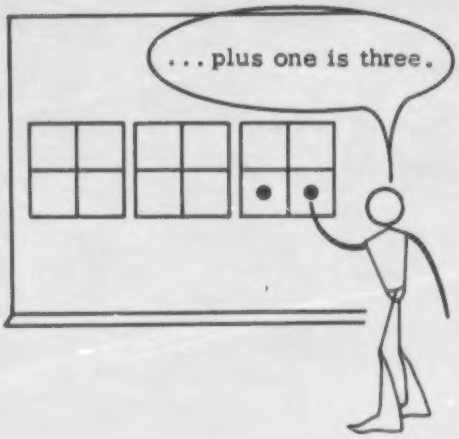
After a student places the checkers this way, lift each checker as you mention its value and then replace it quickly on the Minicomputer.

T: The red square is for which number? (2)

The white square is for which number? (1)

2 + 1 = 3 . 1 + 2 = 3 .




. . . plus one is three.

Remove the checkers from the Minicomputer.

T: Who can put 5 on the Minicomputer?

Ask a student who volunteers, how many checkers are needed. (Two, three, four, and five are all correct answers.) Ask the volunteer to place the checkers on the Minicomputer. Orally add the numbers on the various squares as you did for 3 . If the standard configuration is not suggested, continue asking for another way until you get the standard configuration.[*] You might find it helpful to ask for the same number with one less checker. For example, suppose a student named "Laura" put 5 on the Minicomputer using three checkers.

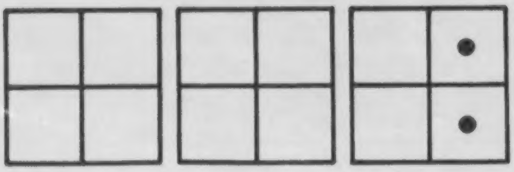
---

[*] The standard configuration of a number on the Minicomputer has at most one checker on a square and has a digit 9 or less on each board. See Appendix A: STANDARD CONFIGURATION and STANDARD MINICOMPUTER MOVES.

N1

T: Laura put 5 on the Minicomputer using three checkers.

Can anyone put 5 on the Minicomputer using only two checkers?



When the checkers are in this position, lift each checker as you mention its value and replace it quickly on the Minicomputer.

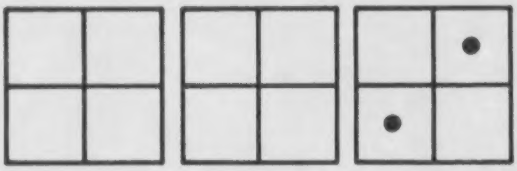
T: The purple square is for which number? (4)

The white square . . . ? (1)

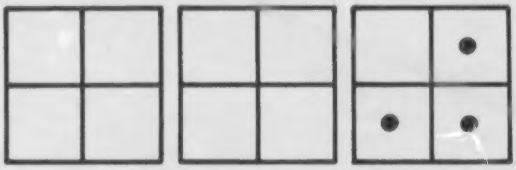
4 + 1 = 5. 1 + 4 = 5.

Remove the checkers from the Minicomputer.

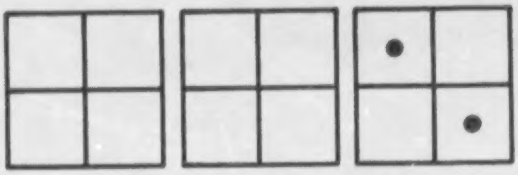
T: Can anyone put 6 on the Minicomputer?



T: Can anyone put 7 on the Minicomputer?

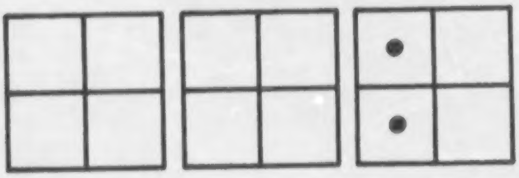


T: Can anyone put 9 on the Minicomputer?



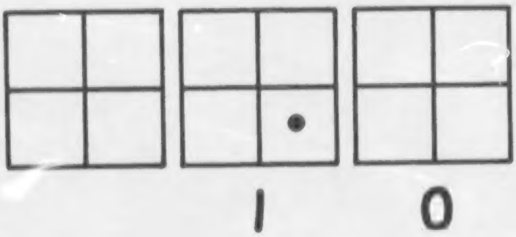
Accept all correct answers. If the standard configuration is not suggested, continue asking for another way (the same number using less checkers) until you get the standard configuration. Remove the checkers from the Minicomputer.

T: Can anyone put 10 on the Minicomputer?



T: There is a way to put 10 on the Minicomputer using only one checker.

Place one checker on the white square of the tens' board. Write "1" below (or above if it is more convenient) the tens' board and "0" below the ones' board.



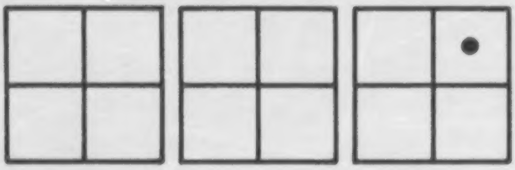
Erase "10", remove the checker, and continue with Exercise 2.

N1

Exercise 1 - B

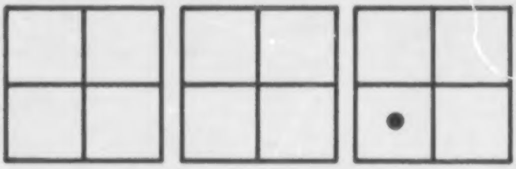
Display three Minicomputer boards.

T: Show me four fingers. This is the number 4 on the Minicomputer.

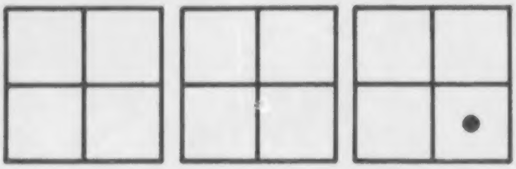


To prevent confusion, always remove the checkers from the Minicomputer before asking the next question.

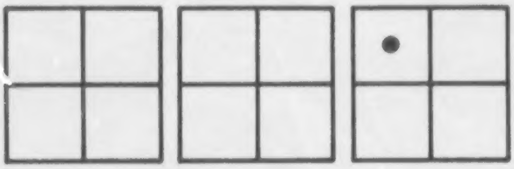
T: Who can put 2 on the Minicomputer?



T: Who can put 1 on the Minicomputer?

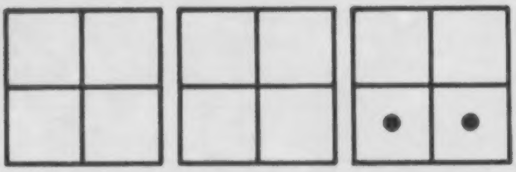


T: Who can put 8 on the Minicomputer?



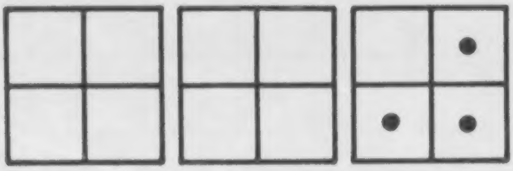
Very quickly review the placement of the checker for 1, 2, 4, and 8.

T: I'll give you two checkers. Can you put 3 on the Minicomputer?



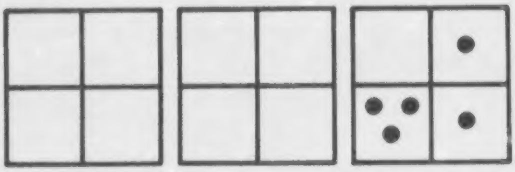
T: Can you put 5 on the Minicomputer with two checkers?
6 with two checkers?
9 with two checkers?
How many checkers do you need to put 7 on the Minicomputer?
(At least three)

Ask someone to put 7 on the Minicomputer with three checkers.



Exercise 2

Put some checkers on the ones' board; for example,



N1

NOTE: You should put the checkers very gradually on the Minicomputer, for example,

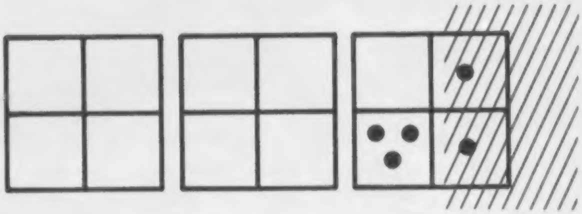
first three checkers on the red square, pause;
then one checker on the purple square, pause again;
then one checker on the white square.

This gives your students a chance to figure out mentally what number it is.

T: Which number is on the Minicomputer? (11)

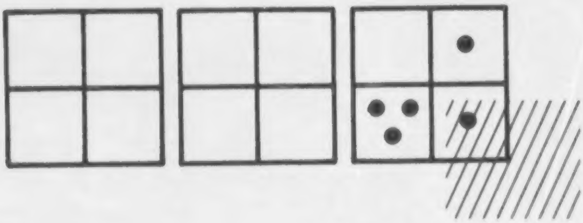
No explanation is necessary if everybody gets the right number. If someone gives the wrong number, you might use this procedure: cover part of the board with a piece of paper to focus the class' attention on certain checkers and gradually uncover the full set of checkers.

Cover the checkers on the purple and the white squares of the ones' board.



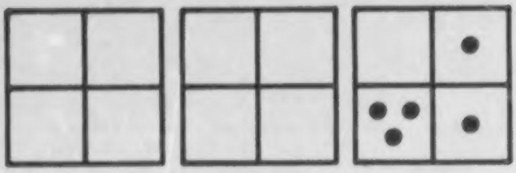
T: What number is this? (6)

Uncover the checker on the purple square of the ones' board.



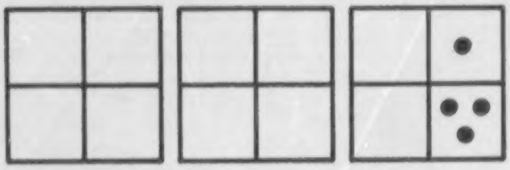
T: And 4 more? (10)

Uncover the ones' board completely.

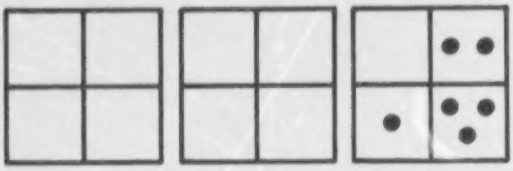


T: And 1 more? (11)

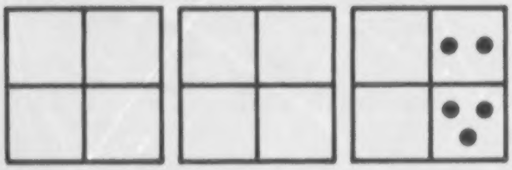
Repeat this activity with other examples; some possibilities are suggested below.



= 7



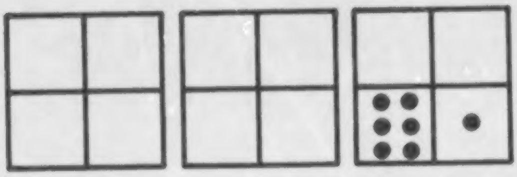
= 13



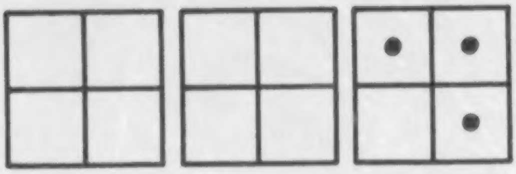
= 11

N1

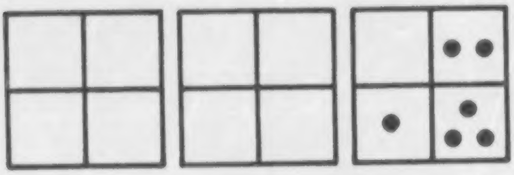
NOTE: It is interesting to present the same number several times, each time with a different number of checkers. For instance, 13 can be represented as:



or



or



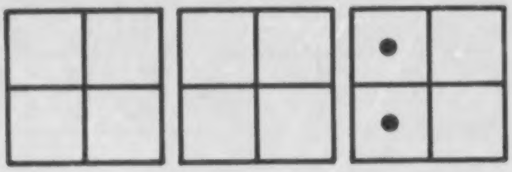
and so on.

Exercise 3

T: Who can put 12 on the Minicomputer?
Can you put 12 on in another way?
Another way?
Another way?

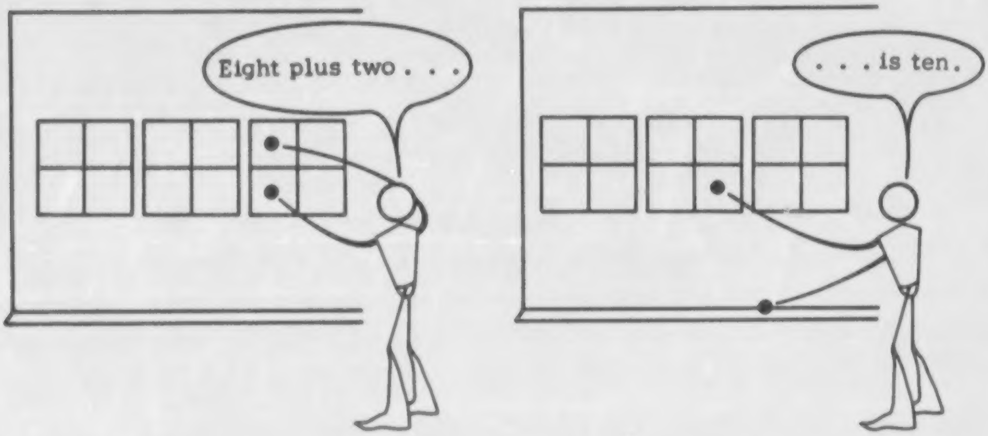
Exercise 4

Put one checker on the 8-square and one on the 2-square.



T: What number is on the Minicomputer? (10)

Pick up the checker on the brown square with one hand and the checker on the red square with the other. Then put one of these checkers on the tens' square and take the other checker away (put it in the chalk tray) . As you are making the play, say, "8 + 2 = 10."[*]



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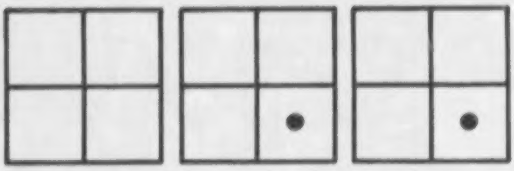
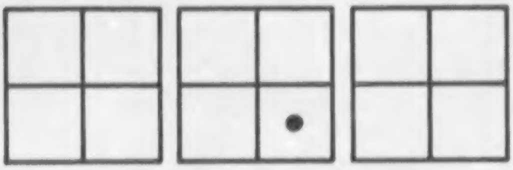
[*] The emphasis here is on the value of the squares on the tens' board, and not on plays.

N1

T: This is a way to put 10 on the Minicomputer with just one checker. I'll give you one more checker. Can you put 11 on the Minicomputer?



Remove the checker on the ones' board.



T: With one more checker, can you show 12 ? (Yes)

Can you show 14 ? (Yes)

Can you show 17 ? (No)

How many checkers do you need? (At least three more)

Can you show 18 ? (Yes)

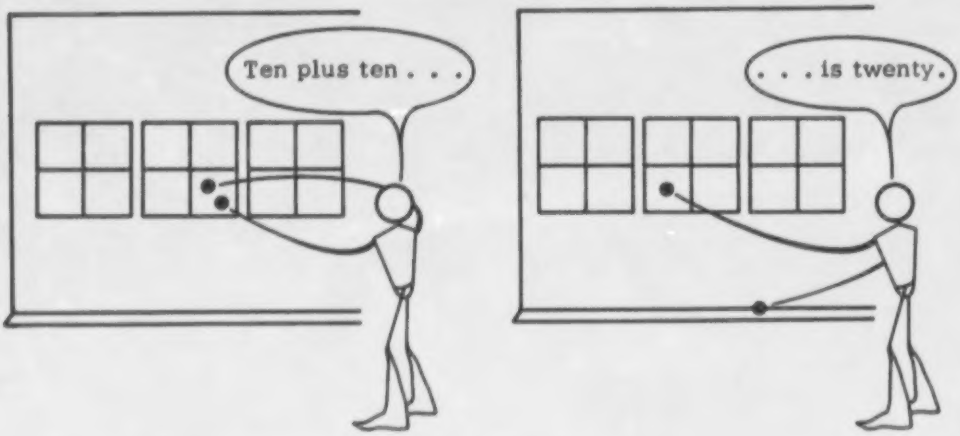
19 ? (No)

NOTE: The question "How many checkers do you need for 17?" has many answers. For the standard configuration you need four checkers, but it is possible to put 17 on the Minicomputer with three checkers: $8 + 8 + 1 = 17$.

Ask someone to put 20 on the Minicomputer using two checkers. If no one volunteers, put two checkers on the 10-square.

T: There is a way to put 20 on the Minicomputer with one checker.

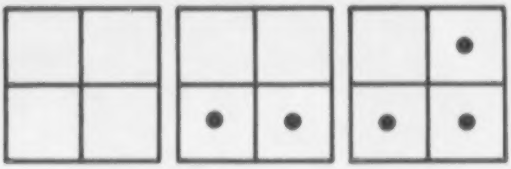
Make the play and say, "10 + 10 = 20."



Remove the checkers and continue with other numbers.

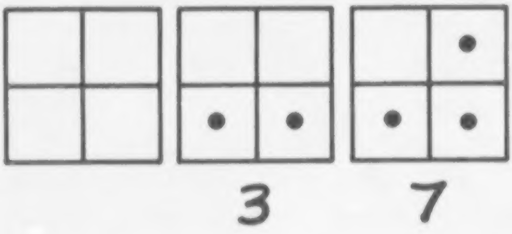
T: Who can put 25 on the Minicomputer? 27? 30?

Put this configuration on the Minicomputer.



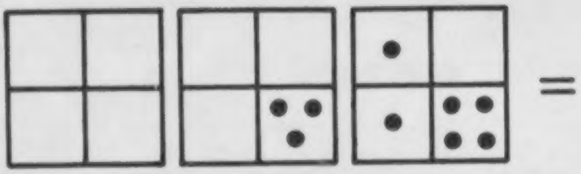
T: What number is this?

When you receive the correct answer, write "3" below the tens' board and "7" below the ones' board.



N1

Put this configuration on the Minicomputer.

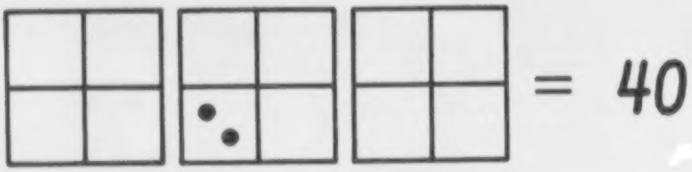


T: What number is this? (44)

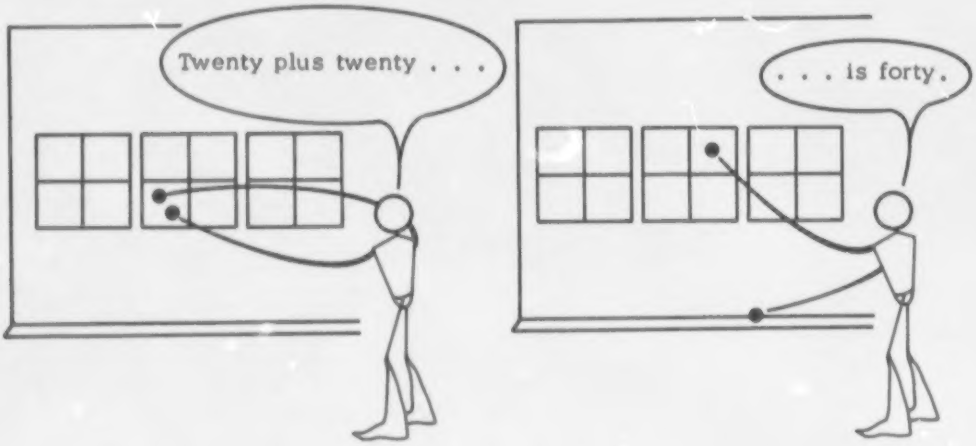
If necessary, make some plays until the class can read the number. You need not get the standard configuration.

Put two checkers on the 20-square.

T: What number is this? (40)

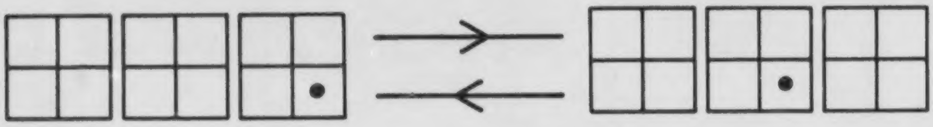


Make the play yourself and say, "20 + 20 = 40."



Remove the checker from the 40-square.

Move a checker back and forth very quickly from the 1-square to the 10-square. Each time you move the checker ask the class which number is on the Minicomputer: "one-ten; one-ten; and so on."



Repeat this with 2 and 20; 4 and 40; 8 and 80.[*]

T: Who can put 47 on the Minicomputer? 53? 96? 32?

Exercise 5

Give a student four checkers to place anywhere on the first two boards of the Minicomputer. You may want to stop a student who begins to put several checkers on the 8-square or the 80-square of the Minicomputer. Tell the class they can whisper to you what they think the number is. If appropriate, ask a student to write the number below the Minicomputer.

NOTE: Often in the description of a lesson it is suggested that you listen to whispers in order to give many students an opportunity to arrive at an answer independently. Let several students who have an answer whisper it in your ear. If appropriate, you will indicate to the student if the answer is correct. In some situations, for example detective stories, you will not indicate if the answer is correct because you want the student to continue to think about the situation.

If the number is too difficult for your students to read, read it yourself and continue the lesson.

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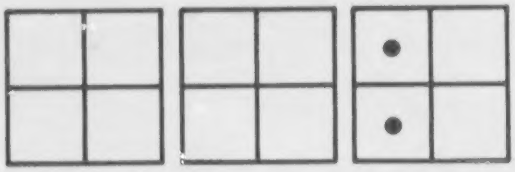
[*] Entry classes might need help with the introduction of the 80-square.

N1

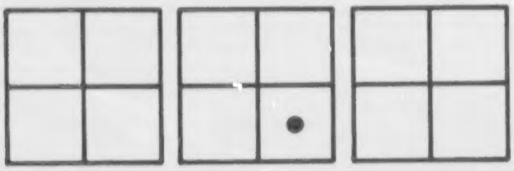
Exercise 6

Put this configuration on the Minicomputer.

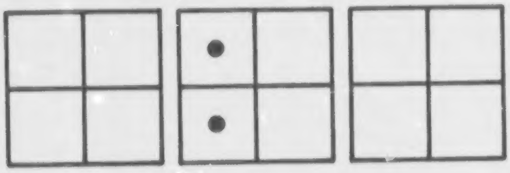


T: What number is this? (10)

Who can put 10 on the Minicomputer with one checker?

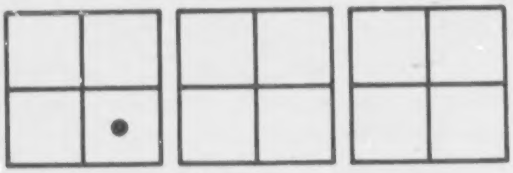


T: What number is this? (100)

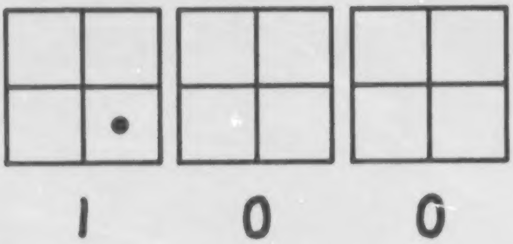


T: How do you know? (80 + 20 = 100)

T: There is a way to put 100 on the Minicomputer with just one checker.



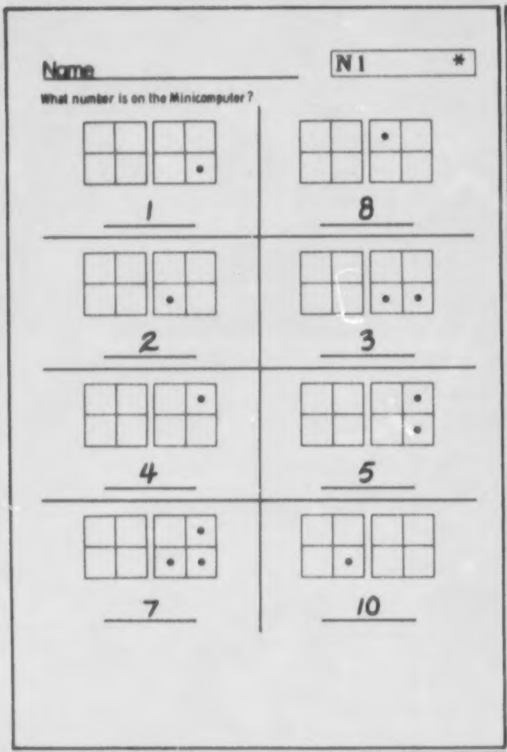
Make the play yourself and say, "80 + 20 = 100." Write "100" below the Minicomputer.



T: Who can put 105 on the Minicomputer? 126 ? And so on.

Occasionally, you may wish to have a student write a numeral below the Minicomputer.

Worksheets N1*, **, ***, and **** are available for independent work.



Name

N1 *

What number is on the Minicomputer?

1

8

2

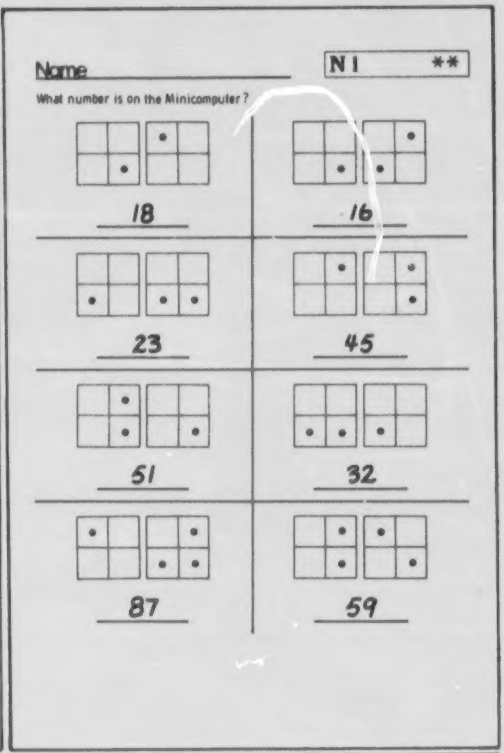
3

4

5

7

10



Name

N1 **

What number is on the Minicomputer?

18

16

23

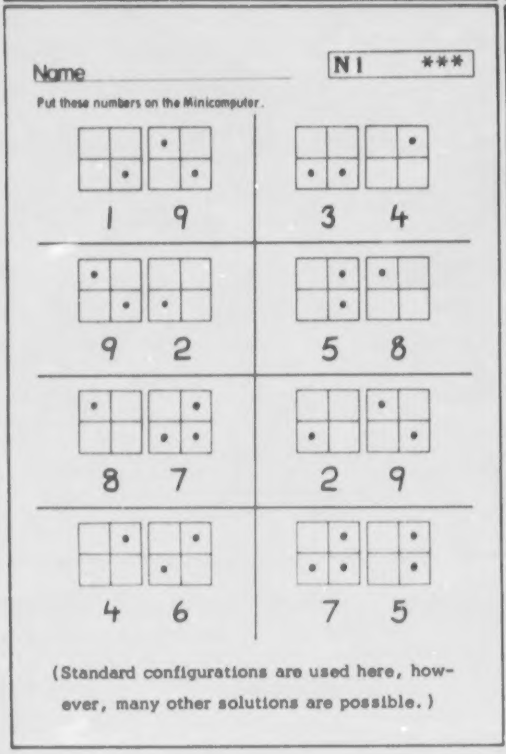
45

51

32

87

59



Name

N1 ***

Put these numbers on the Minicomputer.

1 9

3 4

9 2

5 8

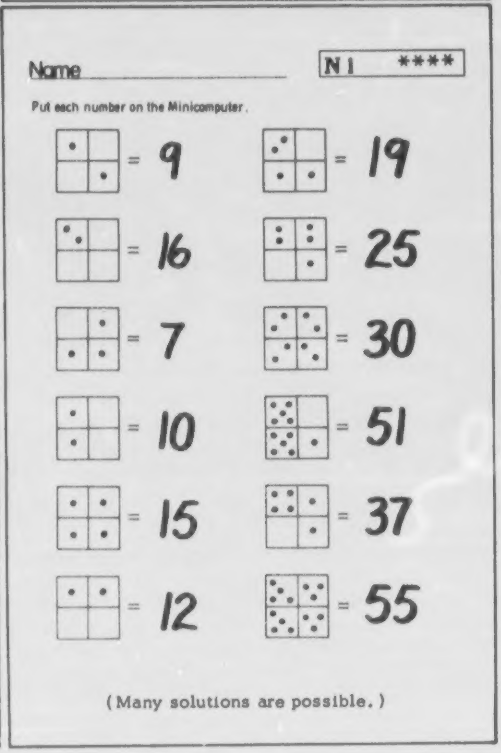
8 7

2 9

4 6

7 5

(Standard configurations are used here, however, many other solutions are possible.)



Name

N1 ****

Put each number on the Minicomputer.

= 9

= 19

= 16

= 25

= 7

= 30

= 10

= 51

= 15

= 37

= 12

= 55

(Many solutions are possible.)

N2 The Functions +3 and -3

CAPSULE LESSON SUMMARY

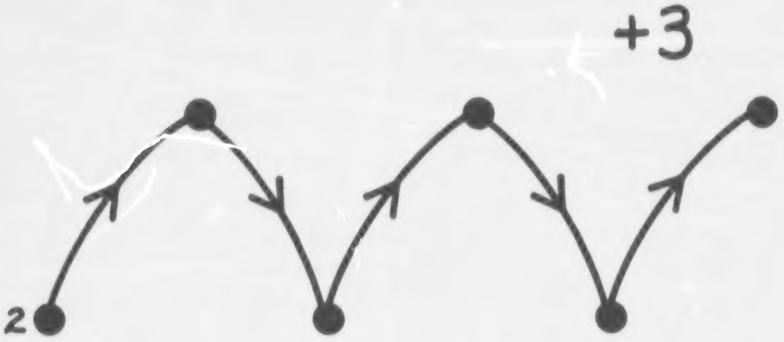
Label the dots and draw the return arrows in a +3 arrow picture.

MATERIALS

Teacher:	Colored chalk
Student:	Pencil; Worksheets N2* and **

DESCRIPTION OF LESSON

Draw this arrow picture on the board.



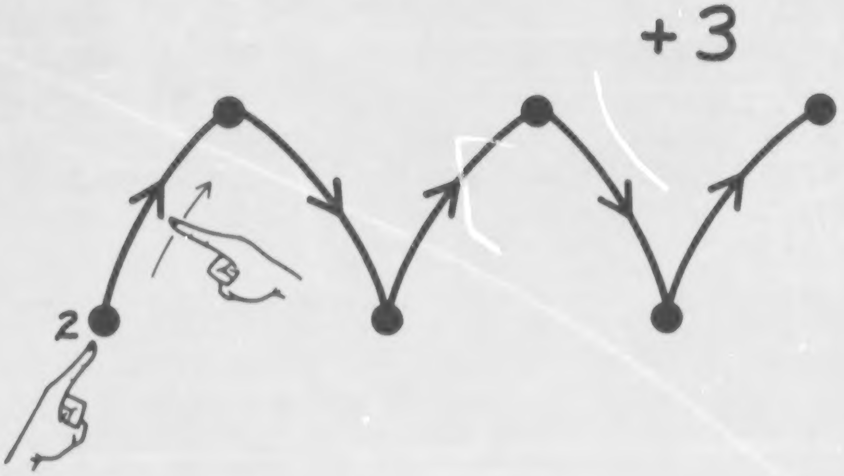
Put your forefinger on the dot for 2 .

T: This dot is for the number 2 . The blue arrow is for +3 .

Put your left forefinger on the dot for 2 and trace the blue arrow starting at 2 with your right forefinger in the direction of the arrowhead as you say:

T: 2 + 3 . What number is this? (5)

N 2



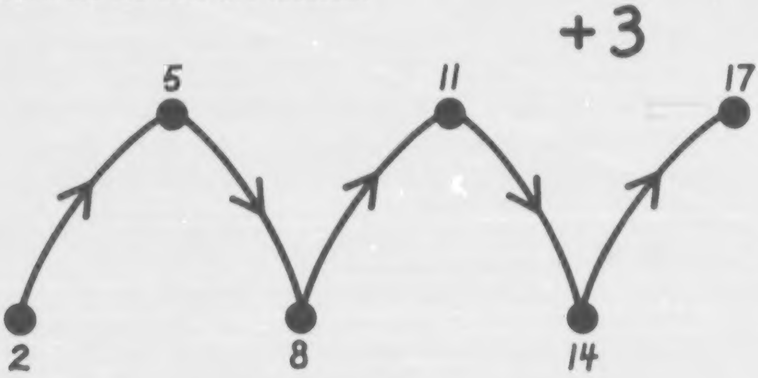
Label the second dot "5". Put your left forefinger on the dot for 5 and trace the blue arrow starting at 5 with your right forefinger as you say:

T: 5 + 3. What number is this? (8)

Label the third dot "8". Point to the unlabeled dots.

T: What are these numbers?

Continue until all the dots are labeled.



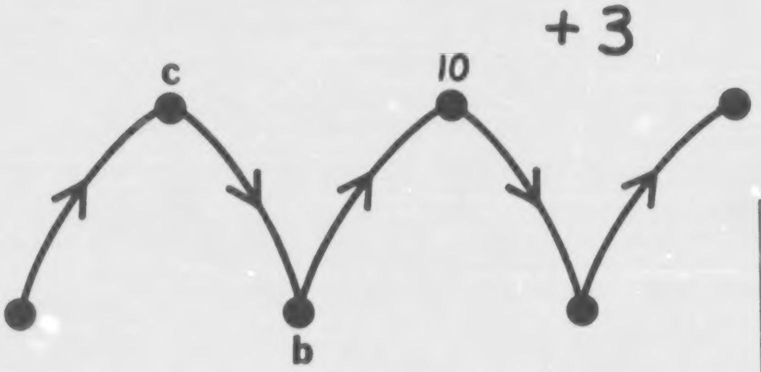
87

T: Could we go on with more +3 arrows? (Yes)

Do not draw more arrows. You could trace more arrows without drawing them.

T: If I keep drawing arrows, do you think we will ever meet the number 20 ? (Yes)

If several students respond correctly but many are uncertain, trace the imaginary arrow and ask the students which number comes next in the picture. Ask the class if you will ever meet the number 25 . (No) How about 30 ? (No) Erase the numerals but not the dots or arrows. Label the fourth dot from the left "10".



Do not write the letters on the board. They are here just to make the description of the lesson easier to follow.

Point to the dot labeled "b" in the preceding illustration.

T: What number is here? Tell me with whispers.

Accept several whispers and then ask a student to answer aloud.

Label the dot.

T: How do you know this is 7 ?

S: 7 + 3 = 10 .

N2

T: Does someone have another way to see that this is 10 ?

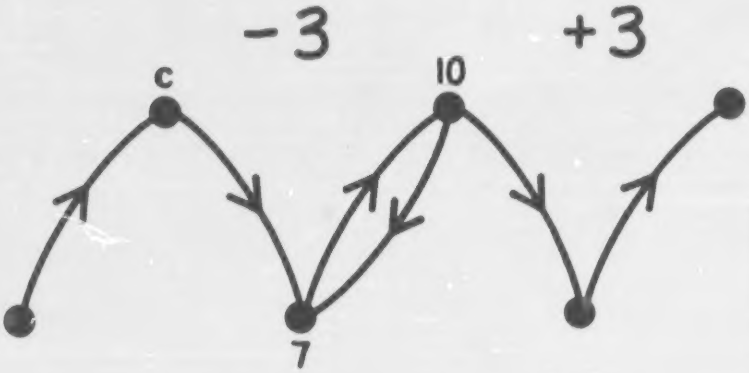
S: 10 - 3 = 7 .

It may be necessary to give the "10 - 3 = 7" observation yourself.

T: We also know that this number is 7 because 10 - 3 = 7 . We could draw a red arrow for -3 .

Put your right forefinger on the dot for 10 . Trace an arrow (starting at 10 and ending at 7) with your left forefinger as you say, "10 - 3 = 7 ."

Draw the return arrow in red from 10 to 7 . Write "– 3" on the board near the arrow picture.



Do not write the letter on the board. It is here just to make the description of the lesson easier to follow.

Point to the dot labeled "c" in the preceding illustration.

T: What number is here?

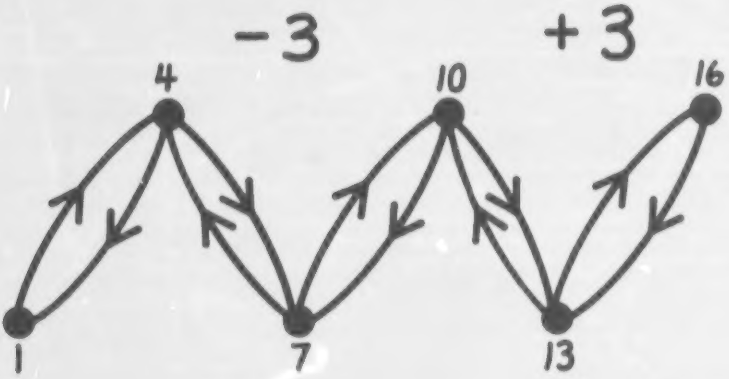
S: 4 , because 7 - 3 = 4 (or 4 + 3 = 7).

T: What are the other numbers in the picture? (1 , 13 , 16)

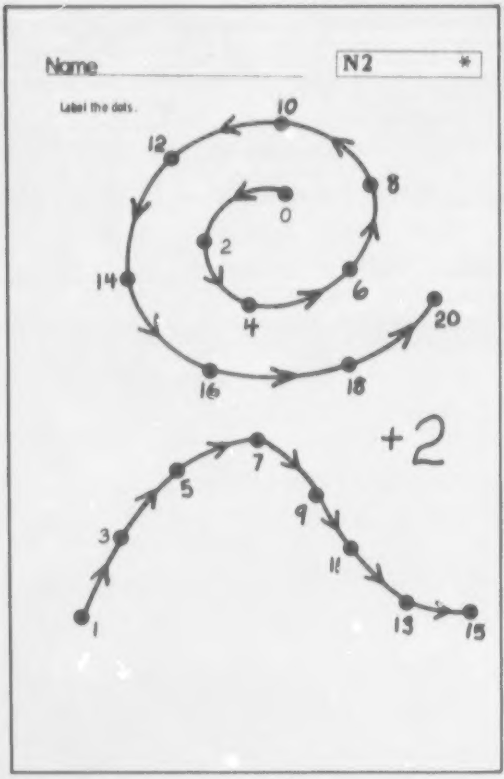
Quickly label the remaining dots.

T: Could we draw more red arrows in this picture? (Yes)

Ask students to draw the red arrows. After each red arrow is drawn, point to its starting number and trace the arrow to its ending number while you say, for example, "13 - 3 = 10." Continue until the arrow picture is completed.



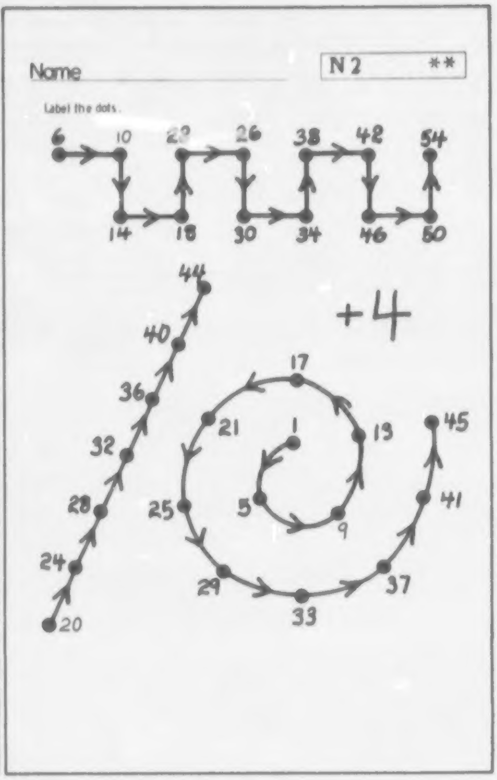
Worksheets N2* and ** are available for independent work.



Name ____________ N2 *

Label the dots.

0
2
4
6
8
10
12
14
16
18
20
+2
1
3
5
7
9
11
13
15



Name ____________ N2 **

Label the dots.

6
10
22
26
38
42
54
14
18
30
34
46
50
44
40
36
32
28
24
20
+4
17
21
1
13
45
5
9
25
41
29
33
37

N3 Minicomputer Introduction #2

CAPSULE LESSON SUMMARY

Practice reading numbers on the Minicomputer. Identify some Minicomputer plays and use them in addition, subtraction, and multiplication calculations. Recall the symbols ">" and "<".

MATERIALS

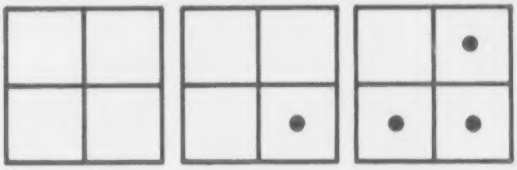
Teacher:	Minicomputer set
Student:	None

DESCRIPTION OF LESSON

Exercise 1

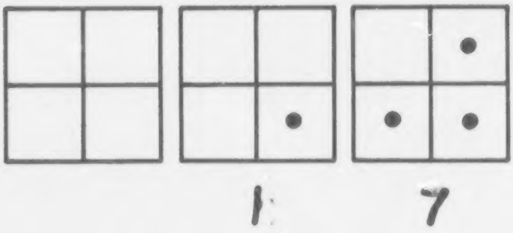
This activity should move quickly. Choose numbers which are appropriate for your class and encourage the slower students to participate.

Display three Minicomputer boards.



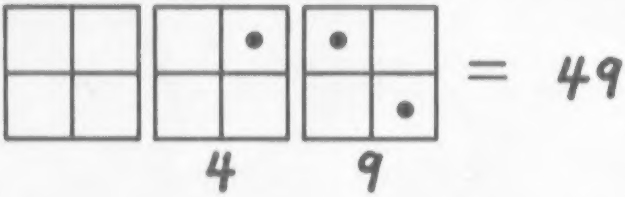
T: What number is on the Minicomputer? (17)

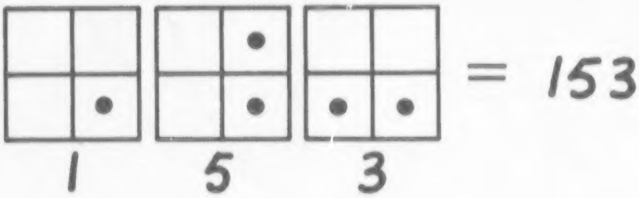
T: Who can write " 17 " below the Minicomputer?

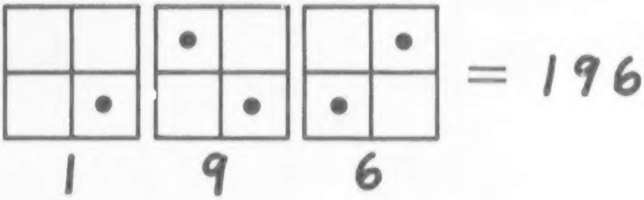


N3

Repeat this activity with other numbers such as those illustrated below. Each time, ask the students to identify the number and then ask someone to write the numeral below the Minicomputer.







Change the activity so that students put numbers on the Minicomputer.

T: Who can put 46 on the Minicomputer? 57 ? 124 ?

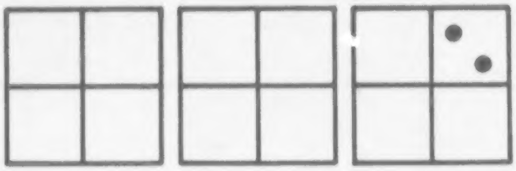
Move checkers quickly from one board to another to review:

1, 10, 100
2, 20, 200
4, 40, 400

8, 80, 800
5, 50, 500
3, 30, 300

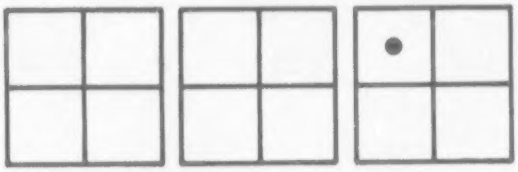
Exercise 2

Put two checkers on the 4-square of the Minicomputer.

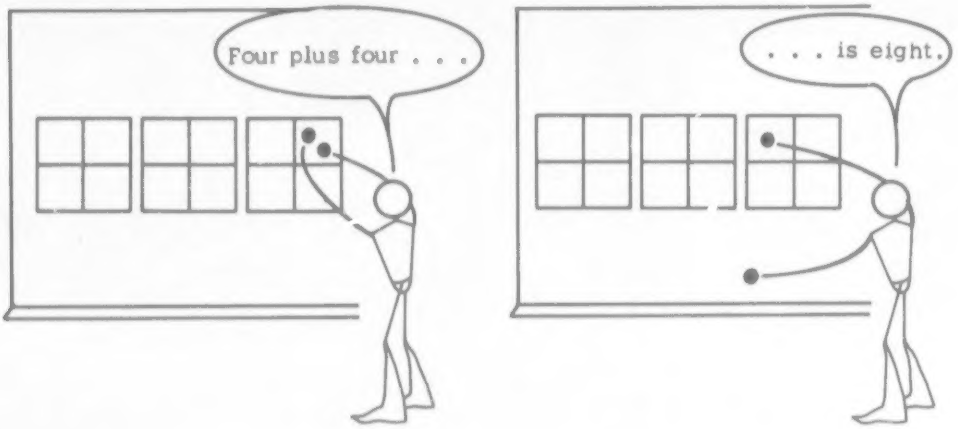


T: What number is this? (8)

Can someone put 8 on the Minicomputer with just one checker?



Repeat the play very clearly and say, "4 + 4 = 8 and 8 = 4 + 4."


Four plus four . . .
. . . is eight.

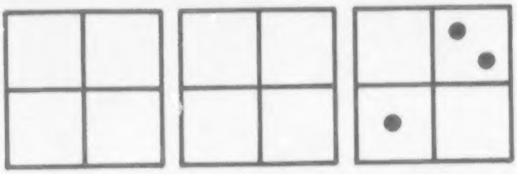
N3

Similarly, demonstrate the following plays:

40 + 40 = 80 and 80 = 40 + 40
100 + 100 = 200 and 200 = 100 + 100
200 + 200 = 400 and 400 = 200 + 200
8 + 2 = 10 and 10 = 8 + 2

Exercise 3

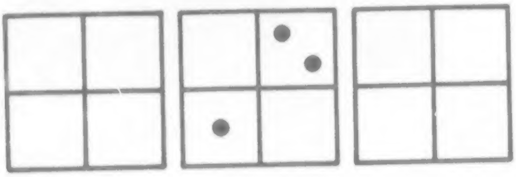
Put this configuration on the Minicomputer.



T: What number is this? (10)

Invite students to make plays until the standard configuration for 10 is on the Minicomputer. Students should name plays before moving any checkers.

Repeat the exercise with this configuration.

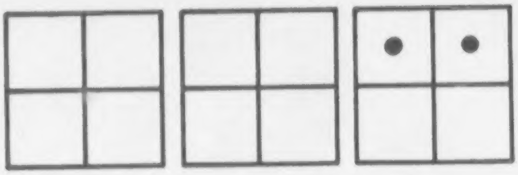


T: What number is this? (100)
What do you notice?

Encourage the students to comment on this new situation and to explain why this number is 100 before asking for volunteers to make plays.

Exercise 4

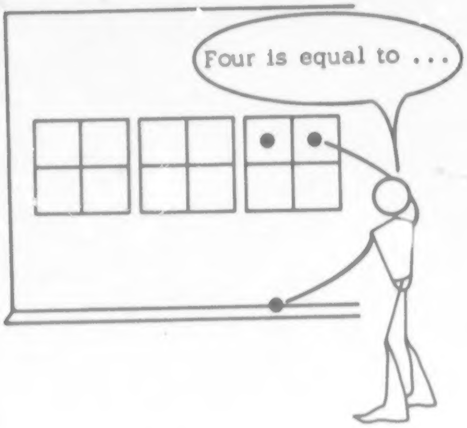
Put this configuration on the Minicomputer.

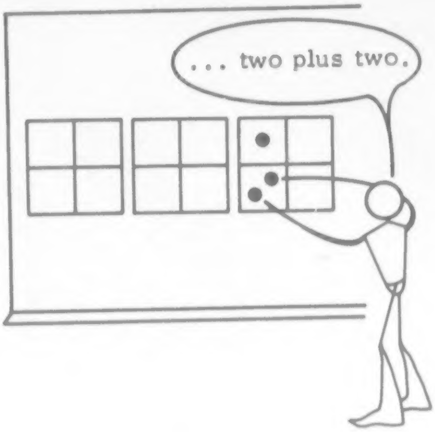


T: What number is this? (12)

T: Without changing the number, I would like to get a checker on the red square so that we can make the jump to the tens' board. What play should we make?[*]

If the "4 = 2 + 2" play is not suggested, make the play yourself.


Four is equal to ...


... two plus two.

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[*] It is possible that a student will suggest an "8 + 4 = 10 + 2" play. Although such plays are not emphasized, they should not be discouraged. In this case, you are interested in demonstrating the "4 = 2 + 2" play. If another valid play is suggested, you should indicate that it is correct, but that you would prefer the "4 = 2 + 2" play for this activity.

T: I want someone to make a play so we will have a checker on the tens' board.

Ask the students to name the play they intend to make before they move any checkers. Do not accept the "2 + 2 = 4" play because you have asked for a play which results in a checker on the tens' board.

NOTE: Backward plays may be difficult for your students at this time. There will be many opportunities to work on these moves in the future.

Exercise 5: Addition Problems

Write this problem on the board.

14 + 22 =

When students respond to these estimation questions, ask them to explain their answers.

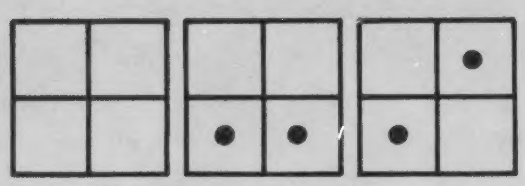
T: Do you think 14 + 22 is more or less than 22 ? (More)
More or less than 30 ? (More) More or less than 50 ? (Less)
We know that 14 + 22 is between 30 and 50 . Would it be more or less than 40 ?

If the students disagree, say "You don't know if it is more or less than 40, but you do know it is between 30 and 50 ." If the class is certain that 14 + 22 is less than 40 , say, " Now you know is is between 30 and 40 . "

T: What number is 14 + 22 ?

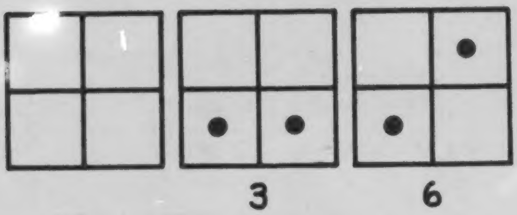
Accept several " guesses " and list them on the board. You could insist that guesses be between 30 and 50, (or between 30 and 40 if this has been determined) and so remind any student who gives a " bad " guess.

T: Who would like to put 14 on the Minicomputer using blue checkers?
Who would like to put 22 on the Minicomputer using red checkers?



Ask a student to write "36" under the Minicomputer and conclude:

$$14 + 22 = 36$$



Point to the list of estimates on the board. Acknowledge anyone who had correctly predicted the answer. If no one had guessed correctly, determine which estimate was the best. Indicate whether certain estimates are more or less than the sum. For example, if you had the following list of estimates, you would indicate that 38 is more than 36 and that 32 is less than 36. However, 38 is the best estimate because it is only 2 more than 36.

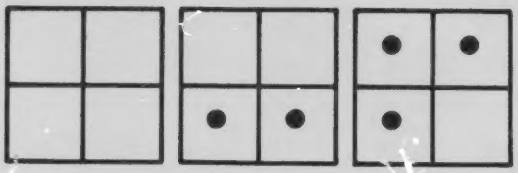
40
$32 < 36$
$38 > 36$
41
45

N3

NOTE: You should use the symbols ">" and "<" throughout the year whenever you have made a list of estimates and wish to decide which is the best estimate. These symbols are introduced in the CSMP First Grade program via stories about "Goldy", a fish who swims toward the school with the largest number of fish. Most entry students will have been introduced to the symbols ">" and "<" in other math programs. There is a short supplemental lesson about Goldy provided in Lesson W4 7-12 (Lesson 2), for classes with little or no experience with these symbols.

Repeat this exercise with 28 + 16 = 44. The class will probably agree that 28 + 16 is between 30 and 50, and may be able to estimate that it will be between 40 and 50.

When 28 and 16 have been put on the Minicomputer, ask if someone could make a play so the number will be easier to read.



When the plays have been made, ask someone to write "36" below the Minicomputer and conclude:

$$28 + 16 = 44$$

Exercise 6: Subtraction Problems

Write this problem on the board.

$$37 - 14 =$$

When students respond to these estimation questions, ask them to explain their answers.

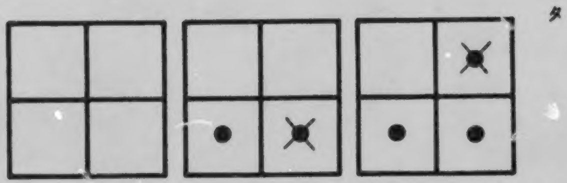
T: Do you think 37 - 14 will be more or less than 37 ? (Less) Less than 30 ? (Yes) Less than 20 ? (No) We know that 37 - 14 is between 20 and 30. What number do you think it is?

Accept several "guesses" and list them on the board. If someone makes a guess which is not between 20 and 30, remind the class that we know 37 - 14 is between 20 and 30 and do not record the guess.

T: Who can put 37 on the Minicomputer?
Who can take away 14 ?

NOTE: When students are asked to put numbers on the Minicomputer, they may or may not show standard configurations for them. The lesson descriptions assume that standard configurations of numbers will be put on the Minicomputer. Adjust the lessons depending upon which configurations your students display.

A student should remove a checker from the 10-square and a checker from the 4-square.



Ask a student to write "23" below the Minicomputer and conclude:

37 - 14 = 23.

---

[*] An " X " through a checker indicates that the checker has been removed from the Minicomputer.

Repeat this exercise with "53 - 42 = 11".

Exercise 7: Multiplication Problems

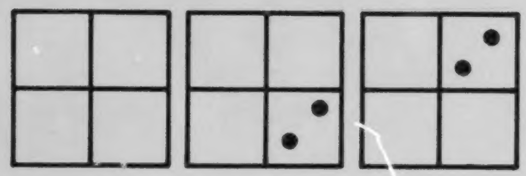
Write this problem on the board.

$$2 \times 14 =$$

You may have to read the problem and to remind the students that "2 x 14" means "14 + 14".

Accept several guesses and list them on the board.

T: Who would like to put 14 on the Minicomputer with blue checkers. Who would like to put 14 on the Minicomputer again with red checkers?



T: What number is on the Minicomputer? (2 x 14, or 28) Who can make some plays so it will be easier to read?

Conclude:

2 x 14 = 28.

Repeat this exercise with "2 x 37 = 74".

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N4 The Functions +5 and -5

CAPSULE LESSON SUMMARY

Practice counting by fives. Collectively, complete an arrow picture with +5 arrows and another with -5 arrows making use of return arrows. Individually, draw other arrow picture using +5 and -5 arrows.

MATERIALS

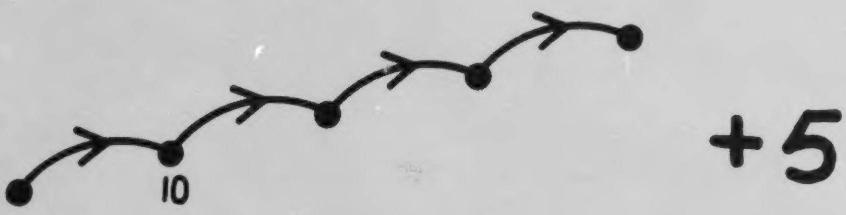
Teacher: Colored chalk

Student: Colored pencils*; pencil; unlined paper

DESCRIPTION OF LESSON

Start the lesson with a mental arithmetic exercise of counting by fives from 5 to 50.

Draw this arrow picture on the board.



T: Where is the smallest number in this arrow picture?
Would someone come to the board and point to the dot for the smallest number?
What is the smallest number in this picture? Show me with your fingers.

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*Crayons can be substituted for colored pencils in any of the lessons.

S: 5 (indicated by five fingers).

T: Who can come to the board and point to the dot for the largest number. What is this number? Tell me with whispers. (25)

Take whispers from several students and then ask someone to give the answer aloud.

Point to the dot to the right of 10 .

T: What number is here? How do you know?

S: 15 , because 10 + 5 = 15 .

Label this dot " 15 " and point to the next dot to the right.

T: What is this number? How do you know?

S: 20 , because 15 + 5 = 20 .

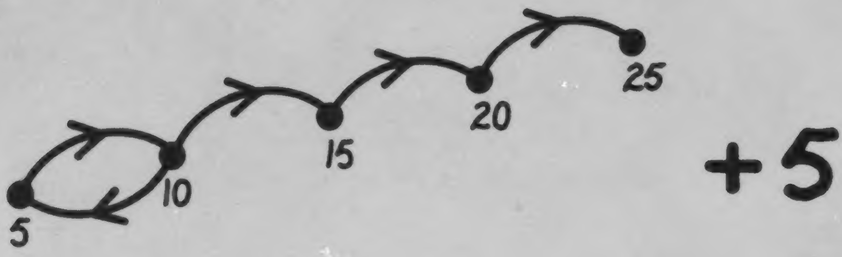
Label this dot and ask for a volunteer to label the last dot to the right. Point to the first dot.

T: What number is here? How do you know?

S: 5 , because 10 - 5 = 5 (or 5 + 5 = 10).

If someone answers " 5 + 5 = 10 " , accept this explanation but ask for another way.

Draw a return arrow in blue from 10 to 5.



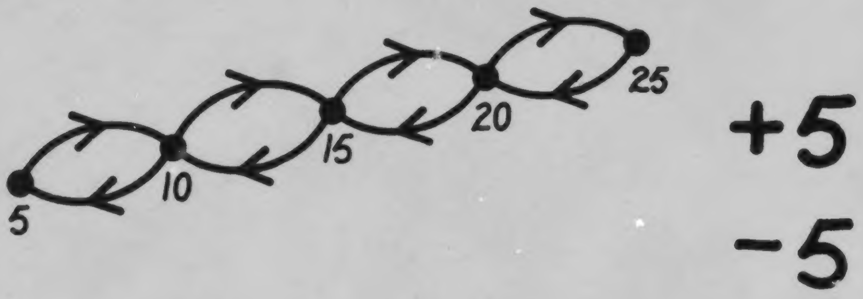
T: What could the blue arrow be for?

S: -5.

Write "-5" on the board near the arrow picture. As you trace the arrow from 10 to 5, say:

T: 10 - 5 = 5.
Could we draw any more blue arrows in this picture?

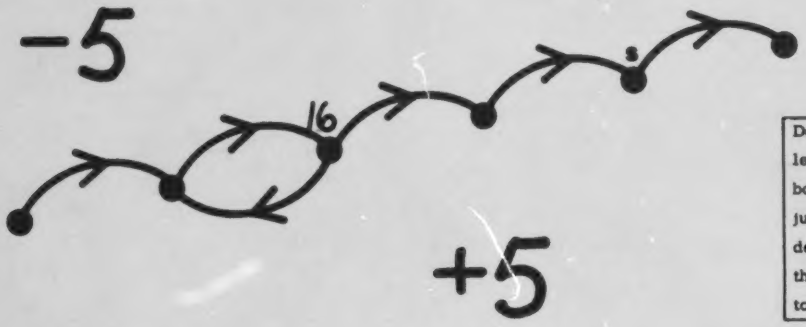
Complete the picture quickly with your class.



T: If we go on drawing red arrows (to the right), would we ever meet 31 ? (No) Would we meet 50 ? (Yes) 63 ? (No) How do you know?

Accept any response which is reasonably correct. It may be difficult for the students to verbalize their ideas.

Erase the board and draw this arrow picture.



Do not write the letter on the board. It is here just to make the description of the lesson easier to follow.

T: What are the blue arrows for? (-5)
Where is the smallest number in this arrow picture?
Where is the largest number in this arrow picture?

Point to the dot labeled " s " in the illustration above.

T: Tell me with whispers what number is here. (6)

Take whispers from several students and then ask someone to answer aloud.

T: Who would like to label a dot?

Occasionally, ask students to explain how they know which numbers correspond to particular dots. Encourage students to use return arrows when they are useful and to draw any missing return arrows to complete the picture.

T: If we go on drawing more red arrows (to the left), do you think we would ever meet 50 ? (No) How do you know?

Encourage students to observe a pattern in the numbers which will be met; that is, they all end in 1 or 6 . Continue these questions if the class is interested.

Erase the board and distribute unlined paper. Ask the students to draw their own arrow pictures using +5 arrows. Allow them to begin with a number of their own choosing and encourage them to continue as long as possible. Students who quickly finish a rather extensive picture could be asked to draw all the -5 arrows in their pictures.

N5

N5 Minicomputer Introduction #3

CAPSULE LESSON SUMMARY

Represent a sum on the Minicomputer by the use of checkers of two different colors. Explore the effect of adding a board to the right. Present a situation where a backward play is used to arrive at the standard configuration of a number. Explore the effect of moving various checkers in a configuration on the Minicomputer -- in each case, does the move increase, decrease, or leave the same number on the Minicomputer? Review plays on the Minicomputer as a way to make numbers easier to read.

MATERIALS

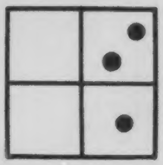
Teacher: Minicomputer set

Student: None

DESCRIPTION OF LESSON

Exercise 1

Display one Minicomputer board.



(Use two different colors for the checkers.)

T: What number is on the Minicomputer?

S: 9.

T: How do you see that 9 ?

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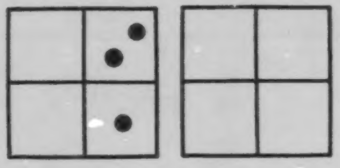
Let students make suggestions, and if it is not suggested that 9 can be seen as 4 (in red) + 5 (in blue), ask:

T: What number do you see in red? (4)
What number do you see in blue? (5)

Write "4 + 5 = 9" on the board.

Without removing the checkers, add a second Minicomputer board to the right.



T: Now, what number is on the Minicomputer?

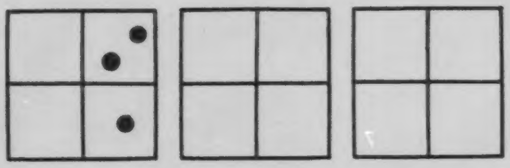
S: 90.

T: How do you see 90 ?

S: 40 in red plus 50 in blue.

Write "40 + 50 = 90" on the board.

Add a third board to the right.



T: What number is on the Minicomputer?

S: 900.

T: How do you see 900?

S: 400 in red plus 500 in blue.

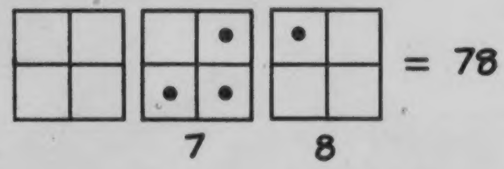
Write "400 + 500 = 900" on the board.

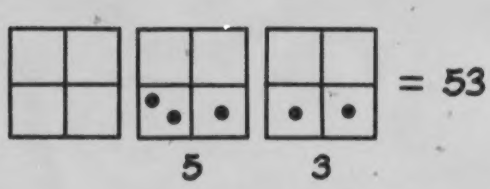
If your class seems to enjoy this exercise, you may go on adding boards (or drawing more boards on the chalkboard when you run out), and asking students to write the calculations:

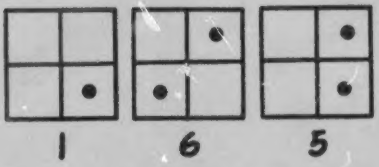
4 + 5 = 9
40 + 50 = 90
400 + 500 = 900
4,000 + 5,000 = 9,000
40,000 + 50,000 = 90,000.

Exercise 2

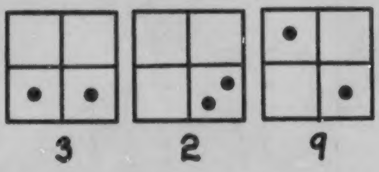
Display three Minicomputer boards. Ask the students to read various numbers you put on the boards, and to write the numerals with each digit centered below the appropriate board.



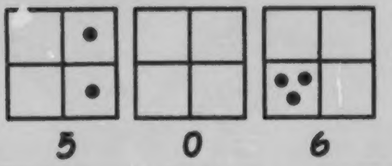




= 165



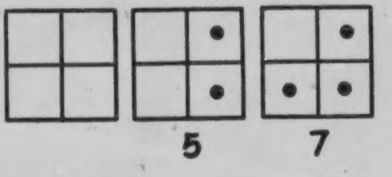
= 329



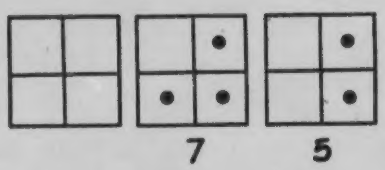
= 506

Vary the activity by asking the students to put various numbers on the Minicomputer and again to write the numerals below the boards. A good sequence of numbers you could ask for is suggested here.

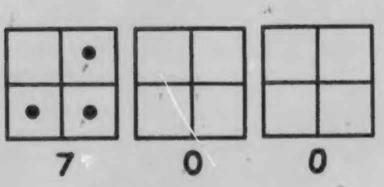
57 =



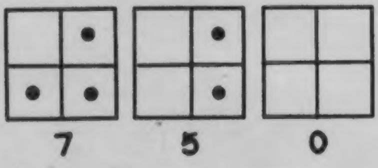
75 =



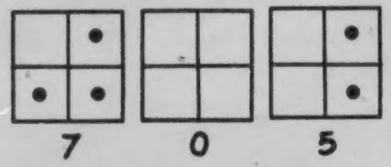
700 =



750 =

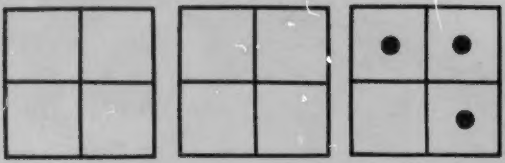


705 =



Exercise 3

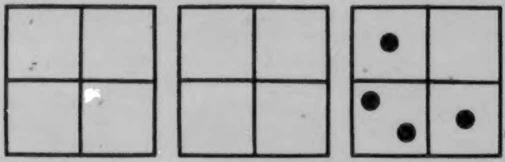
Put this configuration on the Minicomputer.



T: What number is this? (13)

Without changing the number, I want to get a checker on the red square so I can make the jump to the tens' board. What play should I make?

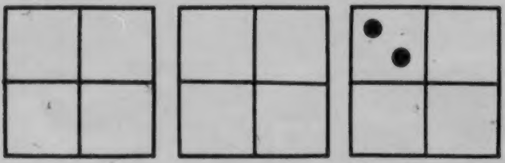
If no one suggests "4 = 2 + 2", suggest it yourself and make the play.



T: Who can make a play so we will have a checker on the tens' board?

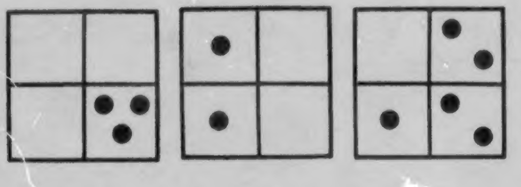
Do not accept the "2 + 2 = 4" play since you have asked for a play which results in a checker on the tens' board.

Repeat this exercise with this configuration.



Exercise 4: Transforming a Number

Put this configuration on the Minicomputer.



T: I put a number on the Minicomputer.
Is this number larger than 100 ? How can you tell?

S: Yes, because there is 300 on the hundreds' board.

T: Is this number larger than 300 ? How can you tell?

S: Yes, because there is 300 on the hundreds' board and more checkers on other boards.

T: Is this number larger than 400 ? How can you tell?

S: Yes, because there is 100 on the middle (tens') board and 300 + 100 = 400 and there are still more checkers on the ones' board.

(To answer these questions, your students could point to some of the checkers or suggest some moves.)

T: For the moment, do not try to find out exactly what number it is. I will move, remove, or take away some checkers. Each time, you should tell me if I get a larger number, a smaller number, or the same number.

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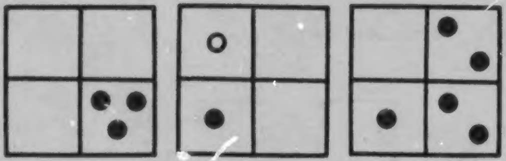
N5

Replace one of the checkers by a checker of another color.



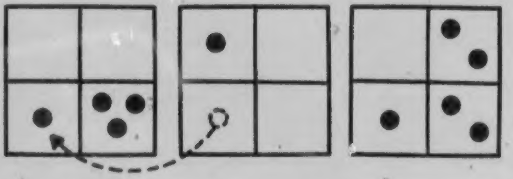
T: What do you think?

S: The same.

T: Why?

S: Because all you did was exchange a checker for a different color checker.

Move the checker from the 20-square to the 200-square.



T: Larger, smaller, or the same?

S: Larger.

T: Why?

S: Because you replaced a checker on the 20-square by a checker on the 200-square.

Return the checkers to their original positions.

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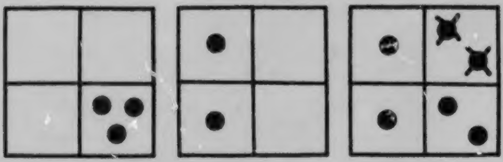
T: Who can move one checker and make the number smaller?

Ask a student to do this and discuss why the move makes the number smaller. Then return the checkers to their original positions.

Remove two checkers from the 4-square and put one checker on the 8-square.



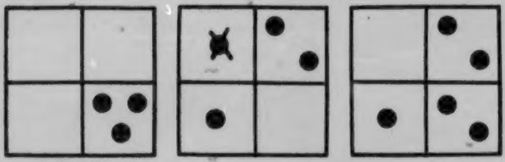
S: The same.

T: Why?

S: 4 + 4 = 8.

Return the checkers to their original positions.

Remove the checker from the 80-square and put two checkers on the 40-square.



S: The same.

T: Why?

S: 80 = 40 + 40.

N5

Return the checkers to their original position.

T: How can we make the number easier to read?
(Make some plays)

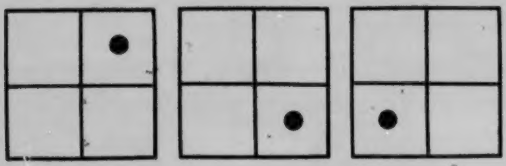
Ask students to name the play they intend to make before they move any checkers. For example,

S: 4 + 4 = 8.

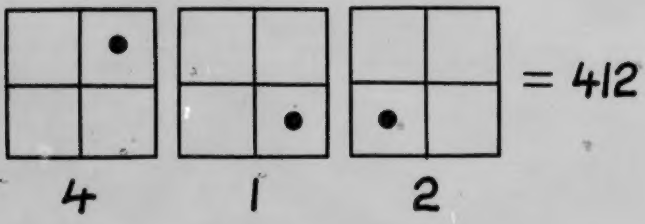
S: 100 + 100 = 200.

Be sure the students understand that any play they make must leave the number the same. Be prepared to make a backward play yourself if one is needed and the students do not suggest one.

Continue until you obtain the standard configuration for 412. Be sure to include any new students in this activity.

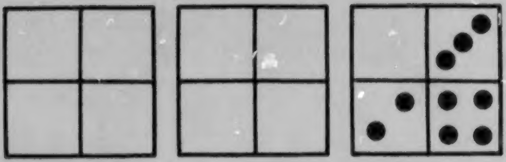


Read the number with the students and write it below the Minicomputer.



Exercise 5

Put the checkers on the Minicomputer gradually, allowing the students to calculate the number mentally.



T: What number is on the Minicomputer? (20)

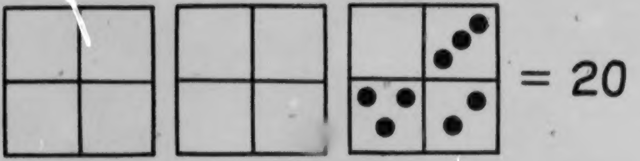
T: How many checkers have I used? (9)

T: What number is one less than 9 ?

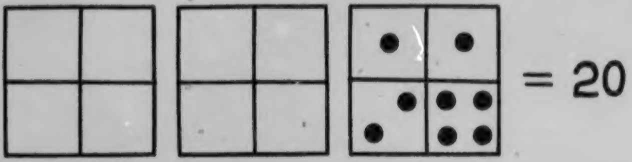
S: 8 .

T: Can you put 20 on the Minicomputer with one less checker (that is, with eight checkers) ?

If the students do not respond, suggest that they make a play. Any forward play they make will solve the problem. For example,



or

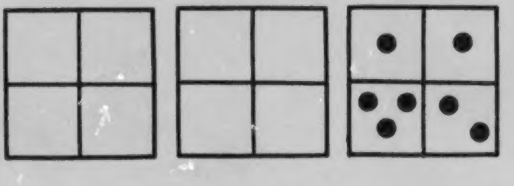


Other solutions are also possible.

N5

T: This is 20 with eight checkers.

Can you put 20 on the Minicomputer with seven checkers?



Other solutions are also possible.

T: Can you put 20 on the Minicomputer with one less checker (i.e. six checkers)?

Do not over extend this activity, but return to it for a few minutes from time to time during the first months of the year.

N6 Twice As Many

CAPSULE LESSON SUMMARY

Repeatedly double a set of dots, going from 1 to 64 dots. Relate this doubling activity to a 2x arrow picture and label the dots.

MATERIALS

Teacher:	Colored chalk; dot sheets; masking tape
Student:	Colored pencils; pencil; unlined paper

NOTE: You will need to prepare three sheets of paper with 16 large dots on each. On each sheet of paper arrange the dots as in the illustration on page 56.

DESCRIPTION OF LESSON

Choose one of your students to be the star of this story.

T: Cheryl is eight years old today. Her aunt has given her one beautiful angel fish. Cheryl watched the fish all morning. She liked to watch it swim around the aquarium. It swam around and around. The angel fish seemed very lonely. Cheryl wished she had at least one more fish.

Her aunt saw that she was sad. Her aunt asked, "Don't you like your angel fish?"

"Yes," said Cheryl, "but I wish I had at least one more fish. It looks very lonely swimming around and around by itself."

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N6

Cheryl's aunt smiled and said: "Your angel fish is lucky. I have a magic wand. When I touch the aquarium with this wand, there will be twice as many fish."[*] Then Cheryl's aunt touched the aquarium with the magic wand. Suddenly there were twice as many fish in the aquarium.

Ask the students how many fish are in the aquarium now.

T: Did Cheryl get her wish? (Yes)

Cheryl was very happy to see two fish in the aquarium. She asked her aunt to touch the aquarium again with the magic wand. So, Cheryl's aunt touched the aquarium again. Immediately there were twice as many fish.

How many fish are in the aquarium now?

S: Four.

T: How many fish did Cheryl get for her birthday?

S: One.

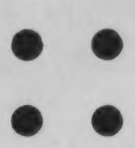
Draw one large dot on the board for the one angel fish Cheryl got for her birthday. Ask what happened when the aquarium was touched once with the magic wand. (Then there were twice as many fish.) Draw another large dot next to first as you say, "2 x 1 = 2 ."

● ●

---

[*] If some of the students have difficulty understanding "twice as many", hold up one finger and then two fingers as you say, "one fish will become two fish."

Ask what happened when the aquarium was touched the second time. (Then there were four fish, twice as many fish.) Draw two more large dots as you say "2 x 2 = 4."



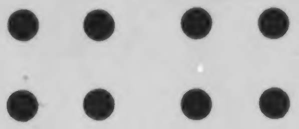
T: Cheryl jumped up and down and sang, "Now I have four beautiful angel fish." It made Cheryl's aunt happy to see Cheryl so excited, so she touched the aquarium again with the magic wand.

Now, how many fish are there in the aquarium?

S: Twice as many as four.

S: Eight.

If students are having difficulty, point to each dot and count by twos to determine the number of fish. On the board, draw four more dots.



T: Cheryl was very excited and cried, "Now my angel fish has seven friends. It will never be lonely again." Cheryl watched her fish swim around and around in the aquarium. Sometimes they seemed to be talking to each other. "These fish are nice," thought Cheryl. "But the magic wand is more fun." She asked her aunt to give her the magic wand.

N6

Ask the students what Cheryl could do with the magic wand, and let them discuss the possibilities for a minute.

T: "Why do you want the magic wand?" asked Cheryl's aunt.

"I want to touch the aquarium 100 times," answered Cheryl.

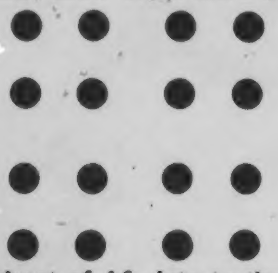
"Oh my! Then there would certainly be too many fish for your aquarium," said Cheryl's aunt.

"I'll only touch it ten times," promised Cheryl.

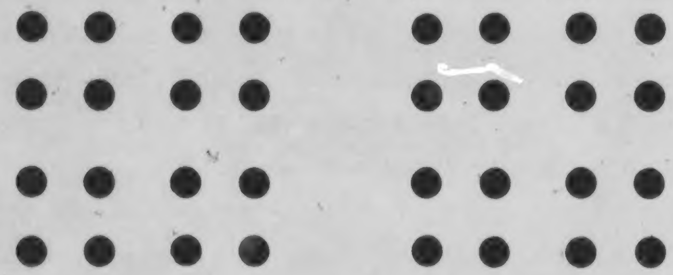
"I'm afraid that even ten times would give you too many fish to fit in one aquarium," answered Cheryl's aunt. "I will let you touch the aquarium just two more times with the magic wand." She gave Cheryl the wand and Cheryl ran to the aquarium.

Ask the students to tell you with whispers how many fish there will be after two more touches. (32)

On the board, draw eight more dots as you say, "2 × 8 = 16 ."



Tape one sheet of 16 dots to the right of your dots as you say, "2 x 16 = 32 ."

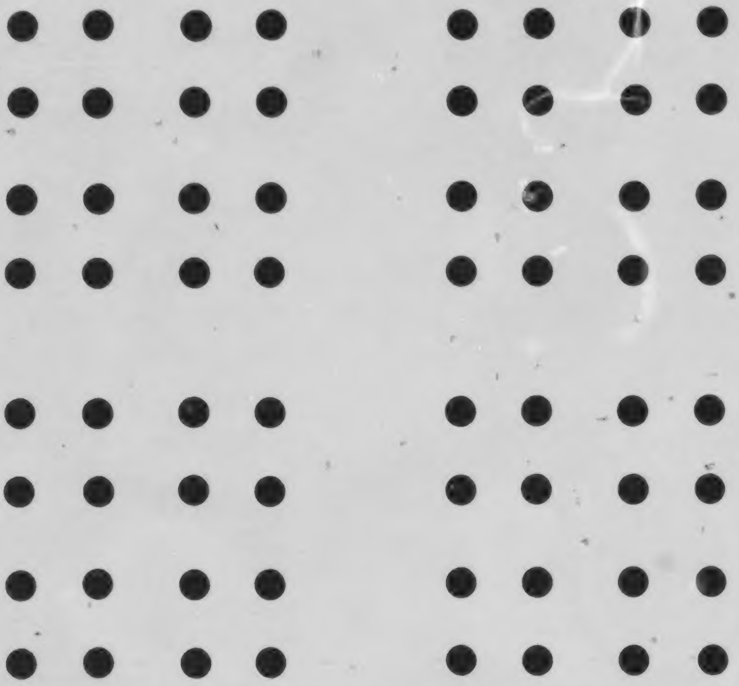


T: Cheryl will have 32 fish if she touches the aquarium two more times.

But Cheryl was a naughty girl. She touched the aquarium two times, and then she touched it a third time. Cheryl was going to touch the aquarium again but she stopped just in time. There were so many fish in the aquarium that they could not swim around anymore.

Ask the students to tell you with whispers how many fish are in the aquarium now. (64)

Tape two more sheets of 16 dots below the other dots on the board as you say "2 × 32 = 64 ."



T: We can tell this story with an arrow picture.

Draw an arrow picture as in the next illustration with five dots, beside the array of dots you have been developing. Leave room for more dots to be added later to the right of the arrow picture.

N6

T: How many fish did Cheryl get for her birthday?

S: One.

Label the first dot on the left "1".

T: How many fish were there after touching the aquarium with the magic wand once?

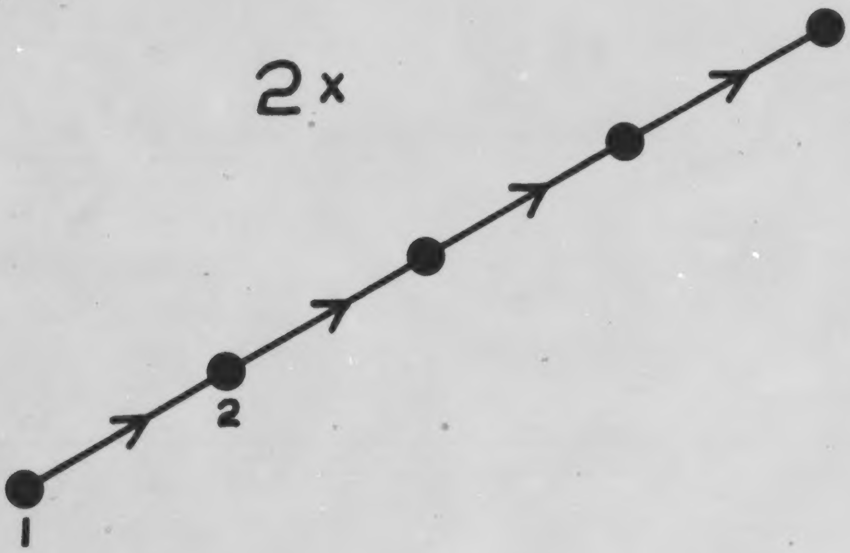
S: Two.

Label the second dot "2". Trace the first arrow from 1 to 2 as you ask:

T: What do you think this arrow could be for?

S: 2x.

If someone says "+1", say: "This first arrow could be for +1, but what else could it be for?" When someone suggest "2x," write the key near the arrow picture in the same color as the arrow. Point to the third dot.



T: What number is this?

S: 4, because 2 × 2 = 4 (or 2 + 2 = 4).

Encourage the explanation, perhaps reminding the class about how many fish were in the aquarium after two touches with the magic wand.

Label this dot. Point to the fourth dot.

T: What number is this?

S: 8, because 2 × 4 = 8 (or 4 + 4 = 8).

Label this dot. Point to the last dot.

T: What number is this?

S: 16.

Label this dot.

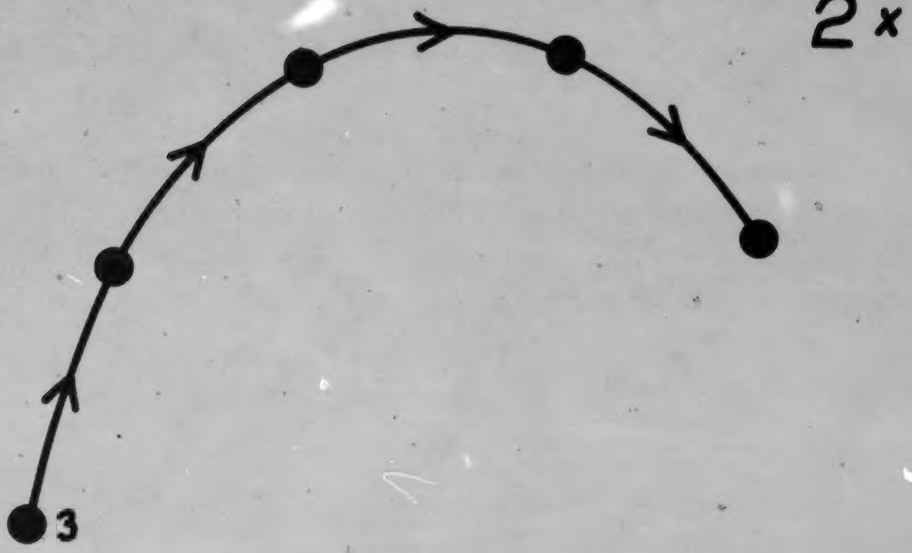
T: How many fish did Cheryl have at the end of our story? (64)
How many more 2× arrows must we draw to reach 64?

S: Two.

Draw two more arrows and label their ending dots with the class.

Erase the board and draw the arrow picture which is on the next page.

N6



Ask the students what the second dot is for.

Distribute unlined paper. Ask students to copy the picture, to label the other dots, and then to extend the arrow picture using more 2x arrows. Some students may wish to use their Minicomputers for the more difficult calculations.

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N7 Addition Problems #1

CAPSULE LESSON SUMMARY

Calculate "1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 " by pairing numbers whose sum is 10. With the Minicomputer, review the effect of adding a board to the right, that is, of multiplying by 10. Look at the many ways a number can be represented on the Minicomputer.

MATERIALS

Teacher: Minicomputer set
Student: Minicomputer set*

DESCRIPTION OF LESSON

Exercise 1: Addition Problems

Choose one of your students to be the star of this story.

T: Last summer Steve went to the beach. Sometimes he swam and sometimes he built castles. One day he found a stick and started writing in the sand. He began to write numbers: 1, 2, 3, 4, ...

Make a list of the whole numbers from 1 to 10 on the board as you say them, leaving space between them.

1 2 3 4 5 6 7 8 9 10

---

*A student's Minicomputer set consists of two individual Minicomputer sheets (each having two boards) and a sheet of individual checkers.

N 7

T: Steve's big brother came along just then and said, "This will keep you busy for a long time." He wrote plus signs between all the numbers with his finger. At the very end, he drew an equals sign and a big question mark.

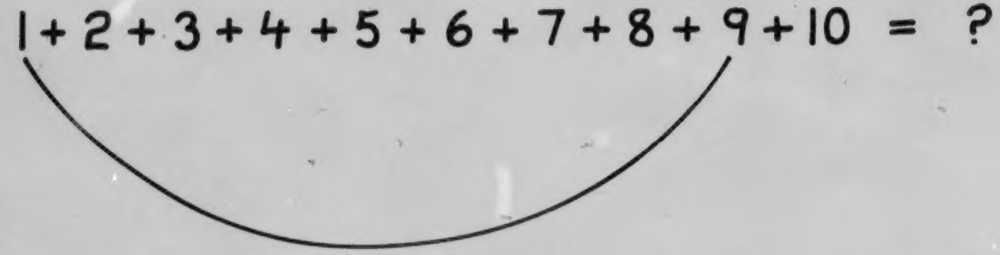
$$1+2+3+4+5+6+7+8+9+10 = ?$$

T: Steve wanted to show his brother how smart he was, so he looked for an easy way to add these numbers. Can you see a good way to pair these numbers so that they will be easy to add?

Accept any suggestions which pair two numbers whose sum is 10. If a student suggests a pair whose sum is not 10, say: "Has that made our problem any easier?" If no one suggests pairing numbers whose sum is 10, continue with the lesson.

T: Soon Steve saw that he could pair 9 and 1. "10 is a very easy number to add to other numbers," he thought.

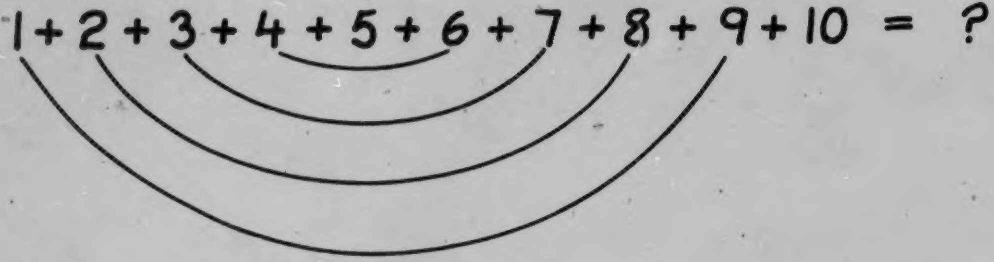
Show the pairing of 9 and 1 in your list.



$$1+2+3+4+5+6+7+8+9+10 = ?$$

T: Which other numbers could Steve pair to make 10?

Encourage students to continue pairing numbers whose sum is 10 until you have all the numbers except 10 in pairs.



T: How did these pairs help Steve solve the problem? (10's are easy to add.)

You may need to suggest this idea yourself.

T: There are five 10's and one 5.

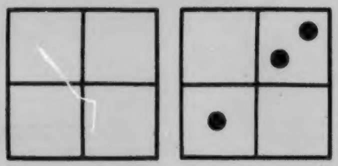
Write this calculation on the board:

$$10 + 10 + 10 + 10 + 10 + 5 = ?$$

This should be enough to convince most of the class that the sum is 55. Afterward, let the students check the calculation on the Minicomputer.

Exercise 2: Multiplying by 10

Put this configuration on the Minicomputer.



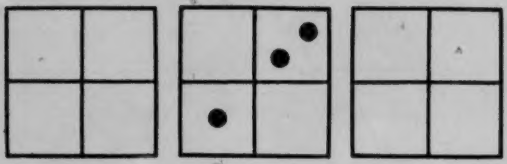
N 7

T: What number is this?
You could make some plays in your head to help you decide.

S: 10.

If several answers are given, let a student make plays until the class agrees that the number is 10.

Return the checkers to their original position. Add a third board to the right.



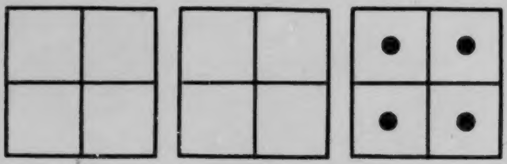
T: What number is this?

S: 100.

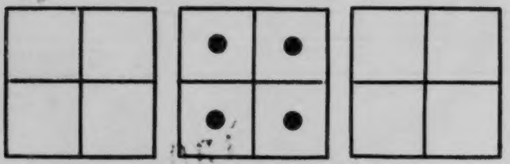
T: What do you notice?

Encourage the students to comment on this situation and to explain how they know this number is 100.

This activity may be repeated with the following configurations.



= 15



= 150

Some students will still need to make moves on the Minicomputer; others will be able to do the calculations mentally. Don't worry. Students will progress at different rates; avoid comparing one with the other.

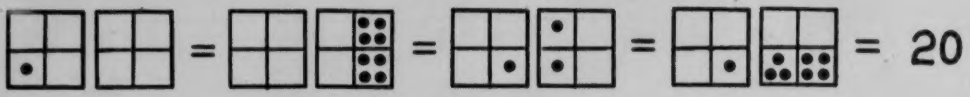
Exercise 3

Distribute individual Minicomputers and checkers. Each student will need two Minicomputer boards (one sheet) and at least 20 positive checkers.

T: There are many ways to put 20 on the Minicomputer.
Try to show some of them.

To be enjoyable, this exercise must move quickly. As students discover different configurations for 20 , let them put their solutions on the demonstration Minicomputer. Do not attempt to get all the possible solutions; there are too many of them.

Possible configurations include:



If there is time remaining, you could repeat this exercise with other numbers (e.g. 50 , 100 , 24 , 35 , 80 , and so on).

N8

N8 The Functions +5, -5, +2, -2

CAPSULE LESSON SUMMARY

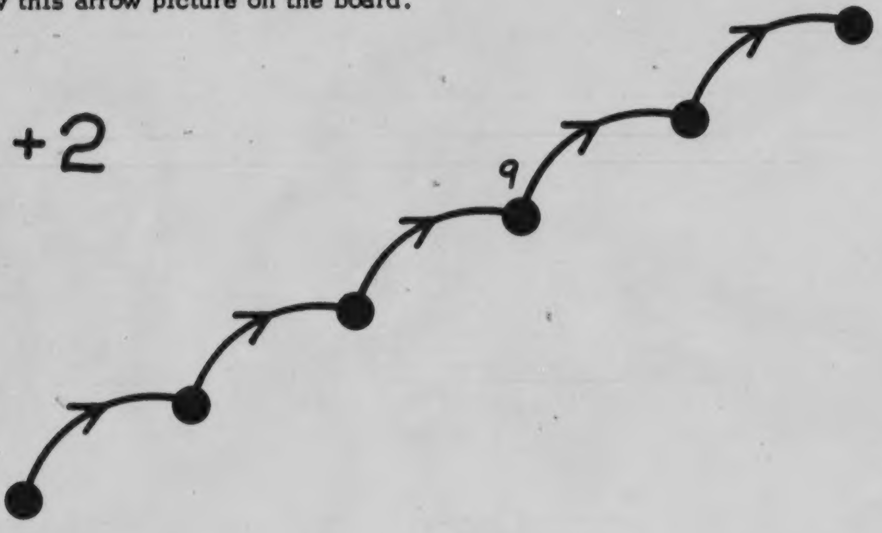
Label the dots and draw return arrows in a +2 arrow picture and in a -5 arrow picture.

MATERIALS

Teacher:	Colored chalk
Student:	Pencil; Worksheets N8*, **, ***, and ****

DESCRIPTION OF LESSON

Draw this arrow picture on the board.



T: Point to the largest number in this arrow picture. Tell me with whispers, what is the largest number. (13)

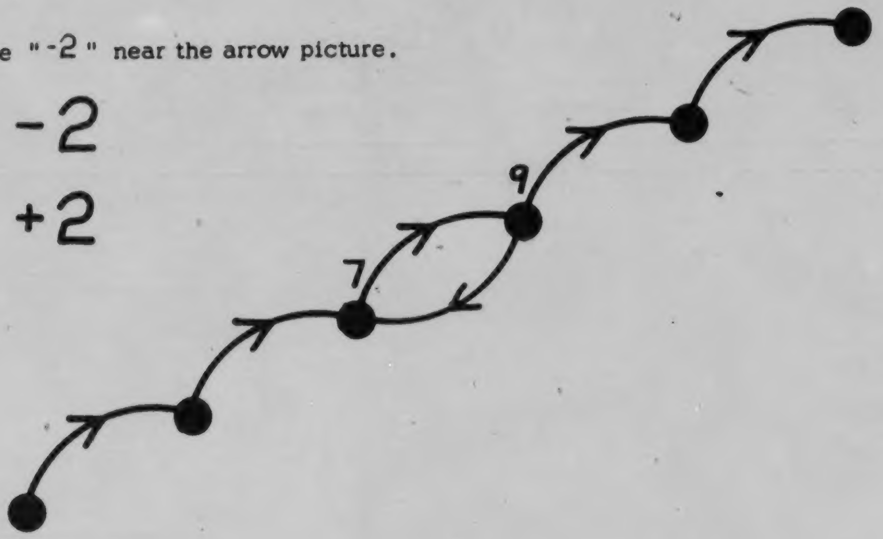
Point to the smallest number. Tell me with whispers, what is the smallest number. (3) What are the other numbers? (5, 7, 11)

With the help of several students, label all the dots. Each time a dot is labeled, ask how we know that dot is for the number suggested. For the dots to the left of the dot for 9 , encourage the students to think of subtraction calculations. For example, when someone suggests that the first dot to the left of 9 should be labeled " 7 " , direct the explanation to mention "9 - 2 = 7". Then ask for a volunteer to first trace and then draw (in blue) the -2 return arrow from 9 to 7.

T: What could the blue arrow be for?

S: -2.

Write "-2" near the arrow picture.



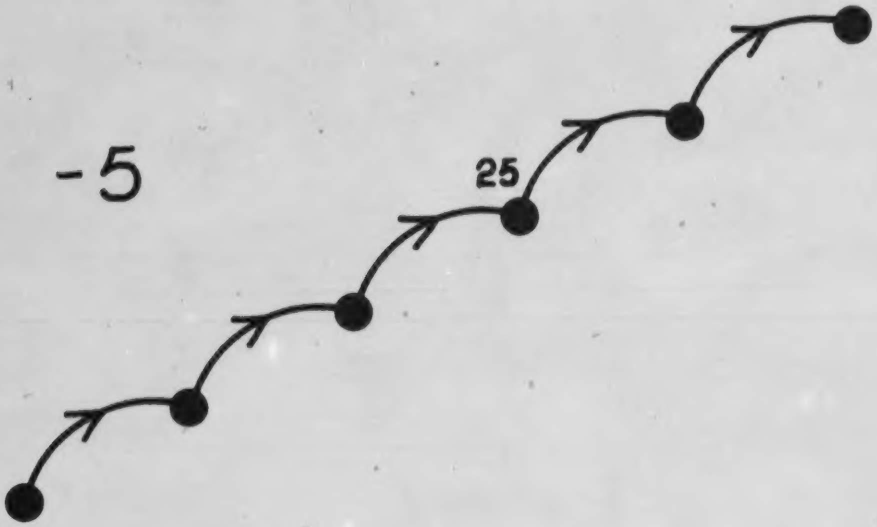
T: Do you see where we could draw another blue arrow?

Let the students suggest as many other blue arrows as they can. You need not insist they find them all immediately. They could continue to label other dots, and then you could return to the request for other blue arrows. Each time a student wants to show where a blue arrow can be drawn, ask the student to trace the arrow and then, if correct, to draw it in blue. This activity should proceed quickly.

N8

T: If I would go on drawing more red arrows (to the right), would we ever meet 20 ? (No)
Would we ever meet 21 ? (Yes) Would we meet 30 ? (No)
Would we meet 40 ? (No) Would we meet 45 ? (Yes)

Ask the students for an explanation for one or two of these questions, but be prepared to help if they have trouble verbalizing at this stage. Erase the board and draw this arrow picture.



T: Point to the largest number in this arrow picture. Tell me with whispers, what is the largest number. (40)
Point to the smallest number. Tell me with whispers, what is the smallest number. (15)

Point to the first dot to the left of 25.

T: What number is this? How do you know?

S: 30 , because 30 - 5 = 25 .

T: Do you know another way to see that it is 30 ?

S: 25 + 5 = 30 .

T: How can we see "25 + 5" in our picture?

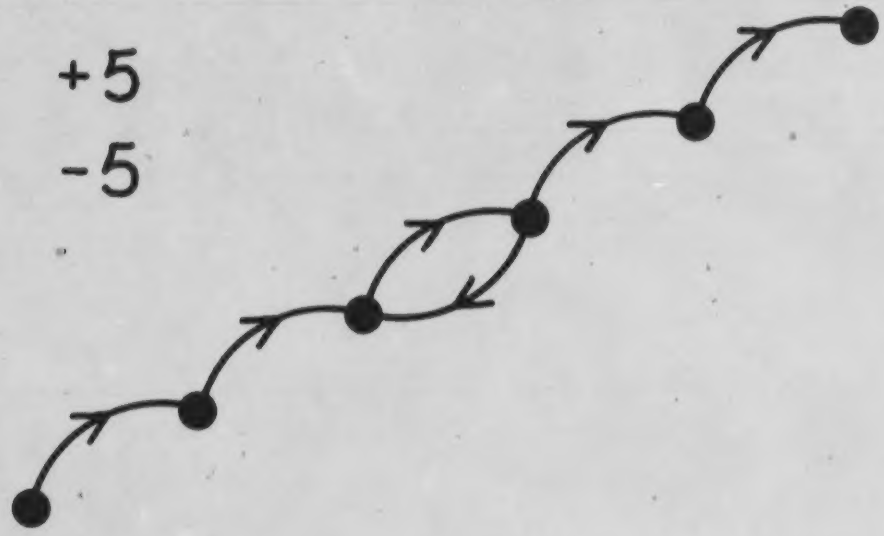
S: Draw a return arrow in another color.

Ask a volunteer to draw the return arrow in red.

T: What could this arrow be for?

S: +5 .

Write "+5" near the arrow picture.



T: Where else could we draw a red arrow?

If the students do not immediately suggest other red arrows, ask them first to label some of the other dots. Then ask them again if they see where more red arrows could be drawn. Complete the arrow picture quickly with the students.

N8

T: If we drew more red arrows (to the left), would we meet the number 100 ? (Yes) 200 ? (Yes) 500 ? (Yes) 101 ? (No)

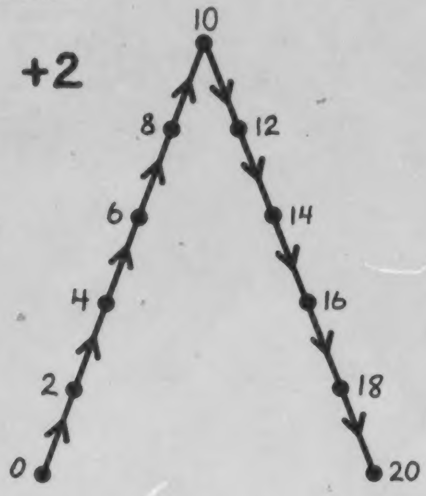
Ask the students for an explanation for one or two of these questions, but be prepared to help if they have trouble verbalizing at this stage.

Worksheets N8*, **, ***, and **** are available for independent work.

Name

N8 *

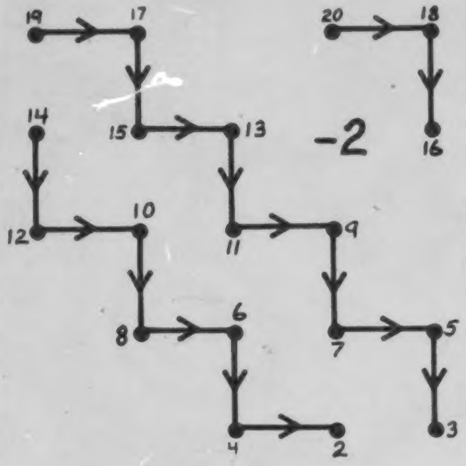
Label the dots.


+2
0
2
4
6
8
10
12
14
16
18
20

Name

N8 **

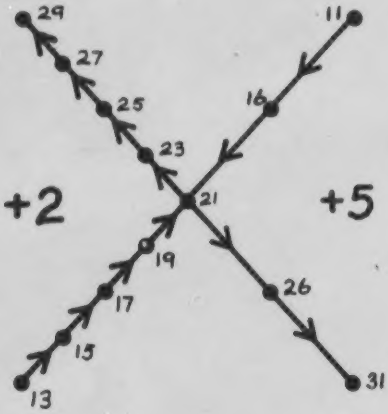
Label the dots.


-2
19
17
20
18
14
15
13
16
12
10
11
9
8
6
7
5
4
2
3

Name

N8 ***

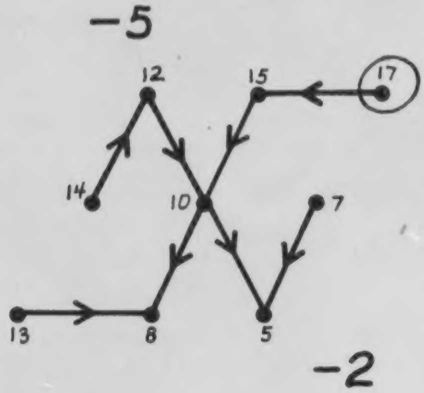
Label the dots.


+2
+5
29
27
25
23
21
19
17
15
13
11
16
26
31

Name

N8 ****

Find 17 in this arrow picture and circle it.


-5
-2
12
15
17
14
10
7
13
8
5

N9 Addition Problems #2

CAPSULE LESSON SUMMARY

Pair numbers whose sum is 20 to simplify an addition problem. Practice putting numbers on the Minicomputer with four boards displayed. Investigate the ways of putting 10 on the Minicomputer using from one to four checkers.

MATERIALS

Teacher:	Minicomputer set; colored chalk
Student:	Minicomputer set

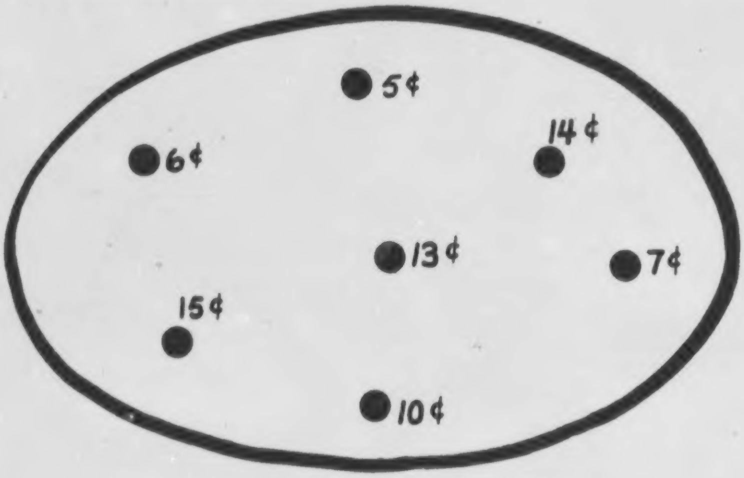
DESCRIPTION OF LESSON

Exercise 1: Addition Problem

Choose one of your students to be the star of this story.

T: Nancy went to the candy store to buy some candy. She bought seven pieces of candy. Inside this blue string are the prices of the candies.

Draw this string picture on the board.

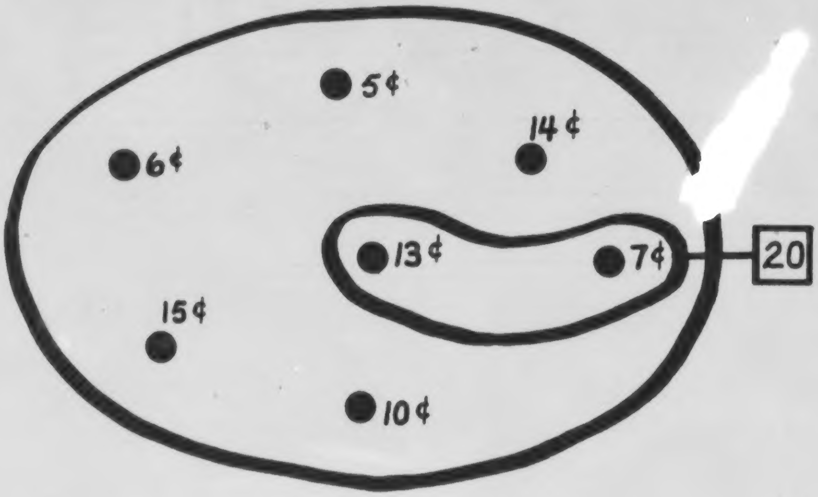


T: How much did Nancy spend?

Let the students react to this problem and let them suggest ways of solving it. Undoubtedly, some of them will suggest using the Minicomputer.

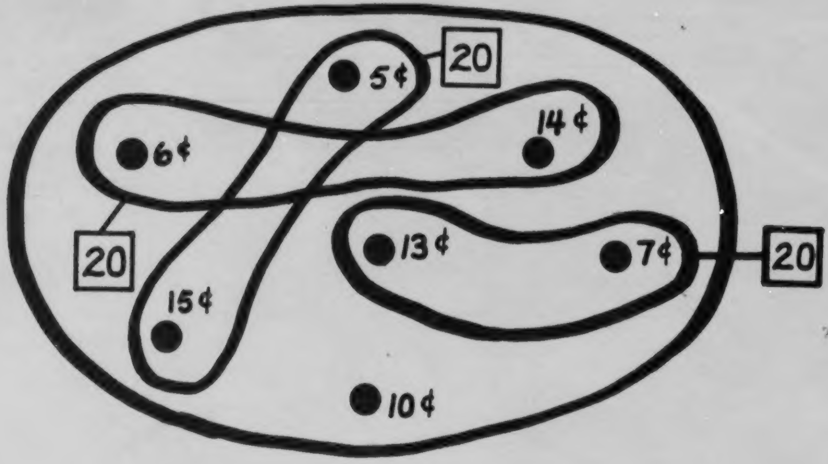
T: We could use the Minicomputer, but first let's try to solve the problem by pairing the numbers so they will be easier to add.

When a student observes, for example, that 13 + 7 = 20 , circle these two numbers.



If no one suggests pairing two numbers whose sum is 20 , comment that 20 is an easy number to add to other numbers and ask the class to find two numbers whose sum is 20 . Continue pairing numbers whose sum is 20 until your picture looks like the one in the next illustration.

N9



T: How can we use this string picture to find what Nancy spent?

S: Add: 20 + 20 + 20 + 10 .

Write " 20 + 20 + 20 + 10 = ? " on the board. Trace the appropriate string as you write each number.

T: How much did Nancy spend? How do you know?

S: 70¢ , because 20 + 20 + 20 + 10 = 70 .

Accept any correct methods and then check the calculation on the Minicomputer.

Exercise 2

Display four Minicomputer boards. Try to include all the students, especially the new students and the weaker students, in this activity.

T: Who can put 15 on the Minicomputer?
Who can write "15" below the Minicomputer?

Always remove the checkers and erase the board before continuing to another number.

T: Who can put 51 on the Minicomputer?
Who can write "51" below the Minicomputer?

Continue this activity with the following sequences of numbers or any similar ones appropriate to the numerical ability of your students.

150	28
105	82
510	280
501	802
1,050	2,800
5,001	8,020

If a student does not include a comma when writing numbers greater than 999, draw it yourself and explain that it helps us to read large numbers.

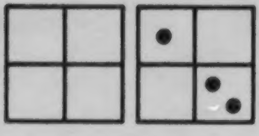
Exercise 3

Display two Minicomputer boards. Each student will need two Minicomputer boards (one individual Minicomputer sheet) and checkers.

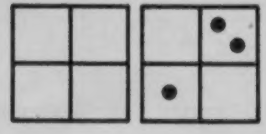
Ask the students to put 10 on their (desk) Minicomputers using exactly one checker. Invite a student to put 10 on the demonstration Minicomputer in the same way. (There is only one way to show 10 with exactly one positive checker.)

Then ask the students to put 10 on their Minicomputers with exactly two checkers. Invite a student to show the solution on the demonstration Minicomputer. (There is only one way to show 10 with exactly two positive checkers.)

Continue this activity with three checkers . . .

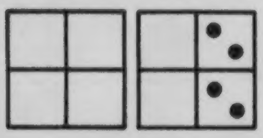


and

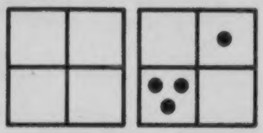


(There are two ways to show 10 with exactly four positive checkers.)

and with four checkers.



and



(There are two ways to show 10 with exactly three positive checkers.)

Do not insist that your class find both ways to show 10 with three or with four positive checkers. If some students have numbers other than 10, tell them what numbers they have and remind them that they are trying to show 10 .

Exercise 4

If your class does well with the previous exercise, you could ask the students to find many ways to put 40 on their Minicomputers. As students find solutions, invite them to show their solutions on the demonstration Minicomputer.

N10 Composition of Functions #1

CAPSULE LESSON SUMMARY

Using arrow pictures, investigate the composition of certain addition functions; for example, +2 followed by +3 and +3 followed by +7 .

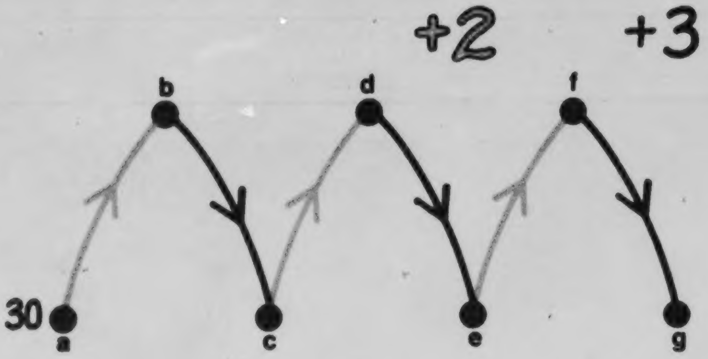
MATERIALS

Teacher: Colored chalk
Student: Colored pencils; pencil; Worksheets N10* and **

DESCRIPTION OF LESSON

Exercise 1

Draw this arrow picture on the board.



Do not write the letters on the board. They are here just to make the description of the lesson easier to follow.

T: Where is the smallest number in this arrow picture? (At a)
Where is the largest number in this arrow picture? (At g)
Tell me with whispers, what is the largest number in this picture. (45)

This last question may challenge even your brighter students. If no one gives the correct answer, wait until c has been labeled and ask them again to tell you with whispers what the largest number is.

T: Where could we draw a +5 arrow? (Write " +5 " near the arrow picture.)

Allow the students to react to this request, but do not allow any new dots to be added to the picture. As students suggest red arrows, let them trace the arrows on the board. Occasionally ask:

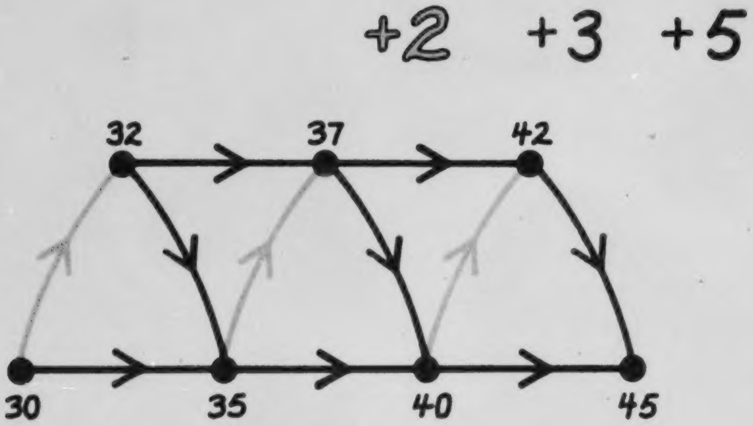
T: How do you know you can draw a red arrow there?

S: Here is 40 (pointing to e) and here is 45 (pointing to g), and 40 + 5 = 45 .

S: A +2 arrow followed by a +3 arrow is a +5 arrow.

Let students draw red arrows which they have traced correctly and label dots which they have referred to numerically.

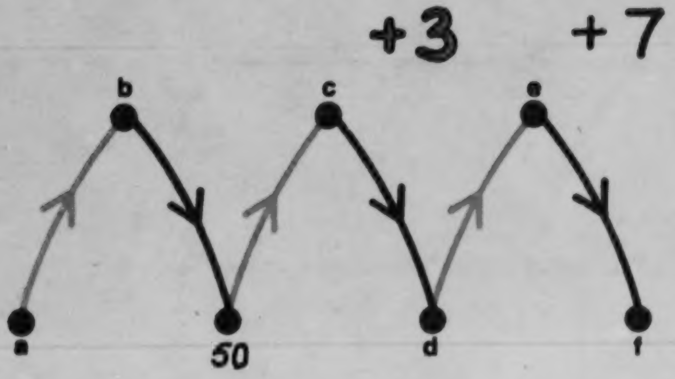
If no one suggests a correct red arrow, ask students to label the dots and then return to the request for red arrows. You might have to draw one of the red arrows yourself. All possible red arrows are shown in the picture below.



Erase the board before going on to Exercise 2 .

Exercise 2

Draw this arrow picture on the board.



Do not write the letters on the board. They are here just to make the description of the lesson easier to follow.

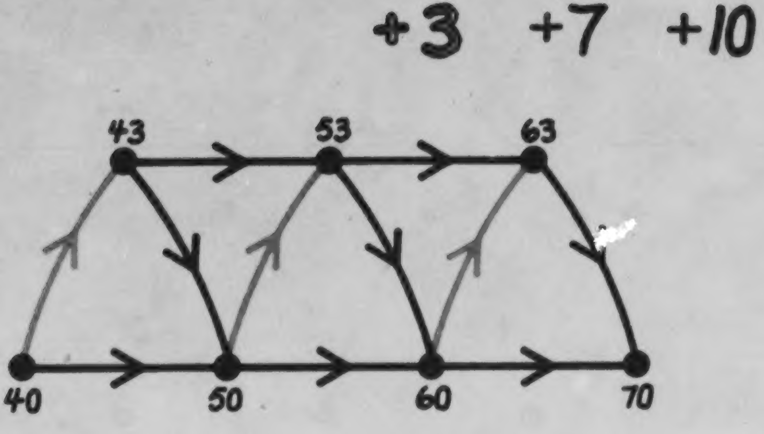
T: Where is the smallest number in this arrow picture? (At a)
Tell me with whispers, what is the smallest number. (40)
Where is the largest number in this arrow picture? (At f)
Tell me with whispers, what is the largest number. (70)

If you have received at least one correct answer to your last request, ask a student to answer aloud. Otherwise, wait until d has been labeled and ask the class again to tell you with whispers what the largest number is.

T: Where could we draw a +10 arrow? (Write " **+ 10** " on the board.)

Continue until all the red arrows in the picture are drawn, and then label the dots with the class. Make use of the red arrows to label dots whenever possible. If students have trouble finding some of the red arrows, suggest labeling the dots first. A completed picture is shown in the next illustration.

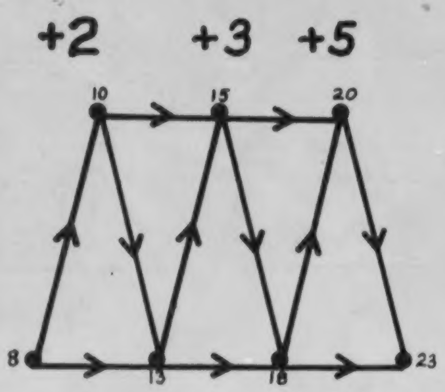
N10


+3
+7
+10
43
53
63
40
50
60
70

Worksheets N10* and ** are available for individual work.

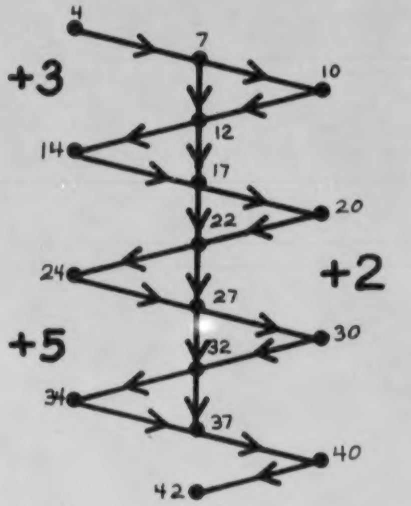
Name ______________________ **N 10** *

Label the dots. Draw all the +5 arrows.


+2
+3
+5
10
15
20
8
13
18
23

Name ______________________ **N10** **

Label the dots. Draw all the +5 arrows.


4
7
10
+3
12
14
17
20
22
24
+2
27
30
+5
32
34
37
40
42

N11

N11 Multiplication and Subtraction #1

CAPSULE LESSON SUMMARY

Tell number stories about a rectangular array of dots. In particular, note that 4 × 3 = 3 × 4 as indicated by the array. Subtract numbers less than 100 with the help of the Minicomputer.

MATERIALS

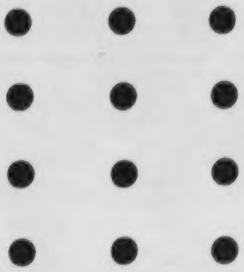
Teacher: Minicomputer set

Student: None

DESCRIPTION OF LESSON

Exercise 1: Multiplication Problems

Draw this pattern of dots on the board. Dots should be large and well-spaced.



T: Look at this set of dots. What do you see?

Let the students express themselves. Many reactions are possible. If a student says, for example, that there are 4 rows with 3 dots in each row, express this as:

$$3 + 3 + 3 + 3$$

or

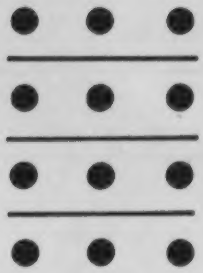
$$4 \times 3 .$$

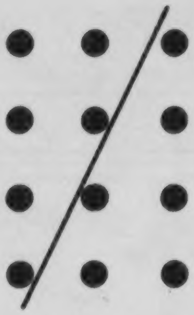
Other examples,

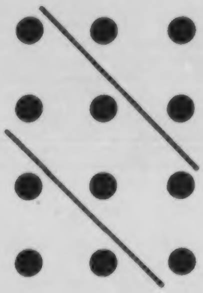
S: I see "4 + 4 + 4" (or "3 x 4").
There are 12 dots.
I count 6 + 6.
I count 1 + 2 + 3 + 3 + 2 + 1.

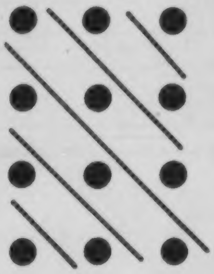
If your students do not give a wide variety of suggestions, you could prompt them by holding a ruler in different positions to partition the array of dots (but drawing no lines on the board).

For instance,

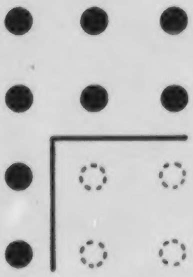


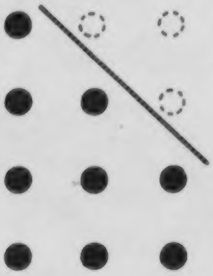


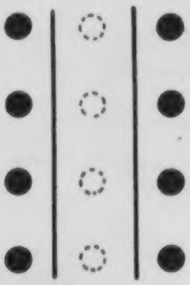


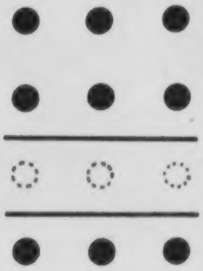


With a sheet of paper, hide some of the dots and ask the class how many of the dots are hidden. Fold the paper if necessary.









Continue presenting many similar situations and let the students suggest some other patterns.

Finally, emphasize the fact that these are two very special number stories related to this picture. Write the number stories on the board.

$$3 \times 4 = 12 \qquad 4 \times 3 = 12$$

Conclude:

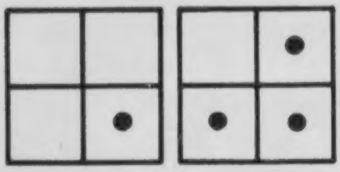
$$3 \times 4 = 4 \times 3$$

Exercise 2: Subtraction Problems

T: Marty has 17 candies. He eats 3 of them. How many are left?

Some students will be able to give an immediate answer. The Minicomputer will help others find the solution almost as quickly.

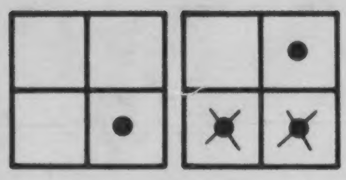
T: Can someone put 17 on the Minicomputer?



= 17

T:. Who can take 3 off the Minicomputer?

Let a student remove the appropriate checkers from the Minicomputer.



T: What number is on the Minicomputer? How many candies did Marty have left?

S: 14 .

Read the number story as you write it on the board.

$$17 - 3 = 14$$

In a similar way, the class can do these calculations by starting with 17 , 75 , and 367 , respectively, on the Minicomputer. For each calculation, no plays are needed. Write each calculation on the board as it is considered.

17 - 4
17 - 1
17 - 5
17 - 6
17 - 10
17 - 12
17 - 13
17 - 15

75 - 40
75 - 30
75 - 50
75 - 70
75 - 4
75 - 5
75 - 21
75 - 24
75 - 34
75 - 61

367 - 100
367 - 60
367 - 7
367 - 24
367 - 45
367 - 27
367 - 240
367 - 205
367 - 241
367 - 300
367 - 242
367 - 246

N12

N12 Introduction to Frames

CAPSULE LESSON SUMMARY

Fill in a box (frame) with a number to make an addition sentence true.

MATERIALS

Teacher: None

Student: Pencil; Worksheets N12*, **, ***, and ****

DESCRIPTION OF LESSON

NOTE: This is a lesson that can be taught with very little talking on your part. Try to let the problems speak for themselves.

Write this open sentence on the board.

$$4 + \square = 9$$

(It is read, "Four plus box equals nine," or "Four plus what number equals nine?")

T: What number goes in the box to make the sentence true?

S: 5.

Write "5" in the box.

$$4 + \boxed{5} = 9$$

Keep the first equation on the board and write this open sentence directly under it.

$$4 + \square = 11$$

T: What number goes in the box to make this sentence true?

S: 7.

Write "7" in the box.

Continue giving problems, each time writing the equations under each other until there is no more space on the board.

$$4 + \square = 9$$

$$4 + \square = 11$$

$$4 + \square = 13$$

$$4 + \square = 15$$

$$4 + \square = 17$$

T: Do you see a pattern?

S: 4 is always at the beginning.

S: Each time the number over there (to the right of the equal sign) is two bigger.

S: The number in the box is also two bigger each time.

N12

If no one volunteers any information, direct the students' attention to what part of the equation stays the same each time, what numbers get bigger, and how the numbers in the boxes are affected. (At some point, you may wish to ask students for a "non-solution", that is, a number that will make the sentence false. This encourages some students to come up with all kinds of interesting "wrong answers" such as 576; or 1,000; or even 1,000,001. Do not spend too long soliciting non-solutions, however, for the main idea here is to find patterns and solutions.)

Repeat this activity using these sequences of problems.

$10 = 3 + \square$

$10 = 4 + \square$

$10 = 5 + \square$

$10 = 6 + \square$

$10 + \square = 15$

$10 + \square = 20$

$10 + \square = 25$

$10 + \square = 30$

$50 + 50 = \triangle$*

$50 + \triangle = 101$

$50 + \triangle = 105$

$50 + 57 = \triangle$

$50 + \triangle = 109$

NOTE: You should vary the difficulty of the problems to fit the abilities of your students. But whatever sequences you make up, encourage students to begin to seek answers to a problem by using information from the previous problem.

For example, if students know that . . .

$$25 + 25 = \boxed{50}$$

---

*The shape of the frame you use (e.g. $\square$, $\triangle$, ⬡) is completely arbitrary. It is wise, however, to avoid circles (○) because they can easily be confused with the number zero.

they can figure out that . . .

$$25 + 26 = \boxed{51}$$

$$25 + 27 = \boxed{52}$$

and so on.

Worksheets N12*, **, ***, and **** are available for independent work.

As a follow-up activity, the students could make up problems of their own using just one frame.

Name ______________ **N 12 ***

Complete these number stories.

$5 + 5 = \boxed{10}$
$5 + 6 = \boxed{11}$
$5 + 7 = \boxed{12}$
$5 + 8 = \boxed{13}$
$5 + \boxed{9} = 14$

$10 + 3 = \boxed{13}$
$10 + 4 = \boxed{14}$
$10 + 5 = \boxed{15}$
$10 + 6 = \boxed{16}$
$10 + \boxed{7} = 17$

$16 + \boxed{1} = 17$
$16 + \boxed{2} = 18$
$16 + \boxed{3} = 19$
$16 + \boxed{4} = 20$
$16 + 5 = \boxed{21}$

$8 + \boxed{2} = 10$
$8 + \boxed{4} = 12$
$8 + \boxed{6} = 14$
$8 + 8 = \boxed{16}$
$8 + 10 = \boxed{18}$

Name ______________ **N 12 ****

Complete these number stories.

$6 + 6 = \boxed{12}$
$7 + 7 = \boxed{14}$
$8 + 8 = \boxed{16}$
$9 + 9 = \boxed{18}$
$10 + \boxed{10} = 20$

$14 - 4 = \boxed{10}$
$13 - 3 = \boxed{10}$
$12 - \boxed{2} = 10$
$11 - \boxed{1} = 10$
$\boxed{10} - 0 = 10$

$10 + 5 = \boxed{15}$
$20 + 5 = \boxed{25}$
$30 + \boxed{5} = 35$
$40 + \boxed{5} = 45$
$\boxed{50} + 5 = 55$

$\boxed{10} = 5 + 5$
$\boxed{10} = 6 + 4$
$\boxed{10} = 7 + 3$
$10 = 8 + \boxed{2}$
$10 = \boxed{9} + 1$

Name ______________ **N 12 *****

Complete these number stories.

$5 + 5 + 5 = \boxed{15}$
$5 + 5 + 6 = \boxed{16}$
$5 + 5 + \boxed{7} = 17$
$5 + 5 + \boxed{8} = 18$
$5 + 5 + 9 = \boxed{19}$

$7 - 3 = \boxed{4}$
$7 - 4 = \boxed{3}$
$7 - 5 = \boxed{2}$
$7 - \boxed{6} = 1$
$7 - \boxed{7} = 0$

$\boxed{100} = 50 + 50$
$\boxed{102} = 51 + 51$
$104 = \boxed{52} + 52$
$106 = \boxed{53} + 53$
$108 = 54 + \boxed{54}$

$23 + 10 = \boxed{33}$
$23 + 20 = \boxed{43}$
$23 + 30 = \boxed{53}$
$23 + \boxed{40} = 63$
$23 + \boxed{50} = 73$

Name ______________ **N 12 ******

Complete these number stories.

$100 + \boxed{100} = 200$
$101 + 101 = \boxed{202}$
$102 + 102 = \boxed{204}$
$103 + \boxed{103} = 206$
$\boxed{104} + 104 = 208$

$50 + \boxed{50} = 100$
$49 + 49 = \boxed{98}$
$48 + 48 = \boxed{96}$
$\boxed{47} + 47 = 94$
$\boxed{46} + 46 = 92$

$30 + 30 = \boxed{60}$
$31 + 29 = \boxed{60}$
$32 + 28 = \boxed{60}$
$33 + 27 = \boxed{60}$
$34 + 26 = \boxed{60}$

$5 + \boxed{5} = 10$
$15 + 15 = \boxed{30}$
$25 + \boxed{25} = 50$
$35 + 35 = \boxed{70}$
$45 + 45 = \boxed{90}$

N13 Multiplication and Subtraction #2

CAPSULE LESSON SUMMARY

Draw a picture with 5 × 3 dots and one with 3 × 5 dots. Conclude that 5 × 3 = 3 × 5. Do some subtraction problems by using backward plays on the Minicomputer.

MATERIALS

Teacher:	Minicomputer set
Student:	Minicomputer set

DESCRIPTION OF LESSON

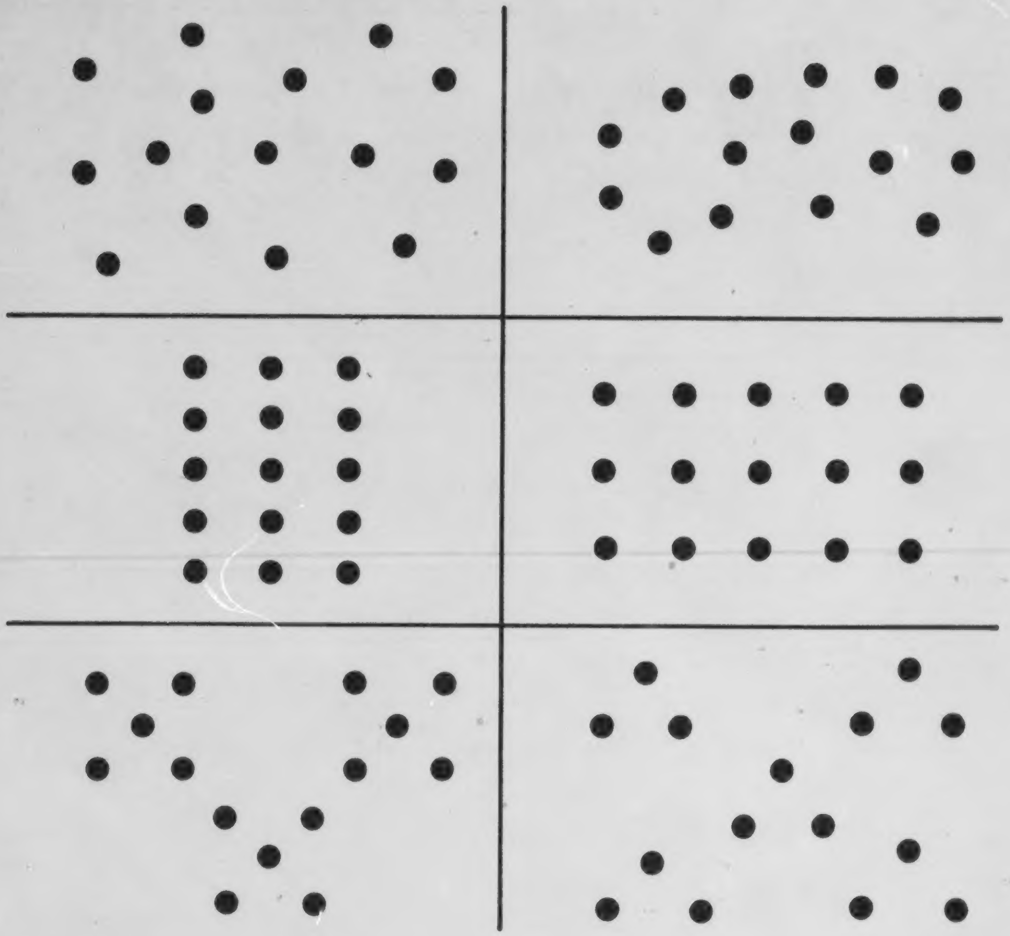
Exercise 1: Multiplication Problems

Ask the students to draw a picture with 5 × 3 dots and a picture with 3 × 5 dots. Let them solve this problem in their own ways, watch how the pictures are drawn; and ask some students to draw one of their pictures on the board. There should be at least one picture organized in rows and columns (5 rows and 3 columns, or 3 rows and 5 columns) put on the board.

If some of your students do not understand what "3 × 5" means, ask someone to explain. You may want to write on the board "3 × 5 = 5 + 5 + 5" and "5 × 3 = 3 + 3 + 3 + 3 + 3". Do not expect all of your students to have this understanding of multiplication. There will be more lessons on multiplication later.

N13

These are some examples of what students might draw.



With your class, count the dots in each of the pictures drawn on the board. Wherever possible, count by fives or by threes.

Ask the students which pictures they like best and why.

Do not criticize the pictures in which the dots are not drawn in rows. They might be interesting from an aesthetic point of view. Be happy if there are 15 dots!

T: How many dots are in your picture with 5 × 3 dots? (15)
How many dots are in your picture with 3 × 5 dots? (15)
Are there any pictures on the board where we can easily see that 5 × 3 = 3 × 5 without counting all the dots?

Any picture which is organized in rows and columns (5 rows and 3 columns, or 3 rows and 5 columns) illustrates nicely that 5 × 3 = 3 × 5, but actually any picture with 15 dots can be used to show this.

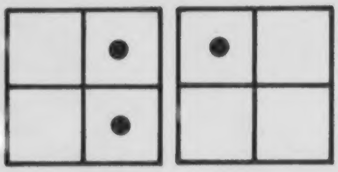
Exercise 2: Subtraction Problems

Display two Minicomputer boards.

T: Nick has 58 marbles. He gave 10 of them to his friend Max. How many marbles are left?

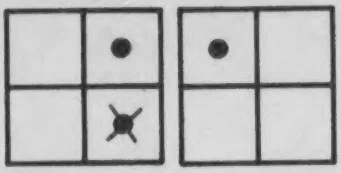
Some students will be able to give an answer immediately. The Minicomputer will help others find the solution almost as quickly.

T: Can someone put 58 on the Minicomputer?



T: Who can take 10 off the Minicomputer?

Let a student remove the appropriate checker from the Minicomputer.



T: How many marbles are left?

Read the number story as you write it on the board.

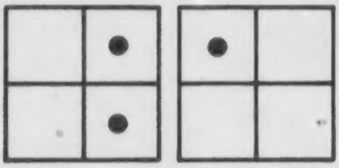
$$58 - 10 = 48$$

T: But what if Nick had given Max 20 marbles . . . ?

Write this number story on the board directly under "58 - 10 = 48".

$$58 - 20 = ?$$

Invite someone to put 58 on the Minicomputer.

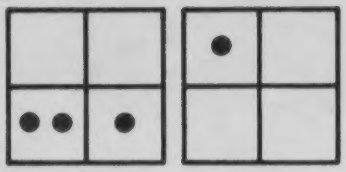


T: Can someone take 20 off the Minicomputer?

S: We do not have a checker on the 20-square.

T: Can we make a play so that we will have at least one checker on the 20-square?

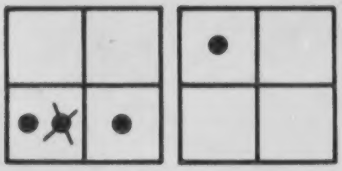
S: Yes, make a backward play.



S: 40 = 20 + 20 .

T: Now it is easy to take 20 off the Minicomputer.

Take a checker off the 20 -square.



T: What number is left on the Minicomputer?

S: 38 .

Complete the number story on the board.

T: Let's suppose Max had needed 24 marbles and that Nick had agreed to give him that many. How many marbles would Nick have had left?

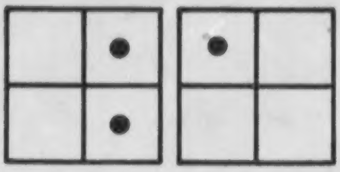
Write this number story on the board under " 58 - 20 = 38 " .

58 - 24 = ?

160

N13

Invite someone to put 58 on the Minicomputer.

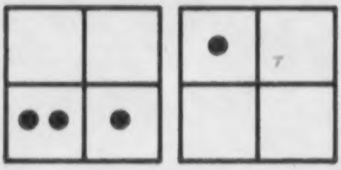


T: Can someone take 24 off the Minicomputer?

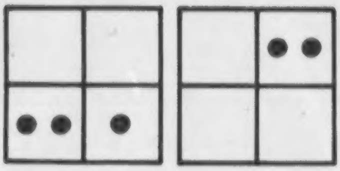
S: We do not have a checker on the 20-square or on the 4-square.

S: We could make some backward plays.

Invite students to make the required plays.



S: 80 = 40 + 40 .



S: 8 = 4 + 4 .

T: Now it is easy to take 24 off the Minicomputer.

Take the appropriate checkers off the Minicomputer.

T: What number did we get?

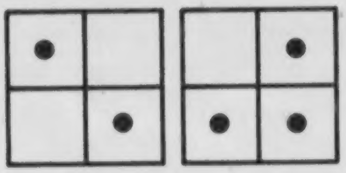
S: 34.

Complete the number story on the board. You will have this list.

58 - 10 = 48
58 - 20 = 38
58 - 24 = 34

For the rest of this exercise, each student will need two individual Minicomputer boards and several positive checkers.

T: Put 97 on your (desk) Minicomputer.



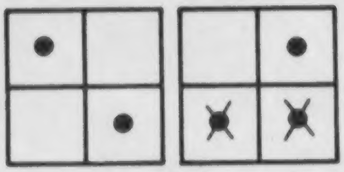
Invite someone to put 97 on the demonstration Minicomputer.

Write this number story on the board near the Minicomputer.

97 - 3 = ?

T: Calculate "97 - 3" on your Minicomputer.

Ask a student to do the calculation on the demonstration Minicomputer.



Conclude "97 - 3 = 94" and complete the number story on the board.

T: You now have 94 on your Minicomputer.

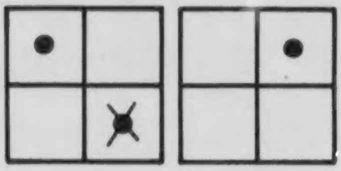
Occasionally check to make sure that the students have the correct number on their Minicomputers.

Write this number story on the board directly under "97 - 3 = 94".

$$94 - 10 = ?$$

T: Calculate "94 - 10" on your Minicomputer.

Ask for a volunteer to do the subtraction problem on the demonstration Minicomputer.



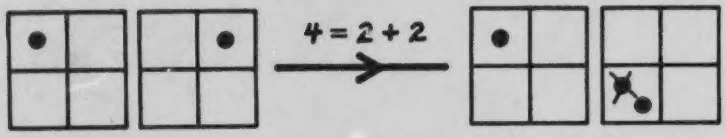
Conclude "94 - 10 = 84" and complete the number story on the board.

T: You now have 84 on your Minicomputer.

Write this number story on the board under "94 - 10 = 84".

$$84 - 2 = ?$$

Ask the students to calculate "84 - 2". A backward play is required before the checker can be removed from the 2-square.



Conclude " 84 - 2 = 82 " and complete the number story on the board. You will have this list.

97 − 3 = 94
94 − 10 = 84
84 − 2 = 82

Continue in the same manner with one or more of the following sequences of subtraction calculations.

22 − 10 = 12
12 − 1 = 11
11 − 11 = 0

82 − 40 = 42
42 − 20 = 22
22 − 10 = 12

63 − 22 = 41
41 − 20 = 21
21 − 11 = 10

59 − 48 = 11
11 − 8 = 3

161

N14

N14 Roads #1

CAPSULE SUMMARY LESSON

Build a road from one number to another using two types of arrows.

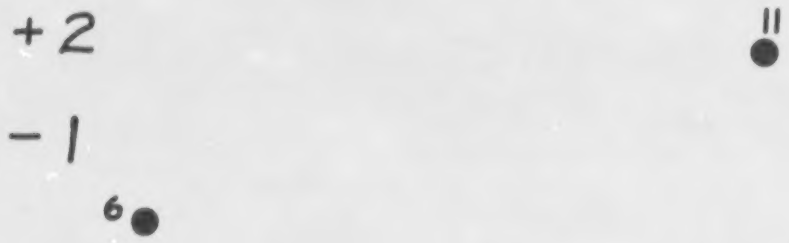
MATERIALS

Teacher: Colored chalk

Student: Colored pencils; pencil; unlined paper

DESCRIPTION OF LESSON

Draw two dots well spaced on the board. Label one of the dots "6" and the other "11".

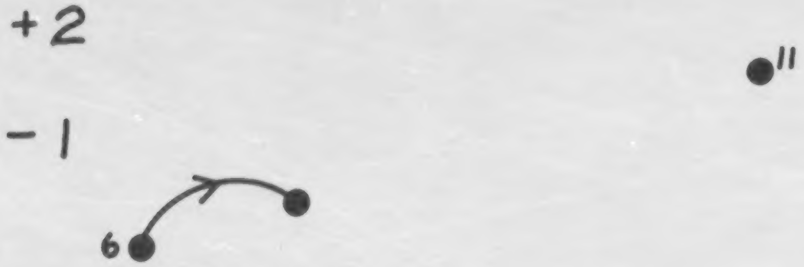


T: Today we are going to build roads. These roads are very special because they are made up of arrows. Let's begin by building a road from 6 to 11. The blue arrows are for +2 and the red arrows are for -1.

You might refer to "+2" and "-1" as the "key" or "color code" for this road.

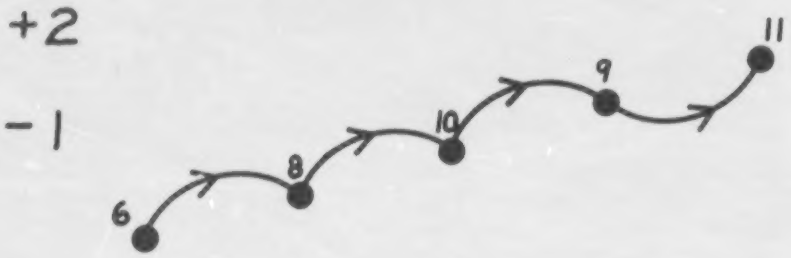
T: Which kind of arrow would you like to start with?

Take whatever suggestion is made (e.g. blue) and draw the first arrow yourself. Point to the ending dot of this arrow.



T: Which number is here? (8 , if the arrow is blue ; 5 , if the arrow is red)

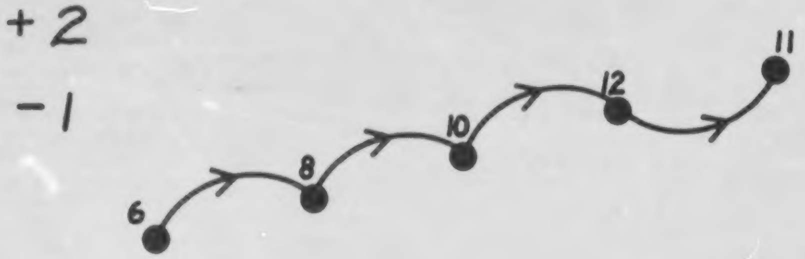
Ask students to complete the road one arrow at a time. One of the many possible roads is shown below.



NOTE: If a student is concentrating on a color pattern for the arrows, you might encourage them to keep in mind what the target number is. In this particular example, alternating +2 and -1 arrows will build a road from 6 to 11 using ten arrows, five blue and five red. There is nothing wrong with such a road, but you might ask if it is possible to build a road using fewer arrows. In this case, there is a shorter road as illustrated above.

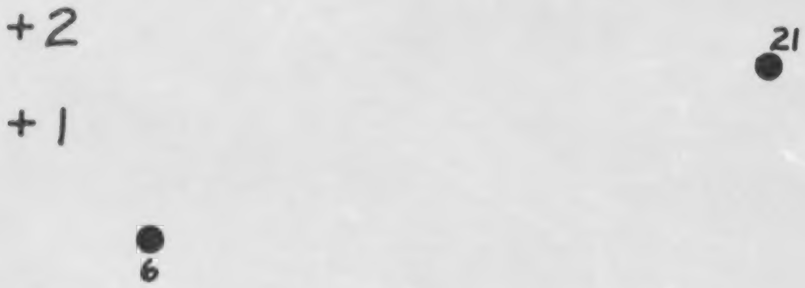
N14

There is no rule against "overshooting" the larger number. For example, here is a perfectly acceptable road.



Construct two or three different roads from 6 to 11 with your class. The purpose of this first collective exercise is to show what a road is and that many different solutions are possible.

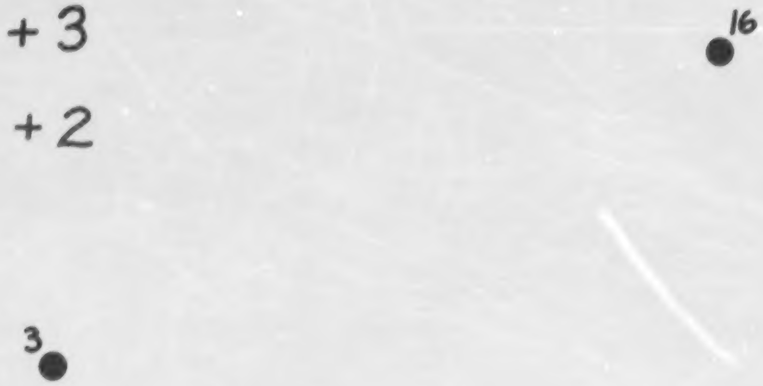
Distribute paper and colored pencils and put the following information on the board.



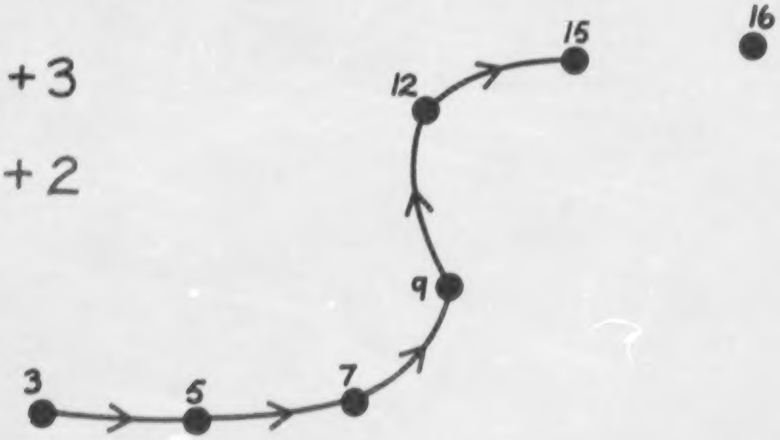
Ask the students to copy what is on the board and to build a road from 6 to 21 using +2 and +1 arrows. Walk around the room as the students work, giving assistance to those who need it. Students who draw cramped pictures should be encouraged to use the whole sheet of paper. As the rest of the class continues to work, ask a couple of students with different numbers of arrows in their roads to copy their pictures on the board. When most of the students have finished, direct the class' attention to the examples on the board. Check each picture on the board with the class to see that it is correct. Point out that there are many correct solutions to this problem.

Afterward, ask which road on the board has the fewest arrows and determine if anyone made a road with even fewer arrows.

Repeat this activity building a road from 3 to 16 using +3 and +2 arrows.

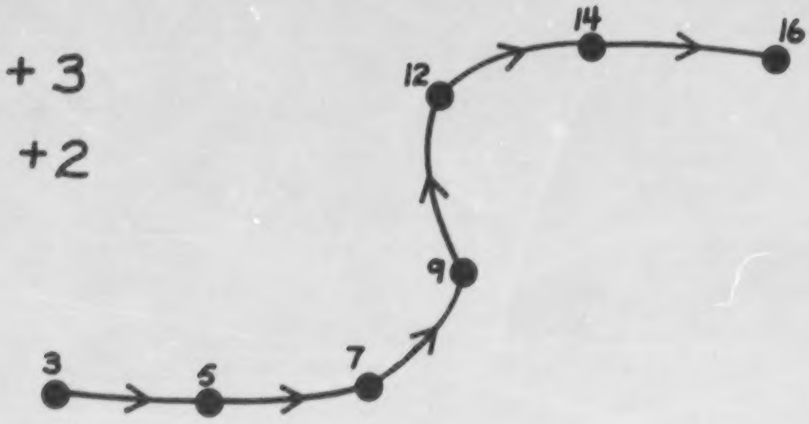


NOTE: A student might draw this road and discover that one can not reach 16 from 15 using only +3 and +2 arrows.



In such a case, you should suggest changing the color of the last arrow. In the preceding picture, changing the arrow starting at 12 from red to blue would enable the student to complete a road with one additional blue arrow.

N14



If there is time remaining, the students could build one or more of the following roads.

1) Build a red-blue road from 17 to 30
using +1 and +2 arrows.

2) Build a red-blue road from 8 to 19
using +3 and −2 arrows.

3) Build a red-blue road from 10 back
to 10 using +3 and −2 arrows.

4) Build a red-blue road from 27 back
to 27 using +3 and −2 arrows.

N15 Addition Problems #3

CAPSULE LESSON SUMMARY

Pair numbers whose sum is 20 to simplify an addition problem. Investigate ways of putting 12 and 16 on the Minicomputer using from two to five checkers.

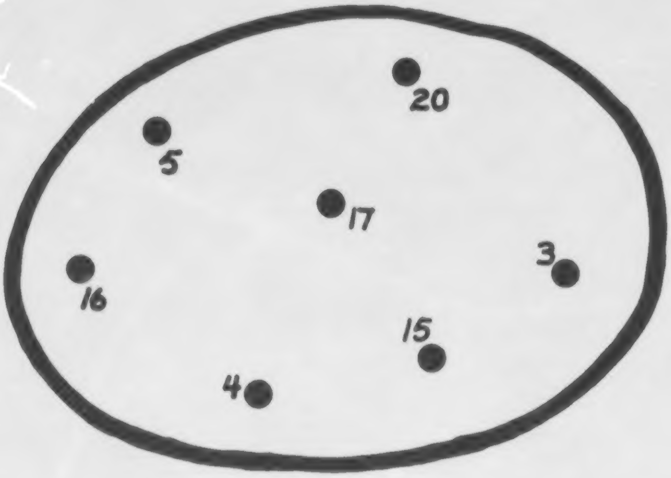
MATERIALS

Teacher:	Colored chalk; Minicomputer set
Student:	Minicomputer set

DESCRIPTION OF LESSON

Exercise 1: Addition Problem

Draw this string picture on the board leaving some space above it.



Tell this story or a similar one. Choose one of your students to be the star of this story.

T: Nicole went to the toy store and bought a lot of little toys. Each of these dots represents the price (in cents) of one toy. How many toys did Nicole buy? (7) How much did Nicole spend?

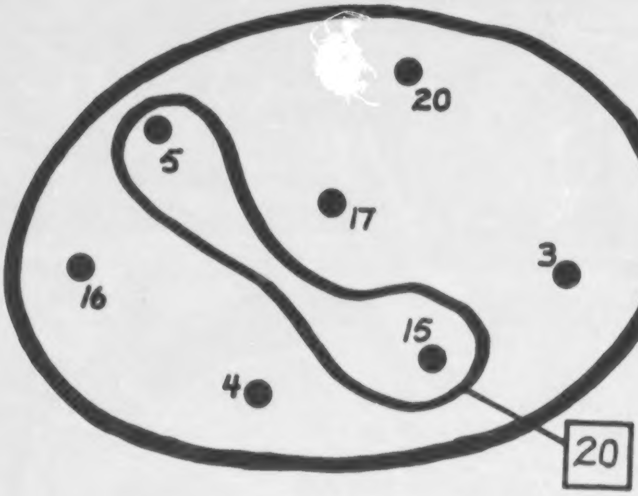
170

N 15

Let the students react to this problem and suggest ways of solving it. Do not expect anyone to solve it completely now. The students might suggest using the Minicomputer.

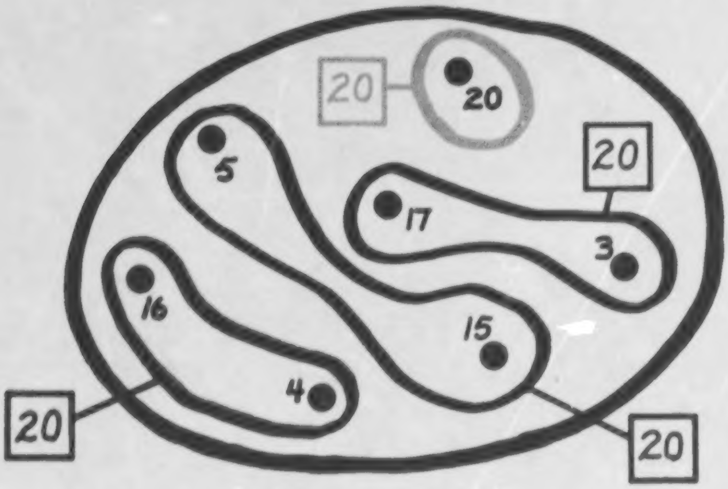
T: Let's try to solve this problem by pairing the numbers so that they will be easier to add.

If a student observes, for example, that 15 + 5 = 20 , circle those two numbers.



If no one suggest pairing two numbers whose sum is 20 , comment that 20 is an easy number to add to other numbers and ask the class to find two numbers whose sum is 20 . Accept all the pairs whose sum is 20 and draw a string around them as they are offered. Ask your class if 20 needs to be paired with another number.

Continue until you have this string picture.



T: How can we use this string picture to find what Nicole spent?

S: Add: 20 + 20 + 20 + 20 .

Above the string picture, write "20 + 20 + 20 + 20 = ? ". Point to the appropriate string as you write each number on the board.

T: How much money did Nicole spend? How do you know?

S: 80¢ , because 20 + 20 + 20 + 20 = 80 .

Accept any correct methods. Then check the calculation "20 + 20 + 20 + 20" on the Minicomputer.

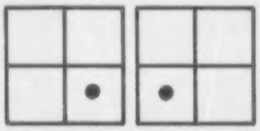
N15

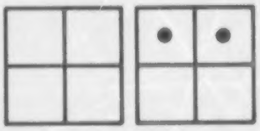
Exercise 2

Distribute two individual Minicomputer boards and some checkers to each student. Display two Minicomputer boards. Ask the students to put 15 on their (desk) Minicomputers. When most of the students have finished, ask someone to put 15 on the demonstration Minicomputer.

Repeat this activity with the number 51 .

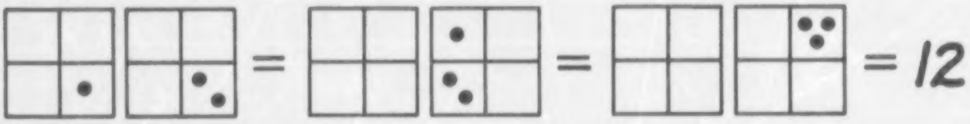
Ask the students to take out two checkers. Tell them to put 12 on their Minicomputers using exactly two checkers. Let students put 12 on the demonstration Minicomputer and ask them to explain their solutions. There are two ways to show 12 with exactly two positive checkers.



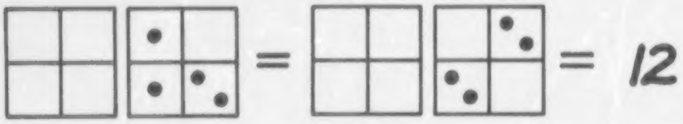


Continue this activity showing 12 on the Minicomputer with three checkers, with four checkers, and with five checkers. Solutions for each of these cases are illustrated below. Do not expect your class to find more than one solution for each case.

Three checkers:

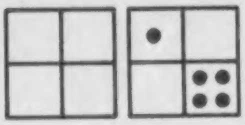
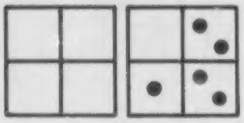
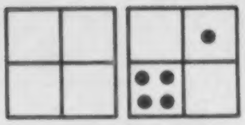


Four checkers:



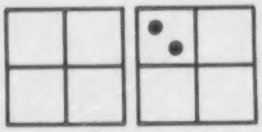
N 15

Five checkers :

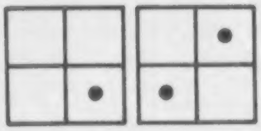
 =  =  = 12

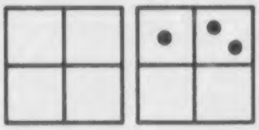
Repeat this activity with the number 16. Solutions are shown below.

Two checkers :

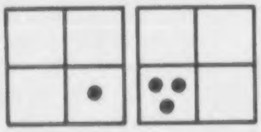


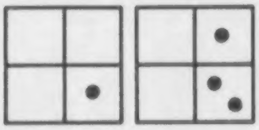
Three checkers :

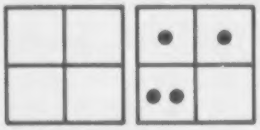


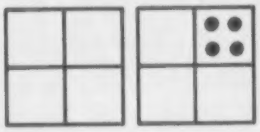


Four checkers :

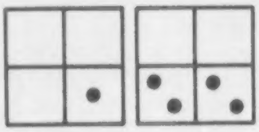


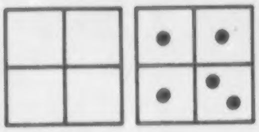


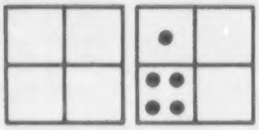


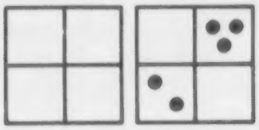


Five checkers :









N16

N16 The Function 2x

CAPSULE LESSON SUMMARY

Ask: How many hands would be raised if everyone in the class puts up one hand? Two hands? Continue with some 2x mental arithmetic and arrow pictures. Note that the only dot with a 2x loop is the number 0.

MATERIALS

Teacher: Colored chalk
Student: Pencil; Worksheet N16*, **, ***, and ****

Exercise 1

The following dialogue assumes there are 23 students present. For your lesson, use the number of students present in your class.

T: Everyone, raise one hand. How many hands are raised?

S: 23.

T: Now, everyone, raise both hands. How many hands are up now? How do you know?

S: 46, because 23 + 23 = 46.

T: 23 + 23 = 46. That is the same as saying that 2 x 23 = 46.

Continue with a mental arithmetic activity involving 2x. On the next page there are some suggested sequences of problems. Write the calculations on the board as you say them and ask for patterns.

175

N 16

2 × 5
2 × 10
2 × 20
2 × 40
2 × 80

2 × 3
2 × 30
2 × 300
2 × 3,000
2 × 3,000,000

2 × 2
2 × 20
2 × 22
2 × 200
2 × 202
2 × 220
2 × 222

2 × 6
2 × 10
2 × 16
2 × 20
2 × 26
2 × 30
2 × 36

Comments:

1. If students have difficulty with "2 × 20", ask for "2 × 2" and then return to "2 × 20". Repeat this strategy as often as necessary.

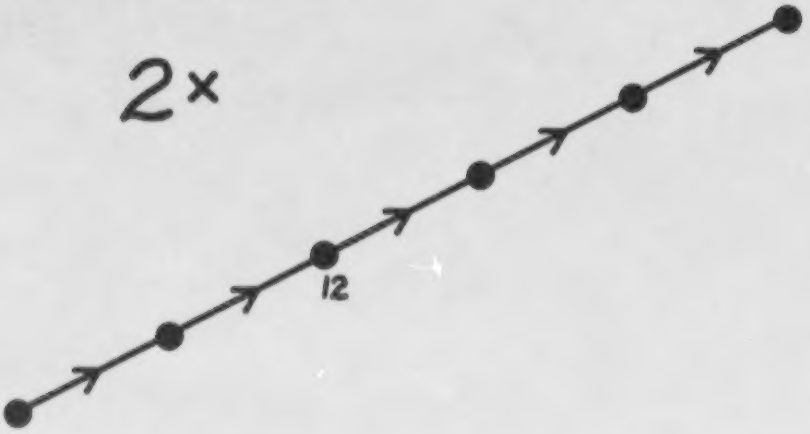
2. If students have difficulty with "2 × 16", ask for "2 × 10" and "2 × 6", and then return to "2 × 16". Repeat this strategy as often as necessary.

3. Do not write the calculations "2 × 3,000,000" on the board. After someone has calculated "2 × 3,000,000" correctly, ask if anyone can write "6,000,000" on the board. Be prepared to help or to write "6,000,000" yourself.

N16

Exercise 2

Draw this arrow picture on the board.



T: What is the red arrow for?

S: 2x .

T: Where is the smallest number? Which number is it?

S (pointing to the lower left dot): 3 .

T: Where is the largest number?
Tell me with whispers, what is the largest number.

S: 96 .

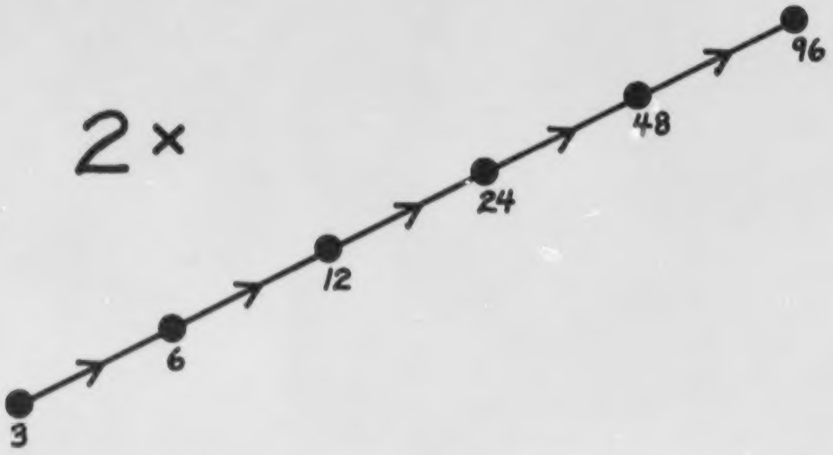
If no one knows what the largest number is at this time, do not tell them. Continue with the lesson, and when the dot for 48 has been labeled, take whispers again for what the largest number is.

177

T: How can we find out what the largest number is?

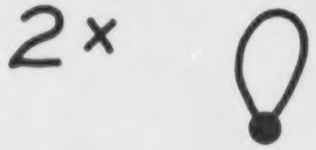
S: Label the dots.

Invite students to label the dots. Occasionally ask them how they know a dot is for a certain number. Continue until all the dots have been labeled.



Exercise 3

Draw this loop on the board.



T: Which number is here (point to the dot)?

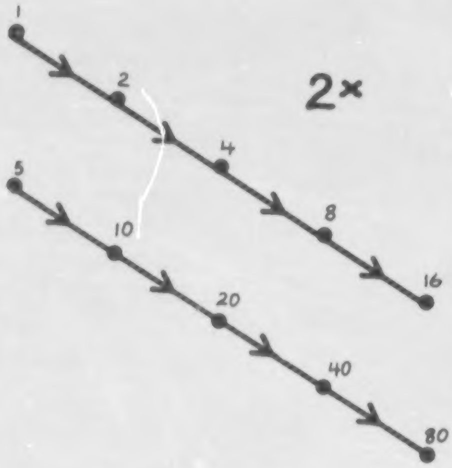
N16

Give the students time to think about this situation. If no one discovers that the number is 0, leave the arrow picture on the board while the students do the worksheets.

Worksheets N16*, **, ***, and **** are available for independent work.

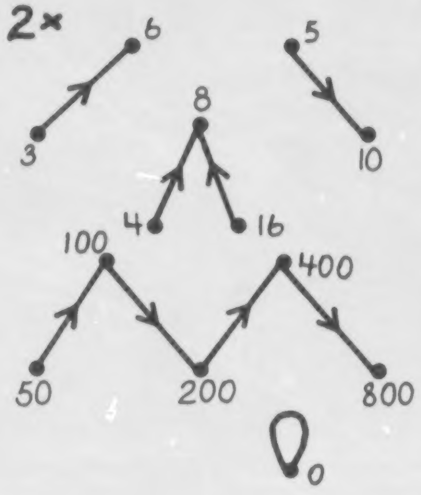
Name ______________ **N16 ***

Label the dots.


1
2
4
8
16
5
10
20
40
80
2×

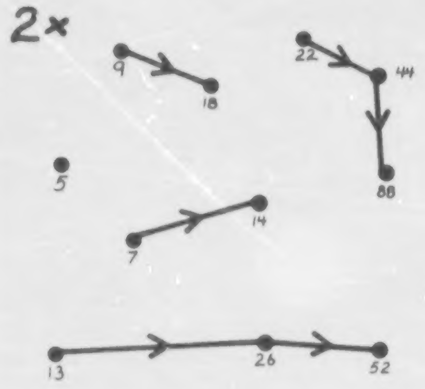
Name ______________ **N16 ****

Label the dots.


2×
3
6
5
10
8
4
16
100
400
50
200
800
0

Name ______________ **N16 *****

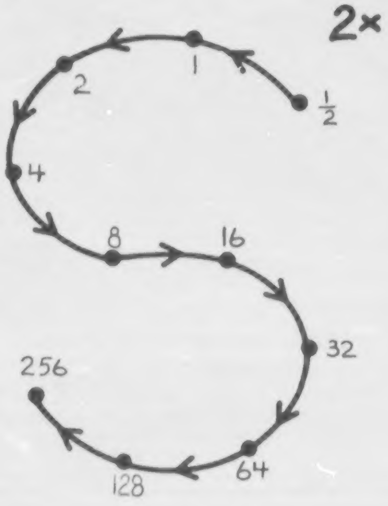
Draw all the 2x arrows.


2×
9
18
22
44
88
5
7
14
13
26
52

You should have six arrows.

Name ______________ **N16 ******

Label the dots.


2×
$\frac{1}{2}$
1
2
4
8
16
32
64
128
256

N17

N17 Addition Problems #4

CAPSULE LESSON SUMMARY

Pair numbers whose sum is 20 to solve an addition problem. Investigate the effect on the value of a configuration when each checker in the configuration is moved from one board of the Minicomputer to the same color square on the next board to its left.

MATERIALS

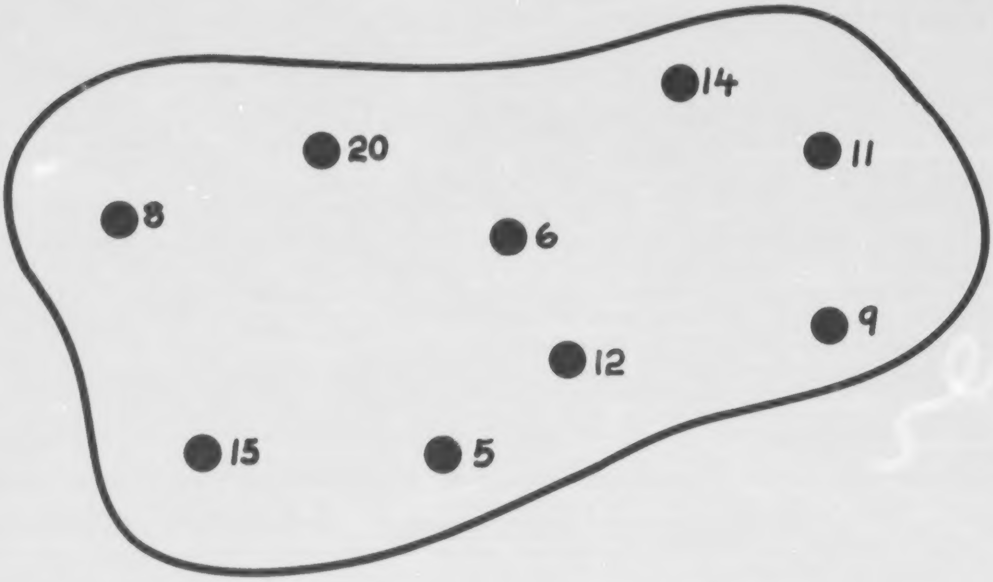
Teacher: Minicomputer set; colored chalk

Student: Pencil; Worksheets N17*, **, ***, and ****

DESCRIPTION OF LESSON

Exercise 1: Addition Problem

Tell this story or a similar one. Choose one of your students to be the star of the story. Draw this string picture on the board, leaving some space above it to write number stories about the picture.



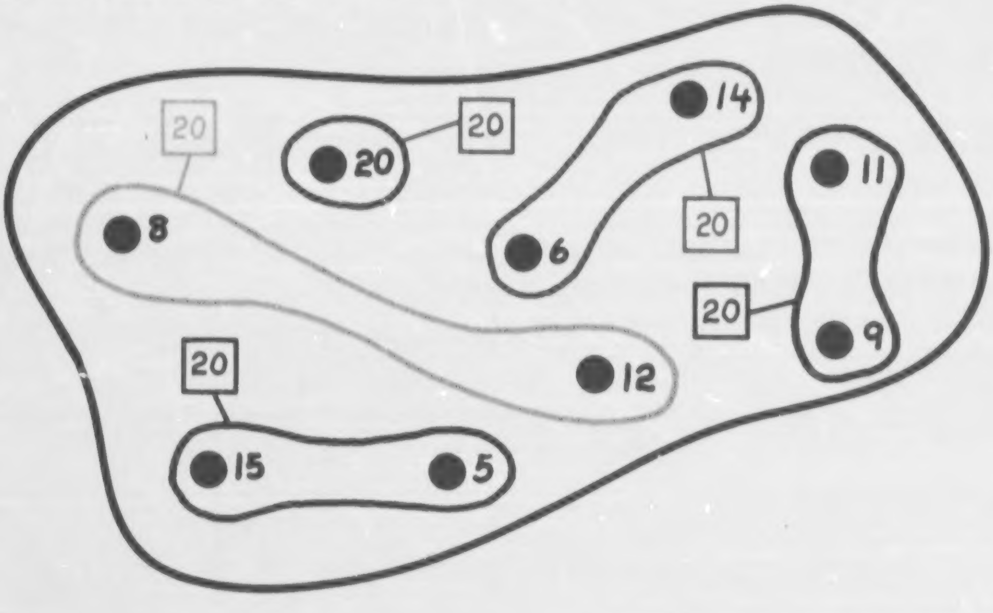
T: Wanda had a party, and she bought each of her guests a balloon. She went to a store which had balloons in many shapes: cat balloons, dog balloons, rabbit balloons, and many others. The dots in this string are for the prices of the balloons that Wanda bought. How many balloons did Wanda buy? (9) How much did Wanda spend on balloons?

Let the students think about the problem and suggest ways of solving it. Do not expect anyone to give a correct solution now. Commend any student who suggests pairing two numbers whose sum is 20 to make the addition easier.

T: Wanda looked at these numbers a long time, and then she found an easy way to add them. She paired numbers whose sum is 20. Does anyone see two numbers whose sum is 20?

S: I see 5 and 15, and 5 + 15 = 20.

Allow the students to make suggestions. Whenever a pair of numbers whose sum is 20 is offered, draw a string around those two prices. Ask if the number 20 needs to be paired with another number.



T: How can we use this string picture to find what Wanda spent?

S: Add: 20 + 20 + 20 + 20 + 20.

Write "20 + 20 + 20 + 20 + 20 = ?" above the string picture. Trace the appropriate string as you write each number on the board.

T: Does anyone know another number story we could write?

S: "5 × 20 = ?".

You may need to give this number story yourself. Record "5 × 20 = ?" below "20 + 20 + 20 + 20 + 20 = ?" on the board.

T: How many cents did Wanda spend? How do you know?

S: 100¢, because 20 + 20 + 20 + 20 + 20 = 100.

Accept any reasonable explanations and then check the answer by counting by twenties. Afterward, calculate "5 × 20" on the Minicomputer. Complete the number stories on the board.

T: Wanda spent 100 cents. How many dollars is 100 cents?

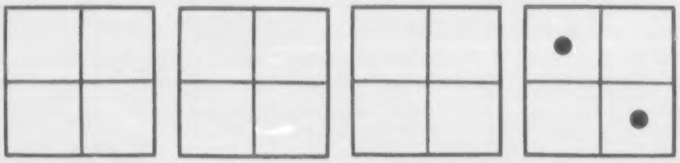
S: One dollar.

T: Who can write "$1.00" on the board?

Your class may need help with the last two questions.

Exercise 2: Multiplying by 10

Put this configuration on the Minicomputer.

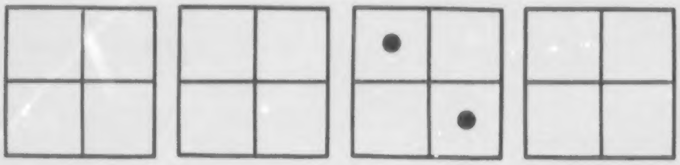


T: What number is this?

S: 9.

Write "9" on the board to one side of the Minicomputer.

Move the checker from the 8-square to the 80-square and move the checker from the 1-square to the 10-square.



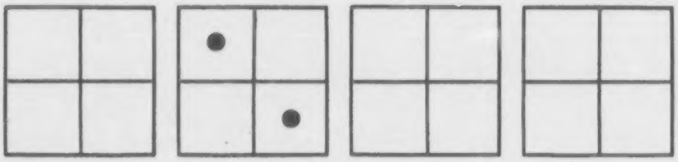
T: What number is this?

S: 90.

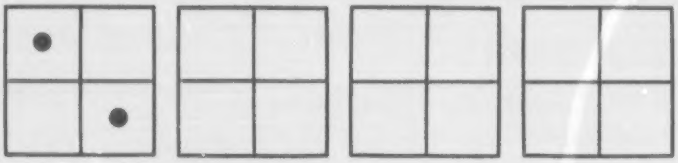
T: Who can write "90" below the Minicomputer?

After "90" has been written correctly below the Minicomputer, erase it and record "90" below "9" on the board.

Repeat this activity with these configurations.



and



N17

You should have the following list of numbers recorded on the board.

9
90
900
9,000

T: Do you see a pattern?

Allow the students to express themselves.

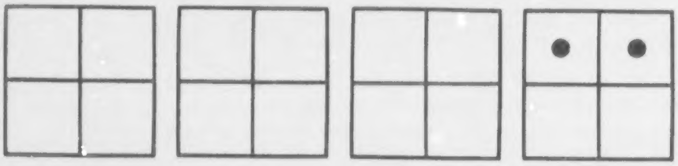
S: They all have a "9" in them.

S: The number of zeros is changing.

T: Why does each of these numbers have one more zero than the number before it?

If a student suggests that there is another zero because all you did was move the checkers to the next board to the left, you should encourage this idea. If none of the students suggest this, continue the lesson.

Erase the list. Put this configuration on the Minicomputer.



T: What number is this?

S: 12.

T: Do you know another way to put 12 on the Minicomputer using exactly two checkers?

S: A checker on the 10-square and a checker on the 2-square.

Let a student point to the squares where the checkers would be.

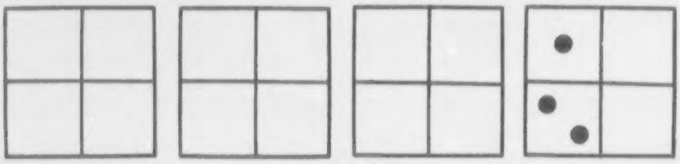
T: Could we make some plays which will result in a checker on the tens' board?

S: A "8 + 2 = 10" play would get a checker on the tens' board.

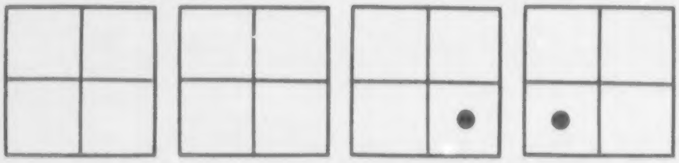
T: We have a checker on the 8-square. How can we get a checker on the 2-square?

S: We could make a "4 = 2 + 2" play.

Invite students to make a "4 = 2 + 2" play ...

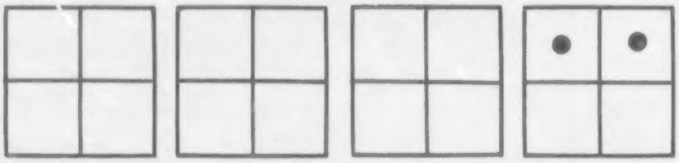


and a "8 + 2 = 10" play.

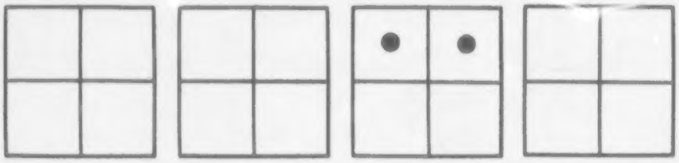


N17

Record "12" to one side of the Minicomputer. Return the checkers to the starting configuration.



Move the checker from the 8-square to the 80-square and move the checker from the 4-square to the 40-square.



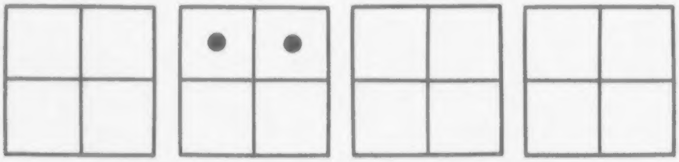
T: What number is this?

S: 120.

T: Do you know another way to put 120 on the Minicomputer using exactly two checkers?

Allow the students to make plays until 120 is in standard configuration. Ask a student to write "120" below the Minicomputer. Return the checkers to the 80-square and 40-square, and record "120" below "12".

Move the checker from the 80-square to the 800-square and move the checker from the 40-square to the 400-square. Erase "120" below the Minicomputer.



T: What number is this? (1,200)

If no one knows that this number is 1,200, continue the lesson.

T: Who can make a play which will make the number easier to read?

Continue until 1,200 is in standard configuration. Ask a student to write the number below the Minicomputer. Return the checkers to the 800-square and the 400-square, and record "1,200" below "120" yourself. You should have this list of numbers recorded on the board.

12
120
1,200

T: Do you see a pattern?

Allow the students to express themselves.

S: They all start with a "1".

S: They all have a "2" in them.

S: They all start with a "12".

S: The number of zeros is changing.

T: Do you know what the next number in this list would be?

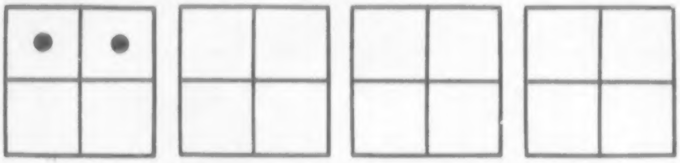
S: 12,000.

If no one suggests 12,000, ask if anyone knows how many zeroes the next number has. After it is written on the board, say the number yourself.

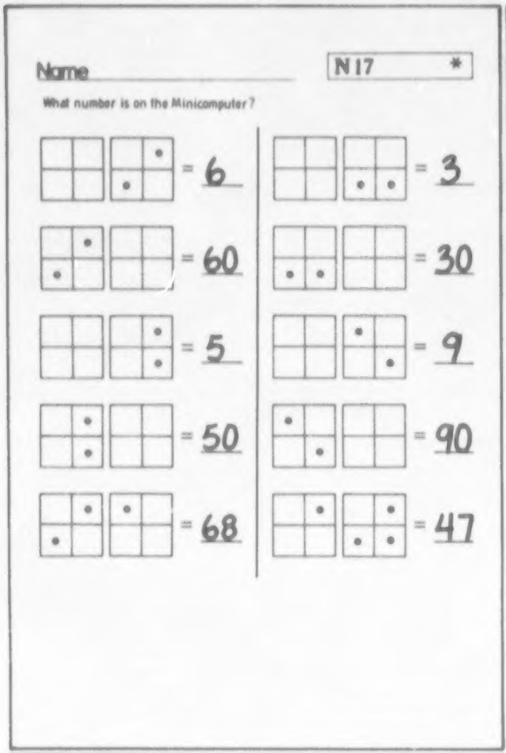
N17

T: Can anyone move the checkers so that 12,000 is on the Minicomputer?

Students should suggest the following configuration or the need for a fifth board to put on the standard configuration.



Worksheets N17*, **, ***, and **** are available for independent work.



Name

N17 *

What number is on the Minicomputer?

= 6

= 60

= 5

= 50

= 68

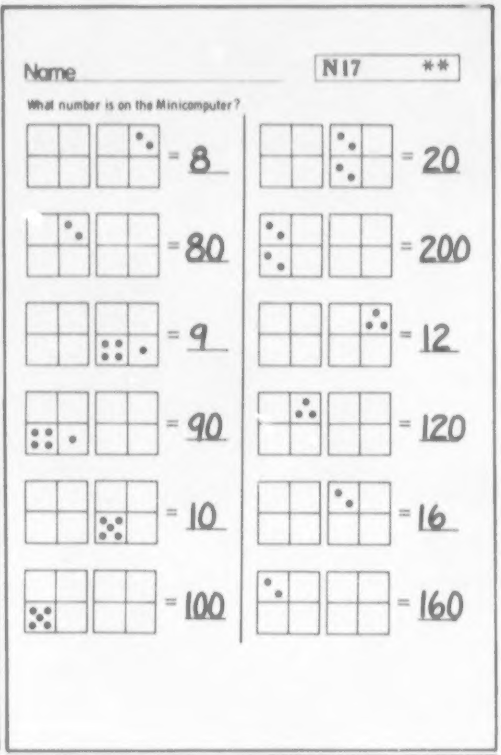
= 3

= 30

= 9

= 90

= 47



Name

N17 **

What number is on the Minicomputer?

= 8

= 80

= 9

= 90

= 10

= 100

= 20

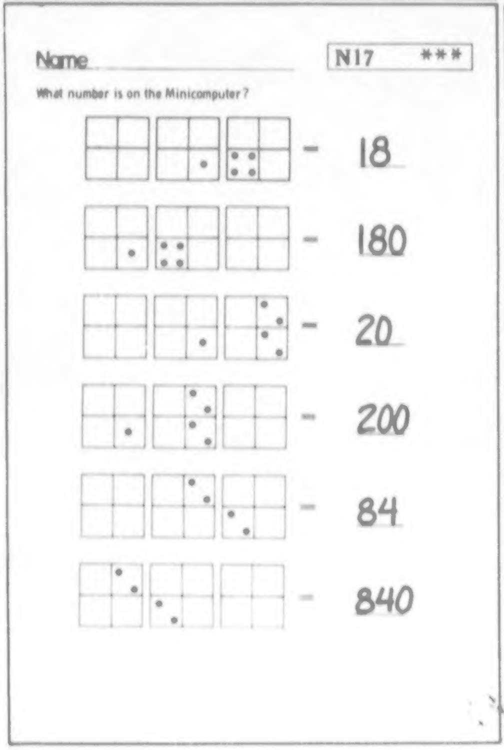
= 200

= 12

= 120

= 16

= 160



Name

N17 ***

What number is on the Minicomputer?

- 18

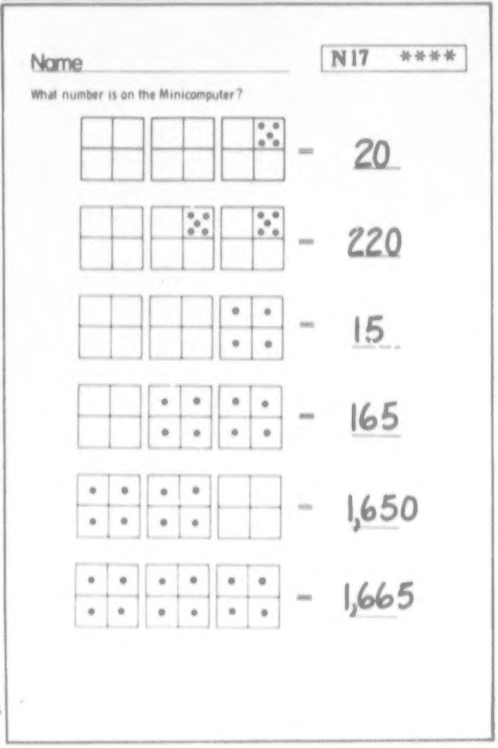
- 180

- 20

- 200

- 84

- 840



Name

N17 ****

What number is on the Minicomputer?

- 20

- 220

- 15

- 165

- 1,650

- 1,665

N18

N18 Becoming Acquainted With 37

CAPSULE LESSON SUMMARY

Get acquainted with the number 37 in a variety of situations — on the Minicomputer, in an arrow picture, and in a string diagram. Discover that 37 is not in a + 2, + 5, or + 3 spiral which starts at 0.

MATERIALS

Teacher: Minicomputer set; colored chalk
Student: Paper; pencil

DESCRIPTION OF LESSON

The goal of this lesson is to become the friend of the number 37. "Why choose 37?" you and your students may ask. Because 37 is an interesting number (as are all numbers) and we want to get to know it a little better.

Exercise 1

T: "I am neither very small, nor very large," says the number 37. "Nevertheless, I am more than 28."

Let students suggest other numbers which 37 is more than.

S: 37 is more than 17.

S: 37 is more than 31.

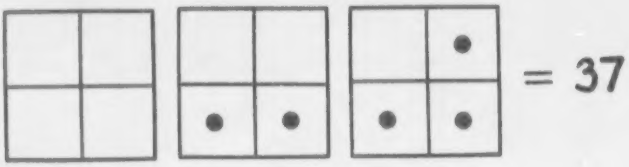
Let students suggest numbers which 37 is less than.

S: 37 is less than 53.

S: 37 is less than 39.

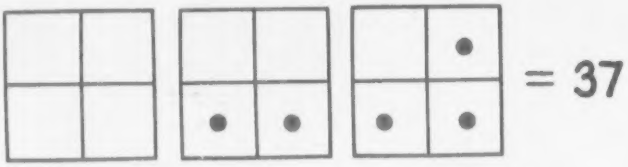
T: "I would like to be on the Minicomputer," says 37.

Let a student put 37 on the Minicomputer with red checkers.



T: "Thank you," says 37. "But it is a little sad to be displayed in only one color."

A student might suggest this configuration.



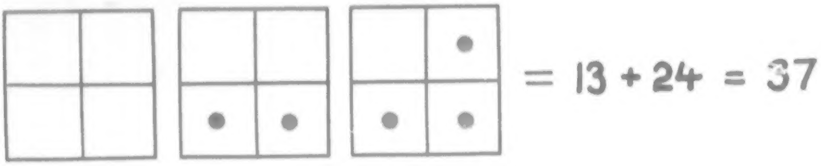
(There are many other possibilities.)

T: "I am happier now. Look at me and give me a new name."

S: 25 + 12.

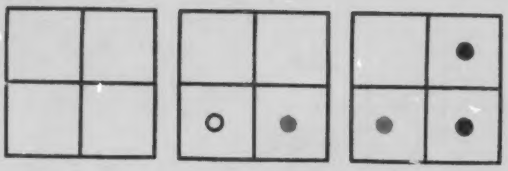
T: Can someone suggest another way of looking at 37 using checkers of two or more colors?

A student might suggest, for example,



N 18

or

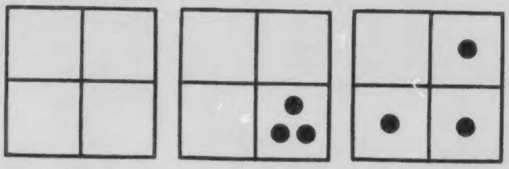


= 20 + 12 + 5 = 37

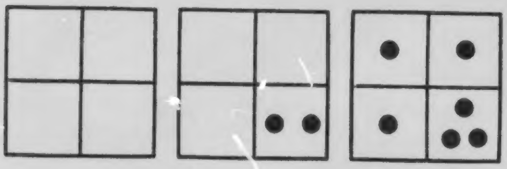
(Note: "O" indicates a yellow checker.)

T: "I'm getting bored with being on the Minicomputer with only five checkers. Can you change me a little?"

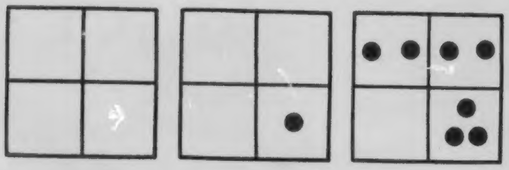
These are some possible configurations for 37 using more than five checkers.



= 37



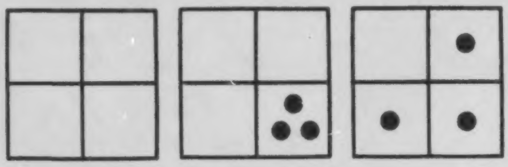
= 37



= 37

T: "Isn't it fantastic!" observes 37. "It's always me, but each time I'm dressed differently."

Ask the students for a number story corresponding to the last configuration put on the Minicomputer. For example,



$$= (3 \times 10) + 7 = 37$$

or

$$(3 \times 10) + 4 + 2 + 1 = 37$$

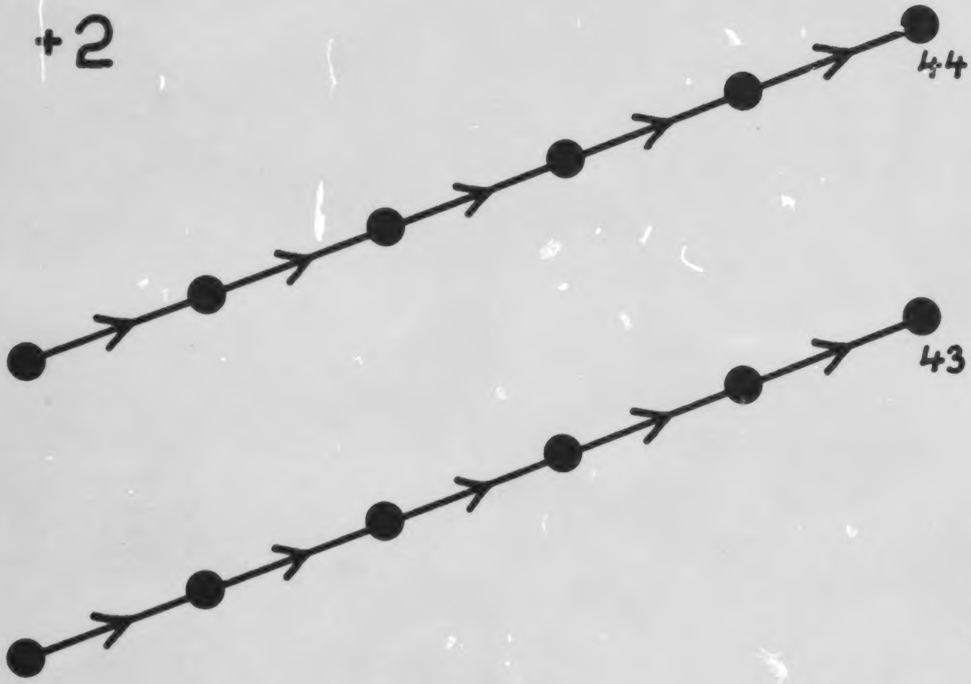
or

$$10 + 10 + 10 + 7 = 37$$

and so on

Exercise 2

Draw this arrow picture on the board.

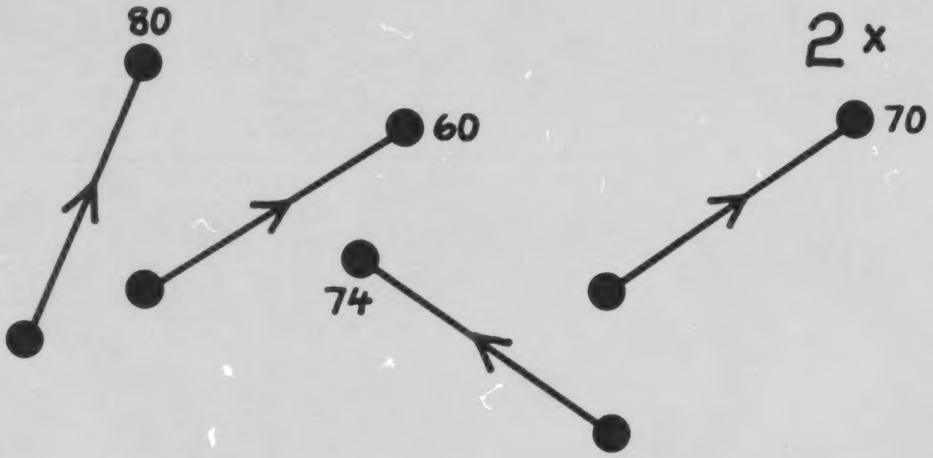


N18

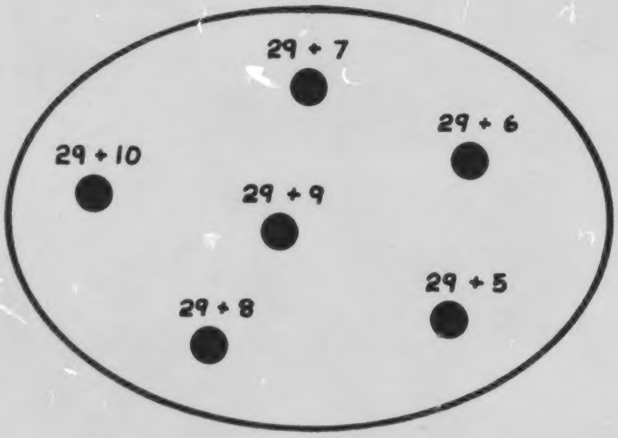
T: "I am hidden on the board," shouts 37. "I am one of these dots. Can you find me?"

Ask the students to reproduce this picture on a sheet of paper and to circle the number 37.

A similar activity can be done with an arrow picture using 2 x arrows . . .


80
60
2 x
70
74

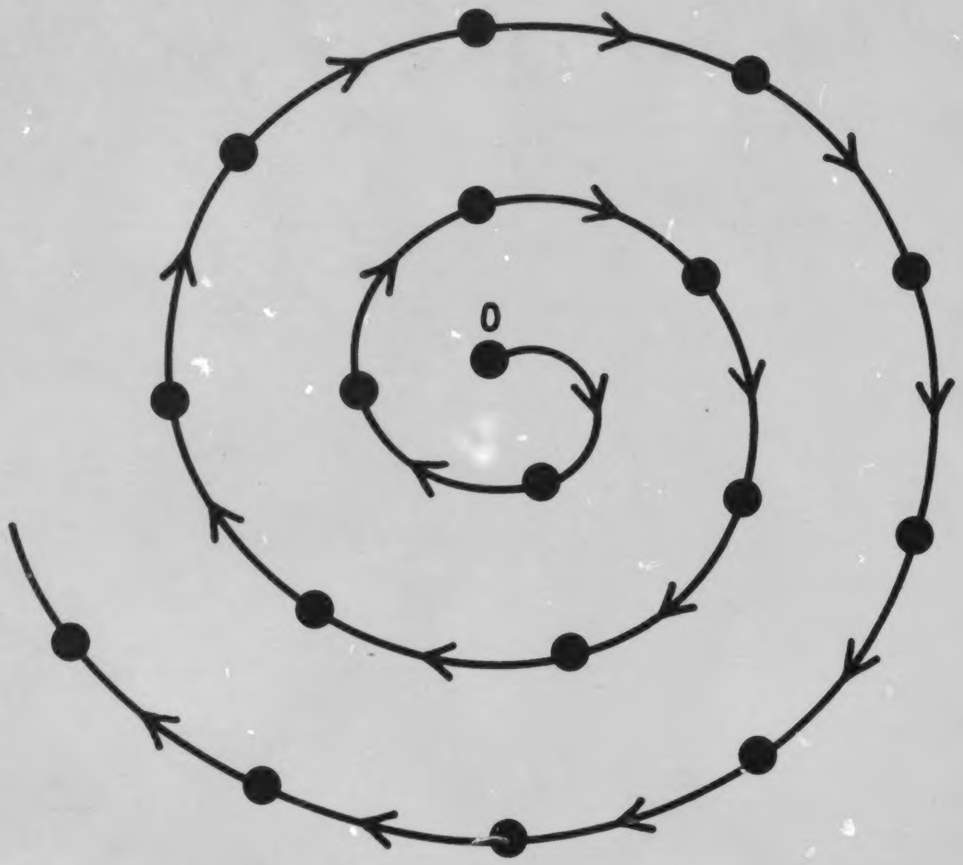
or with a string picture.


29 + 7
29 + 6
29 + 10
29 + 9
29 + 5
29 + 8

Let the students work at their own pace. It is not necessary to solve all these problems. Allow enough time for Exercise 3.

Exercise 3

Draw this spiral at the board.



T: "If the red arrows were for + 2, would I be on this spiral?" asks 37.

Let the students discuss the problem. Do not allow them to label the dots in the picture.

S: Impossible, because 37 is not an even number (or because 37 is an odd number).

N18

T: "If the red arrows were for + 3, would I be on this spiral?" asks 37.

Count by threes starting at 0 with your class: "0, 3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36, 39, ..."

S: 37 isn't one of these dots because we skipped over it.

T: "If the red arrows were for + 5, would I be on this spiral?" asks 37.

After counting by fives with the class, conclude that 37 can not be on the spiral.

End the lesson with this open question for the students to think about by themselves.

T: "And if the arrows were for + 4, or for + 6, or for + 7, or for + 10, . . . ?" continues 37.

N19 Multiplication and Subtraction #3

CAPSULE LESSON SUMMARY

Give number stories for a 7 × 4 array of dots. Note that 7 × 4 = 4 × 7. Do a multiplication problem which is suggested by a particular configuration on the Minicomputer. Subtract on individual Minicomputers.

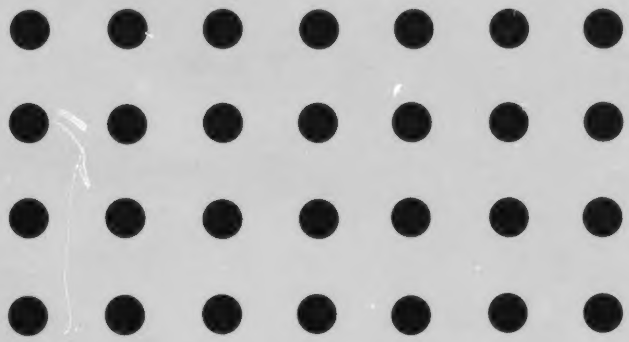
MATERIALS

Teacher: Minicomputer set
Student: Minicomputer set

DESCRIPTION OF LESSON

Exercise 1: A Multiplication Problem

Draw this pattern of dots on the board.



T: Look at this pattern of dots. What do you see?

Let the students express themselves. Many reactions are possible. If a student says there are 4 rows with 7 dots in each row, express this as:

$$7 + 7 + 7 + 7$$

or

$$4 \times 7.$$

N19

Other possibilities:

S: I see "4 + 4 + 4 + 4 + 4 + 4 + 4".
There are 28 dots.
I see "2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2".
I see "14 + 14".
"7 x 4".

Help the students to look at this picture in different ways by holding a ruler (but drawing no lines) in different places to partition the array of dots.

With a sheet of paper, hide some of the dots and ask the students how many dots are hidden. Occasionally ask students why they know how many dots are hidden.

Then, emphasize the fact that these are some very special number stories related to this picture and write these on the board.

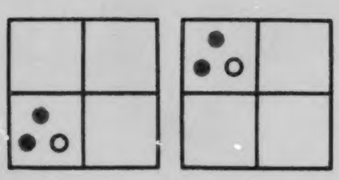
$$4 \times 7 = 28 \qquad\qquad 7 \times 4 = 28$$

Conclude:

$$4 \times 7 = 7 \times 4$$

Exercise 2: Another Multiplication Problem

Display two Minicomputer boards.



(Note: "O" indicates a yellow checker.)

T: Tell me what you see on the Minicomputer.

Let the students express their observations. For the moment, do not let them make plays on the Minicomputer.

If not already suggested, ask what number is in red, what number is in blue, and what number is in yellow. (28 in red, 28 in blue, 28 in yellow) Conclude "3 x 28" is on the Minicomputer and write this expression on the board.

Now ask what number is in red, in blue, and in yellow on each board individually. Conclude (3 x 20) + (3 x 8) is on the Minicomputer and write it under "3 x 28".

$$3 \times 28$$

$$(3 \times 20) + (3 \times 8)$$

T: What number is 3 x 20? (Point to where this is written on the board.)

S: 60.

T: What number is 3 x 8? (Point to where this is written on the board.)

S: 24.

T: So what number is (3 x 20) + (3 x 8)? (Point to where this is written on the board.)

S: 60 + 24.

S: 84.

Let the class check the result by making plays on the Minicomputer. Be prepared to suggest backward plays if needed.

Exercise 3: Subtraction Problems

Display two Minicomputer boards.

Each student will need two individual Minicomputer boards and several positive checkers.

T: Put 76 on your Minicomputer.

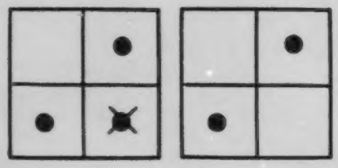
Accept but do not encourage non-standard configurations. Ask a student to put 76 on the demonstration Minicomputer.

Write this story on the board to one side of the Minicomputer.

$$76 - 20 = ?$$

T: Calculate "76 - 20" on your Minicomputer.

Ask a student to take 20 off the demonstration Minicomputer. Conclude that 76 - 20 = 56 and complete the number story on the board.



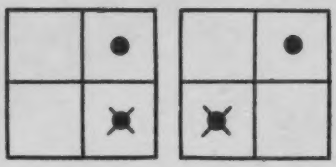
$$76 - 20 = 56$$

T: You now have 56 on your Minicomputer.

Occasionally check to make sure the students have the correct number. Write "56 - 12 = ?" on the board under "76 - 20 = 56".

T: Calculate "56 - 12" on your Minicomputer.

Ask for a volunteer to do the subtraction on the demonstration Minicomputer. Conclude that 56 - 12 = 44 and complete the number story on the board.

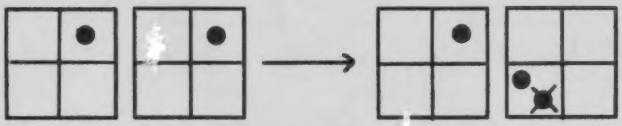


76 - 20 = 56
56 - 12 = 44

Continue with this calculation.

44 - 2 = ?

This calculation will require a backward play before a checker can be taken off the 2-square.



76 - 20 = 56
56 - 12 = 44
44 - 2 = 42

Continue this activity with the following sequences of subtraction problems or any similar ones appropriate to the numerical ability of your students.

42 - 20 = 22
22 - 0 = 22
22 - 22 = 0

69 - 28 = 41
41 - 20 = 21
21 - 10 = 11

55 - 41 = 14
14 - 8 = 6
6 - 4 = 2

97 - 40 = 57
57 - 15 = 42
42 - 22 = 20

N20

N20 Frames of the Same Shape

CAPSULE LESSON SUMMARY

Introduce number stories with more than one frame of the same shape.

MATERIALS

Teacher: Chalk

Student: Worksheets N20* and **; pencil

DESCRIPTION OF LESSON

Put this open sentence on the board.

$$\square + \square = 20$$

T: Whenever we have frames of the same shape in a sentence, we will put the same number in each frame. This is a rule we always follow. What number can we put in each box to make a true number story?

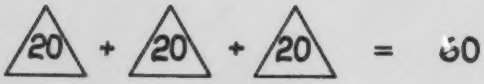
S: 10.

Fill in the boxes.

$$\boxed{10} + \boxed{10} = 20$$

Continue in a similar manner with other problems, each time filling in the frames with students' suggestions. You may need to remind your class that the same number goes in each frame.

$$\boxed{5} + \boxed{5} + \boxed{5} = 15$$



Suppose a student offers an incorrect answer.

$$\square + \square + 7 = 25$$

T: What number goes in each box to make a true sentence?

S: 10?

T: Let's check that.

Fill in the boxes.

$$\boxed{10} + \boxed{10} + 7 = 25$$

T: What number is 10 + 10?

S: 20.

T: And what number is 20 + 7?

S: 27.

T: But we need the sum to be 25. Is 10 too big or too small to put in the box?

S: Too big.

T: Try another number.

Erase "10" from the boxes and ask for another suggestion. Fill in the boxes and check the calculations each time.

N 20

Continue the lesson with other sequences of open sentences. Encourage students to find patterns and to explain them to the class.

□ + □ = 20

□ + □ = 22

□ + □ = 24

□ + □ = 26

⋮

△ + △ = 40

△ + △ = 50

△ + △ = 60

△ + △ = 70

⋮

43 = △ + △ + 3

63 = △ + △ + 3

83 = △ + △ + 3

103 = △ + △ + 3

123 = △ + △ + 3

⋮

□ + □ + 5 = 25

□ + □ + 5 = 27

□ + □ + 5 = 29

□ + □ + 5 = 31

□ + □ + 5 = 33

⋮

N 20

Change the number of boxes in a sequence of open sentences, but keep the sum of the boxes fixed.

$$\square + \square = 28$$

$$\square + \square + \square + \square = 28$$

$$\square + \square + \square + \square + \square + \square + \square = 28$$

$$\square + \square + \square + \square + \square + \square + \square + \square + \square + \square + \square + \square + \square + \square = 28$$

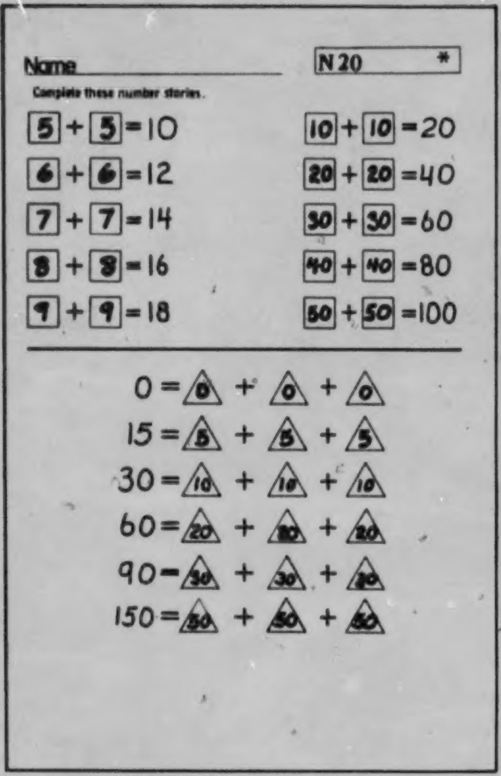
Challenge your class with these open sentences.

$$\triangle + \triangle = \triangle$$

$$\triangle + \triangle + \triangle + \triangle + \triangle + \triangle + \triangle = \triangle$$

(The only whole number that makes these last two equations true is 0.)

Worksheets N20* and ** are available for independent work. Remind the students that in each equation, frames of the same shape should be filled in by the same number. Students who finish early can make up (and answer) frame equations of their own.



Name ______________________ N 20 *

Complete these number stories.

$\boxed{5} + \boxed{5} = 10$ $\boxed{10} + \boxed{10} = 20$

$\boxed{6} + \boxed{6} = 12$ $\boxed{20} + \boxed{20} = 40$

$\boxed{7} + \boxed{7} = 14$ $\boxed{30} + \boxed{30} = 60$

$\boxed{8} + \boxed{8} = 16$ $\boxed{40} + \boxed{40} = 80$

$\boxed{9} + \boxed{9} = 18$ $\boxed{50} + \boxed{50} = 100$

$0 = 0 + 0 + 0$

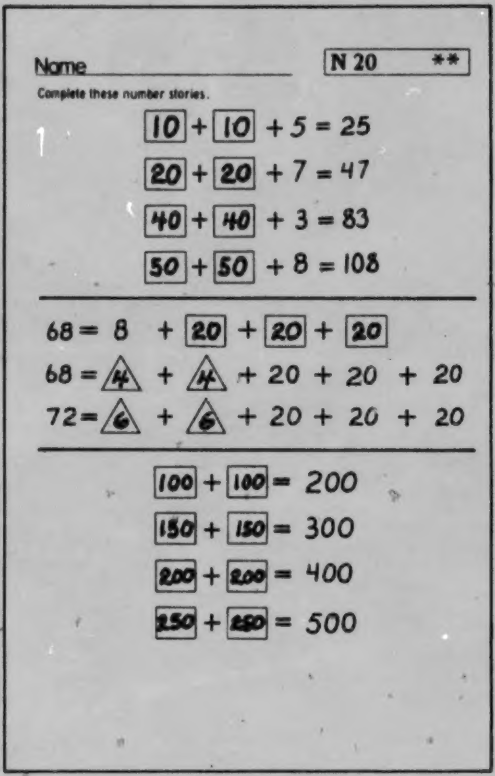
$15 = 5 + 5 + 5$

$30 = 10 + 10 + 10$

$60 = 20 + 20 + 20$

$90 = 30 + 30 + 30$

$150 = 50 + 50 + 50$



Name ______________________ N 20 **

Complete these number stories.

$\boxed{10} + \boxed{10} + 5 = 25$

$\boxed{20} + \boxed{20} + 7 = 47$

$\boxed{40} + \boxed{40} + 3 = 83$

$\boxed{50} + \boxed{50} + 8 = 108$

$68 = 8 + \boxed{20} + \boxed{20} + \boxed{20}$

$68 = 4 + 4 + 20 + 20 + 20$

$72 = 6 + 6 + 20 + 20 + 20$

$\boxed{100} + \boxed{100} = 200$

$\boxed{150} + \boxed{150} = 300$

$\boxed{200} + \boxed{200} = 400$

$\boxed{250} + \boxed{250} = 500$

N21 Multiplication and Subtraction #4

CAPSULE LESSON SUMMARY

Practice multiplication in some mental arithmetic activities. Extend experiences with multiplication and subtraction on the Minicomputer.

MATERIALS

Teacher: Minicomputer set

Student: Pencil; Worksheets N21*, **, ***, and ****; Minicomputer set (optional)

DESCRIPTION OF LESSON

Exercise 1

Very quickly do some mental arithmetic by asking the following sequence of questions.

T: 2 × 4 = ?
2 × 8 = ?
2 × 10 = ?
2 × 0 = ?

2 × 10 = ?
2 × 4 = ?
2 × 14 = ?

If no one responds to "2 × 14 = ?", ask again for "2 × 10 = ?" and "2 × 4 = ?", and then return to "2 × 14 = ?". Ask students who answer correctly to explain to the class how they knew the answer.

Repeat this activity with the following sequences.

2 × 10 =
2 × 3 =
2 × 13 =

2 × 10 =
2 × 5 =
2 × 15 =

2 × 30 =
2 × 1 =
2 × 31 =

2 × 20 =
2 × 5 =
2 × 25 =

If your class is unable to solve some of these problems, do the calculations on the Minicomputer.

You may wish to continue with more difficult calculations, depending on the abilities of your students.

2 × 50 =
2 × 6 =
2 × 56 =

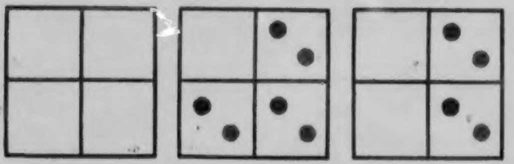
2 × 100 =
2 × 4 =
2 × 104 =

2 × 10 =
2 × 7 =
2 × 17 =

2 × 20 =
2 × 8 =
2 × 28 =

Exercise 2: Multiplication Problems

Put this configuration on the Minicomputer.



T: What do you see on the Minicomputer?

Let the students express their observations. For the moment, do not let them actually make any plays on the Minicomputer.

If not already suggested, ask what number is in red and what number is blue. (75 in red; 75 in blue) Conclude that 2 × 75 is on the Minicomputer and write "2 × 75" on the board.

Now ask what number is in red and what number is in blue on each of the boards individually. Conclude that (2 × 70) + (2 × 5) is on the Minicomputer and write "(2 × 70) + (2 × 5)" under "2 × 75".

$$2 \times 75$$
$$(2 \times 70) + (2 \times 5)$$

T: What number is 2 × 70? (Point to this calculation on the board.)

S: 140.

T: What number is 2 × 5? (Point to this calculation on the board.)

S: 10.

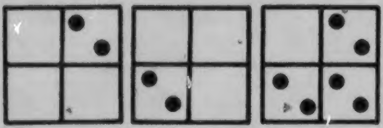
T: So what number is (2 × 70) + (2 × 5)? (Point to this calculation on the board.)

S: 140 + 10.

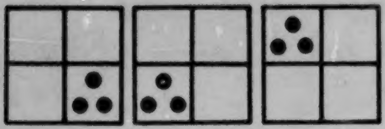
S: 150.

Let the students check the result by making plays on the Minicomputer.

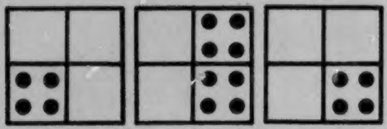
Continue this activity with the following or similar calculations.



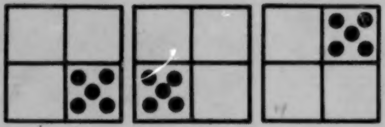
$$= 2 \times 427 = (2 \times 400) + (2 \times 20) + (2 \times 7)$$
$$= 800 + 40 + 14$$
$$= 854$$



$$\begin{aligned} &= 3 \times 128 = (3 \times 100) + (3 \times 20) + (3 \times 8) \\ &= 300 + 60 + 24 \\ &= 384 \end{aligned}$$



$$\begin{aligned} &= 4 \times 251 = (4 \times 200) + (4 \times 50) + (4 \times 1) \\ &= 800 + 200 + 4 \\ &= 1{,}004 \end{aligned}$$



$$\begin{aligned} &= 5 \times 124 = (5 \times 100) + (5 \times 20) + (5 \times 4) \\ &= 500 + 100 + 20 \\ &= 620 \end{aligned}$$

Exercise 3: Subtraction Problems

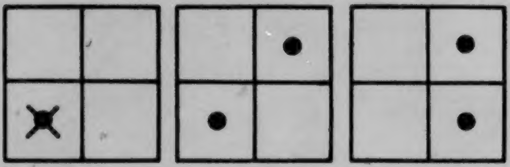
Put 265 in standard configuration on the Minicomputer.

T: Does anyone know what number 265 - 200 is? (65)

Ask any student who answers correctly for an explanation. Let a student put 265 on the Minicomputer. Write "265 - 200 = ?" on the board.

T: Who can take 200 off the Minicomputer?

The student should remove the checker from the 200-square.



T: What number is on the Minicomputer now?

S: 65.

Complete the number story on the board.

265 - 200 = 65

Write "265 - 41 = ?" on the board under "265 - 200 = 65". Put 265 on the Minicomputer. Let students predict the answer and then ask someone to take 41 off the Minicomputer.

T: What number is 265 - 41?

S: 224.

Complete the number story on the board.

265 - 41 = 224

Write "265 - 10 = ?" on the board under "265 - 41 = 224". Put 265 on the demonstration Minicomputer.

T: Why is this problem more difficult?

S: Because we have no checker on the 10-square.

S: We need to make a backward play.

T: Which play should we make?

S: "20 = 10 + 10".

N21

Invite someone to make the backward play and to take 10 off the Minicomputer. Complete the number story on the board.

$$265 - 10 = 255$$

Continue this activity with the following or similar calculations which are appropriate for the abilities of your students.

T: 265 - 14 = ?

265 - 120 = ?

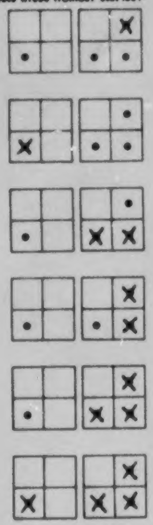
265 - 42 = ?

265 - 115 = ?

Worksheets N21*, **, ***, and **** are available for independent work. Allow the students to use their individual Minicomputers, if they wish, when doing subtraction calculations.

Name ______________ **N 21 ***

Complete these number stories.



27 - 4 = 23

27 - 20 = 7

27 - 3 = 24

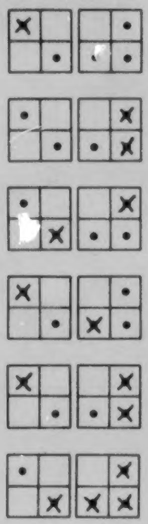
27 - 5 = 22

27 - 7 = 20

27 - 27 = 0

Name ______________ **N 21 ****

Complete these number stories.



97 - 80 = 17

97 - 5 = 92

97 - 14 = 83

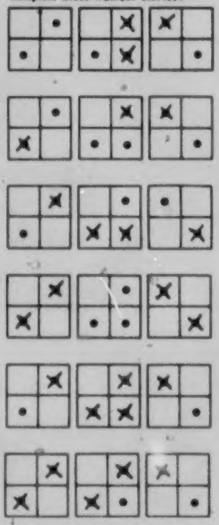
97 - 82 = 15

97 - 85 = 12

97 - 17 = 80

Name ______________ **N 21 *****

Complete these number stories.



679 - 58 = 621

679 - 248 = 431

679 - 431 = 248

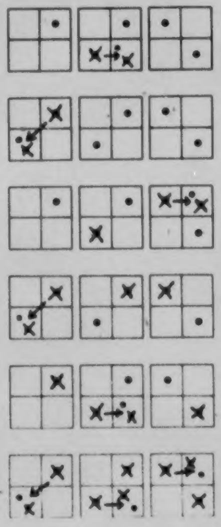
679 - 609 = 70

679 - 478 = 201

679 - 668 = 11

Name ______________ **N 21 ******

Complete these number stories.



469 - 10 = 459

469 - 200 = 269

469 - 24 = 445

469 - 248 = 221

469 - 411 = 58

469 - 255 = 214

N 22

N22 Composition of Functions #2

CAPSULE LESSON SUMMARY

Using arrow pictures, investigate the composition of certain addition and subtraction functions; for example, - 1 followed by + 6 and + 3 followed by + 7. Introduce a game in which the players build a road using arrows for + 3 and + 2.

MATERIALS

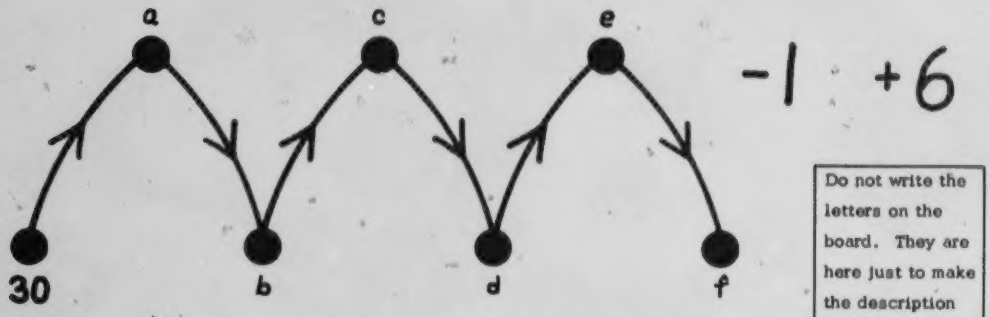
Teacher: Colored chalk

Student: Colored pencils; pencil; unlined paper

DESCRIPTION OF LESSON

Exercise 1

Draw this arrow picture on the board.



Do not write the letters on the board. They are here just to make the description of the lesson easier to follow.

T: Where is the smallest number in this arrow picture? (At a)
Would someone come to the board and point to this dot?
Tell me with whispers what the smallest number is. (29)

T: Where is the largest number in this arrow picture? (At f)
Would someone come to the board and point to this dot?
Tell me with whispers what the largest number is. (45)

If no one knows what the largest number is, wait until d has been labeled and again ask the students to whisper to you what the largest number is.

T: Where could we draw a green + 5 arrow? (Write " + 5" near the arrow picture.)

As students suggest green arrows, let them trace the arrows on the board. Occasionally ask:

T: How do you know you can draw a green arrow there?

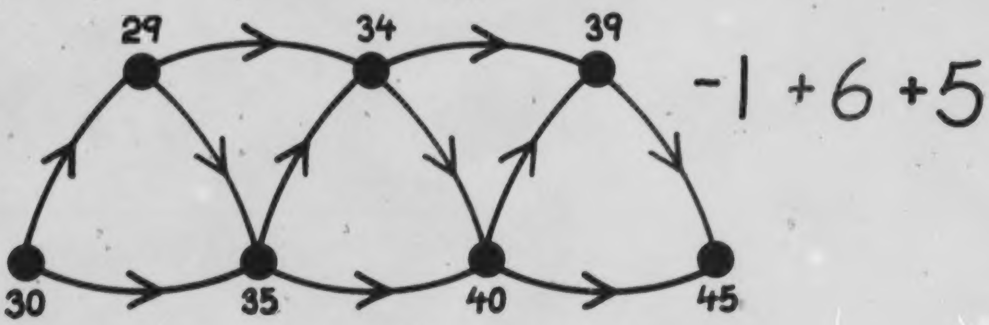
S: Here is 35 (pointing to b) and here is 40 (pointing to d) and 35 + 5 = 40.

S: - 1 followed by + 6 is the same as + 5.

Let students draw any green arrows which they have correctly traced and label any dots which they have referred to numerically.

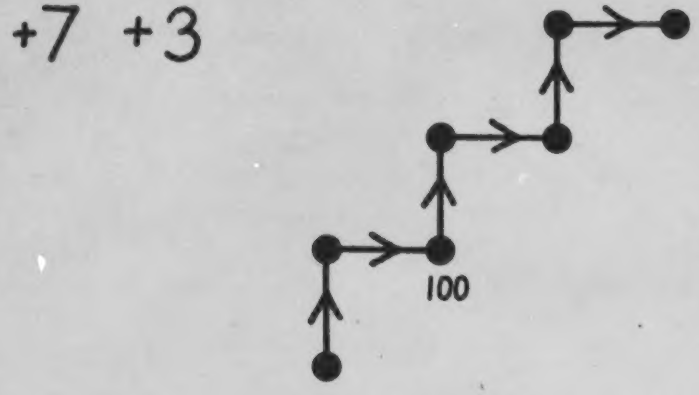
If no one suggests a correct green arrow, label the dots quickly with the students' help and then return to the request for green arrows. If necessary, draw one of the green arrows yourself.

Complete the arrow picture.



Exercise 2

Repeat Exercise 1 using this arrow picture.



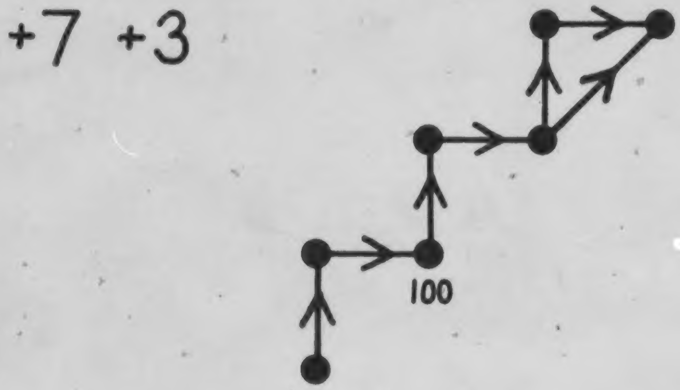
Variation of Exercise 2

If your class had no difficulty with Exercise 1, you may want to use this variation of Exercise 2.

After drawing the arrow picture on the board, ask:

T: What are the blue arrows for? (+7)
What are the red arrows for? (+3)

Draw a green arrow as illustrated.



T: What could the green arrow be for?

S: +10.

T: How do you know the green arrow could be for +10?

S: I figured out what the dots are.

S: +3 followed by +7 is the same as +10.

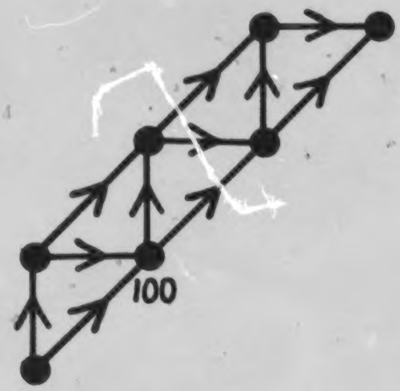
Write "+10 " near the arrow picture.

T: Can we draw any more green arrows?

Let students trace and then draw green arrows in the picture.

+7 +3

+10



If all the green arrows in the picture are not suggested, continue with the lesson and return to the problem of finding green arrows after the dots have been labeled.

T: Point to the smallest number in this arrow picture.
Tell me with whispers, what is the smallest number. (90)
Point to the largest number in this arrow picture.
Tell me with whispers, what is the largest number. (120)

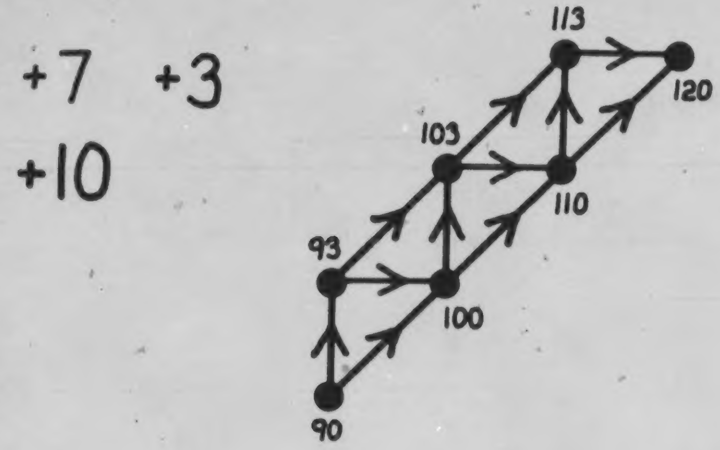
N 22

After listening to several whispers, ask a student to give the answers aloud.

T: How do you know the smallest number is 90?
How do you know the largest number is 120?

Encourage the idea of subtracting 10 from 100 to get 90, and the idea of counting by tens from 100 to 120.

Complete the arrow picture.



Point to "113".

T: If we drew another green + 10 arrow starting at 113, what number would be at the end of the arrow? (123)
And after that? (133, 143, 153, ...)

Point to "93", then trace with your finger a green + 10 arrow ending at 93.

T: If we drew another green + 10 arrow here, what would be the starting number of this arrow? (83)
And before that? (73, 63, 53, ...)

Exercise 3

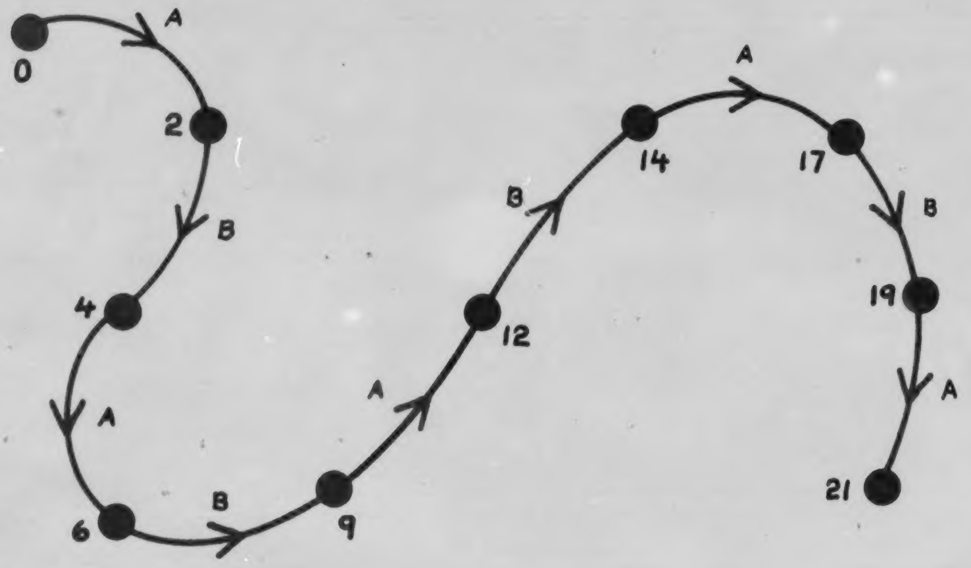
NOTE: This is good activity to use whenever you have some extra time.

Write "+3" and "+2" on the board. Draw one dot and label it "0".

T: This game is called "20" because the first person to go over 20 loses. The players take turns drawing one arrow until one of them meets a number which is more than 20. When it is your turn, you may draw either a red + 3 arrow or a blue + 2 arrow. Label the ending dot of each arrow you draw. We will play a game on the board to help you understand the rules.

Choose two students to play the game at the board. Give each student a piece of red and a piece of blue chalk and make sure they understand they can use either blue or red arrows. A sample game is shown below. The players will be called "A" and "B".

+2 +3



N22

Player A was forced to go over 20 and therefore lost. Player B could also have won by using a +3 arrow from 17 to 20.

When the game at the board is finished, ask each student to find a partner. Every student will need a regular pencil, a red pencil, and a blue pencil. Each pair will need a piece of paper. If you wish, you can ask the students to change opponents after each game.

N23 Addition Problems #5

CAPSULE LESSON SUMMARY

Pair numbers whose sum is 30 to solve an addition problem. Put 20 on the Minicomputer in a variety of configurations, looking at number stories for each representation.

MATERIALS

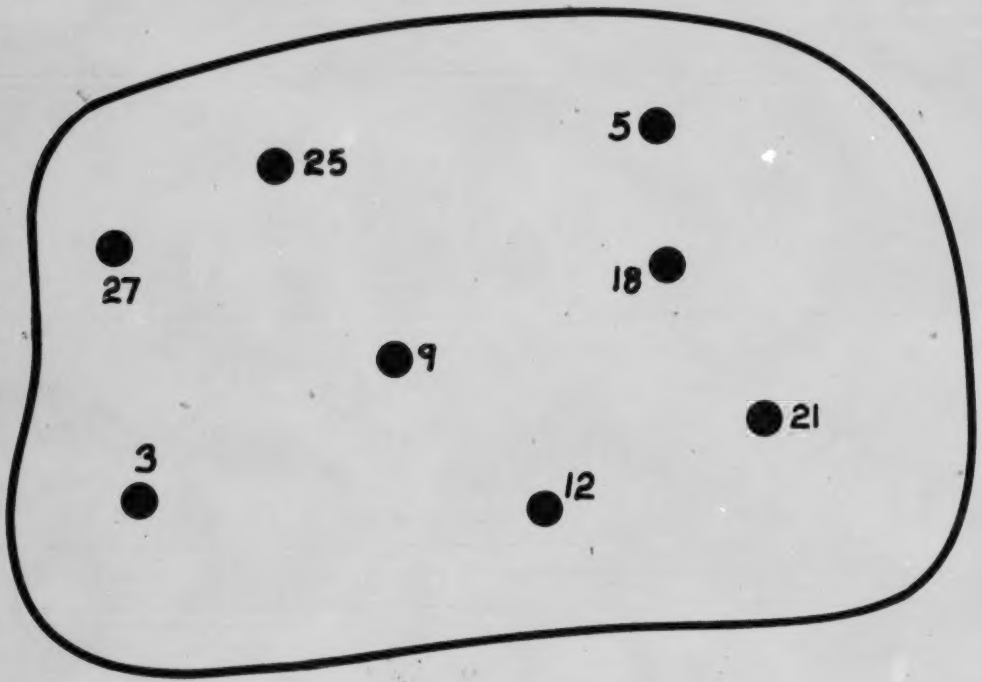
Teacher: Minicomputer set; colored chalk

Student: Minicomputer set

DESCRIPTION OF LESSON

Exercise 1: Addition Problem

Draw this string picture on the board, leaving space above it to write number stories about the picture.



222

N23

Tell your students this story or a similar one. Choose one of your students to be the star of the story.

T: Jeff's club had a bake sale last Saturday.

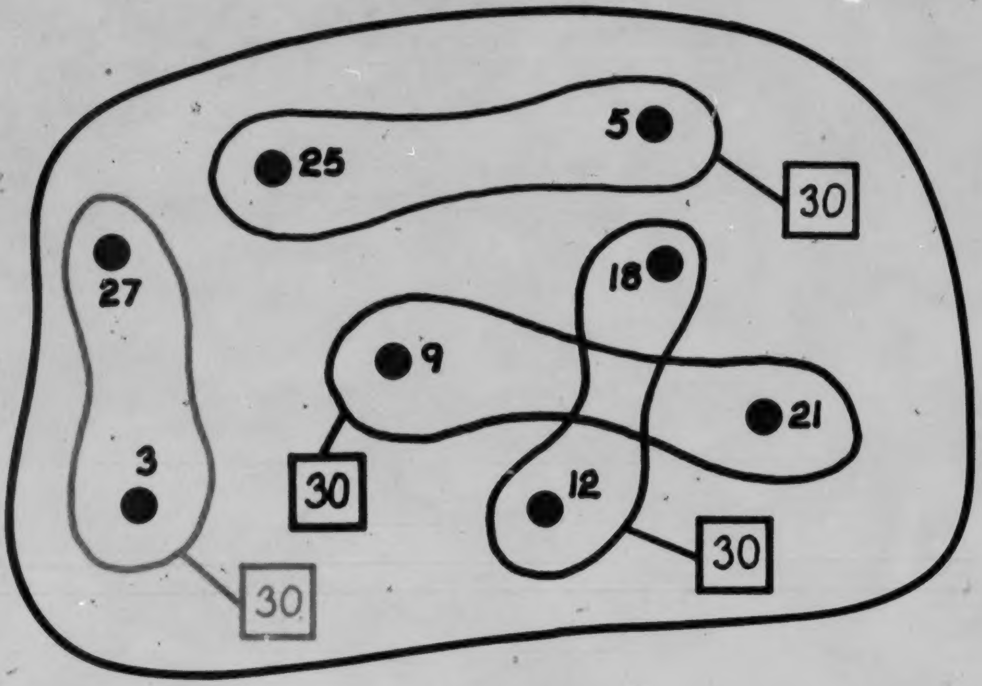
The class may want to discuss why Jeff's club might have had a bake sale and what might have been sold.

T: Jeff made some chocolate chip cookies for the bake sale and his mother made some brownies. Jeff worked at the bake sale for awhile making change and wrapping the food for the customers. Before Jeff went home, he bought some things for his family. Each of these dots represents the price in cents of one of the pastries Jeff bought. How many pastries did Jeff buy? (8) How much did Jeff spend at the bake sale?

Let the students think about the problem and suggest ways of solving it, but do not expect them to solve it now. Encourage the idea of pairing two numbers whose sum is 30.

T: Jeff looked at these numbers a long time before he saw an easy way to add them. He paired numbers whose sum is 30. Does anyone see two numbers whose sum is 30?

Allow the students to make suggestions. Whenever a pair of numbers whose sum is 30 is suggested, draw a string around those two prices. Continue until all the numbers in the string picture are paired.



T: How can we use this string picture to find what Jeff spent?

S: Add: 30 + 30 + 30 + 30.

Above the string picture, write "30 + 30 + 30 + 30 = ?". Point to the appropriate string as you write each number on the board.

T: Does anyone know another number story we could write about this string picture?

S: "4 × 30 = ?".

You may need to suggest this number story yourself. Record "4 × 30 = ?" below the "30 + 30 + 30 + 30 = ?".

T: How many cents did Jeff spend at the bake sale?

224

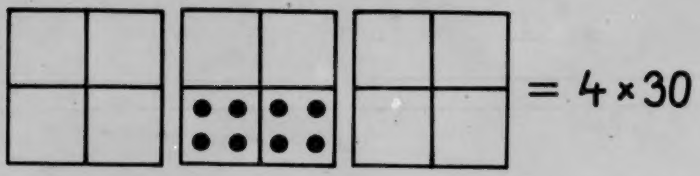
S: 120¢.

Ask any students who respond correctly to explain how they calculated the answer.

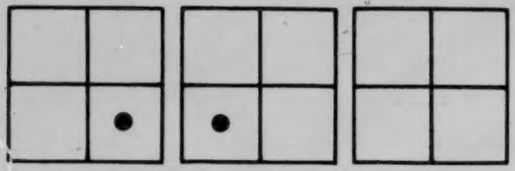
T: Let's calculate "4 x 30" on the Minicomputer.
How many times should we put 30 on the Minicomputer?

S: 4.

Display three Minicomputer boards and ask for a volunteer to put 4 x 30 on the Minicomputer.



Let students make plays until the standard configuration for 120 is on the Minicomputer.



Ask a student to write the number below the Minicomputer and ask another to complete the number stories.

T: Who can read these number stories?

S: 30 + 30 + 30 + 30 = 120.
4 x 30 = 120.

T: Jeff spent 120 cents at the bake sale. Does anyone know another way to say how much money 120 cents is?

S: One dollar and twenty cents.

T: Who can write "\$1.20" on the board?

The class may need help with the last two questions.

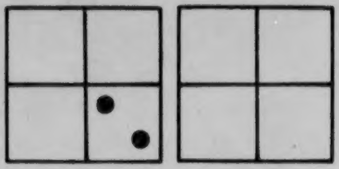
Exercise 2

Each student will need two individual Minicomputer boards and some checkers. Display two Minicomputer boards.

Ask the students to put 20 on their (desk) Minicomputers in a variety of configurations. Observe the students working and help anyone having difficulty.

Choose several students to put their configurations for 20 on the demonstration Minicomputer. Each time a configuration is displayed, call the class' attention to it and ask for an appropriate number story.

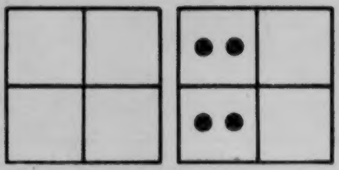
These are some of the possibilities and several of the number stories corresponding to each.



$$= 10 + 10 = 20$$

or

$$2 \times 10 = 20$$



$$= (8+2) + (8+2) = 20$$

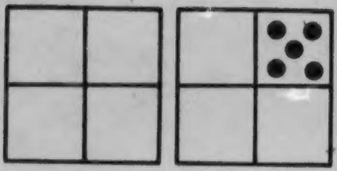
or

$$(8+8) + (2+2) = 20$$

or

$$(2 \times 8) + (2 \times 2) = 20$$

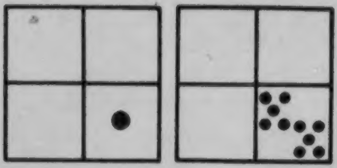
N 23



$= 5 \times 4 = 20$

or

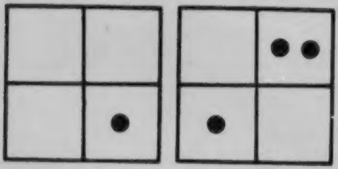
$4 + 4 + 4 + 4 + 4 = 20$



$= 10 + 10 = 20$

or

$10 + (10 \times 1) = 20$



$= 10 + 4 + 4 + 2 = 20$

or

$10 + (2 \times 4) + 2 = 20$

or

$10 + 6 + 4 = 20$

N24 Adding and Subtracting Three

CAPSULE LESSON SUMMARY

Count forward and backward by threes. Examine the relationship between the number stories "1 + 3 = 4" and "4 - 3 = 1" by the use of a simple arrow diagram. Make use of return arrows to label some of the dots in an arrow picture using + 3 arrows and in another using - 3 arrows. Individually, draw arrow pictures using + 3 and - 3 arrows.

MATERIALS

Teacher: Colored chalk; 0-109 Chart (optional)
Student: Colored pencils; pencil; unlined paper

DESCRIPTION OF LESSON

Exercise 1

Divide your class into three or four groups in a manner natural to the seating arrangement. Indicate an order in each group, i.e., which student is first, second, and so on.

T: We are going to count forward by threes starting at 0. I will start by saying "0", then the first student in a group will say "3," the next "6," the next "9" because 9 is three more than 6, and so on. Try to remember the number you say.

Let each group take its turn.

Now reverse the counting. Starting with the last student in each group, count backward by threes by asking students to recall their numbers. For example, a group of seven students would count:

21, 18, 15, 12, 9, 6, 3

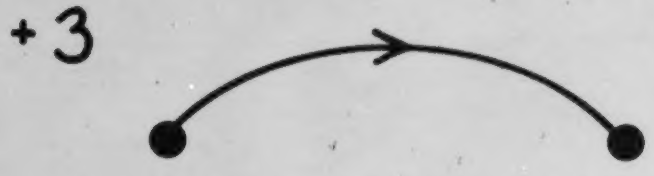
... and you would say "0."

If a student is having difficulty with either activity, hold up three fingers and count (by ones) forward or backward, as the case may be, from the previous number. The 0-109 Number Chart also could be used in a similar manner.

If your class is capable, you may wish to have the entire class participate as a single group in counting forward and backward by threes.

Exercise 2

Draw this arrow picture on the board.



T: What is this red arrow for?

S: + 3.

T (pointing to the dots): Do you know what these numbers are?

S: No.

T (pointing to the dot on the left): If this number were 2 (trace the red arrow and point to the dot on the right), what number would this be?

S: 5, because 2 + 3 = 5.

T (pointing to the dot on the left): If this number were 5, what would the other number be? (8)
If this number were 8? (11)
If this number were 0? (3)
If this number were 100? (103)

Very obviously, change the direction of your question.

T (pointing to the dot on the right): If this number were 4, what would the other number be? How do you know?

S: 1, because 4 - 3 = 1.

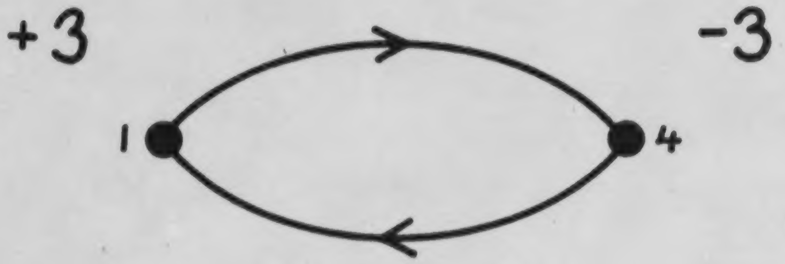
If the student answers "1 + 3 = 4", ask if someone could find the number another way.

Label the dots and trace an arrow from 4 to 1.

T: If we drew a blue arrow from 4 to 1, what could it be for?

S: -3.

Draw the return arrow in blue and write "-3" near the arrow picture.



T: What number stories are told by this arrow picture?

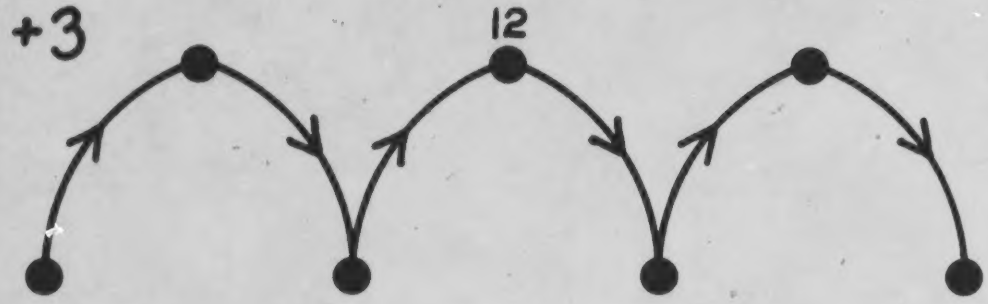
S: 1 + 3 = 4 and 4 - 3 = 1.

Erase "1" and "4". Alternating dots, assign a number to a dot and ask what the other number in the picture would be. Adjust the difficulty of the calculations to the abilities of your students.

N24

Exercise 3

Draw this arrow picture on the board.



T: Point to the largest number in this arrow picture. Tell me with whispers what the largest number in this arrow picture is.

S: 21.

T: Point to the smallest number in this arrow picture. Tell me with whispers what the smallest number in this arrow picture is.

S: 3.

It is not necessary for you to reveal immediately the identities of the smallest and largest numbers to the class. When these dots are labeled, acknowledge any students who had whispered the correct numbers to you earlier and ask how they found their answers.

Ask for volunteers to label the dots in this arrow picture. Occasionally ask students to explain how they know which numbers correspond to particular dots. Continue until the dot to the left of 12 is labeled "9". Trace the red arrow from 9 to 12.

T: How do you know that the starting number for this arrow is 9?

S: 12 - 3 = 9.

If a student answers "9 + 3 = 12," ask if someone can find this number another way.

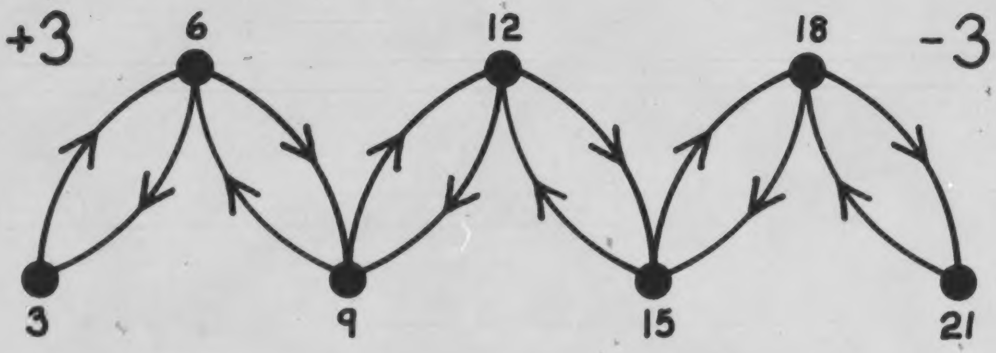
Draw a blue arrow from 12 to 9.

T: What could this blue arrow be for?

S: -3.

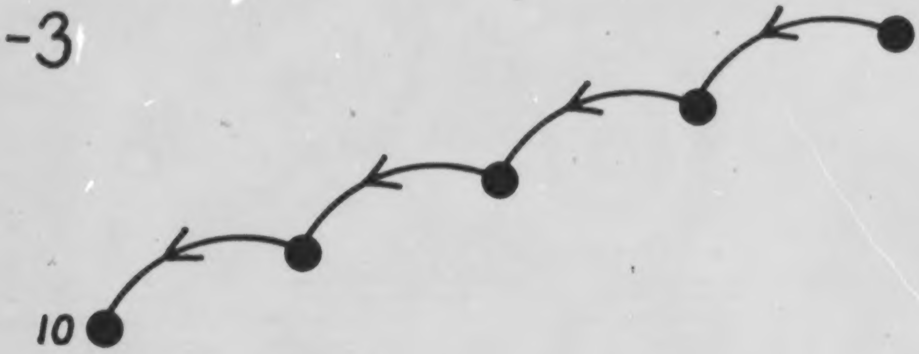
Write "-3 " near the arrow picture.

Invite students to draw return arrows and label dots until the arrow picture is complete.



Exercise 4

Draw this arrow picture on the board.



N24

T: Point to the smallest number in this arrow picture. Tell me with whispers what the smallest number in this arrow picture is.

S: 10.

T: Point to the largest number in this arrow picture. Tell me with whispers what the largest number in this picture is.

S: 22.

This last question is difficult. Ask any students who answer correctly to explain their methods of calculating the largest number.

T: This arrow road ends at the number 10. What can we do that will help us calculate these other numbers?

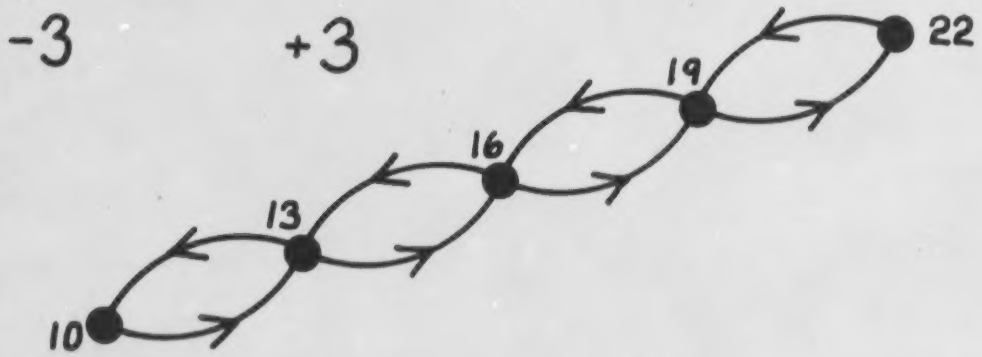
S: Draw return arrows.

You may need to suggest drawing return arrows yourself.

T: What would the return arrows be for?

S: +3.

Write "+3" near the arrow picture and invite students to either draw a +3 arrow or label a dot. Complete the arrow picture.



N 24

Distribute to each student a red pencil, a blue pencil, and a piece of unlined paper. Ask the students to draw their own arrow pictures using + 3 and - 3 arrows. Some students will need help getting started. Encourage them to draw the appropriate return arrows. Ask students who finish an extensive picture quickly to draw another arrow picture labeling the dots with larger numbers.

N 25

N25 Addition Problems #6

CAPSULE LESSON SUMMARY

Use the Minicomputer to explore the relationship between calculations such as "3 × 4", "3 × 40", and "3 x 400". Add the odd numbers between 0 and 40 by pairing numbers whose sum is 40.

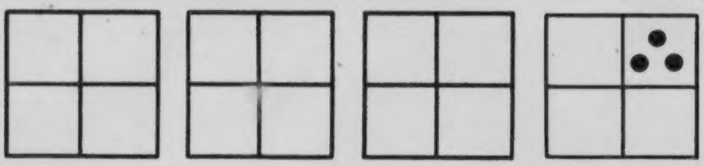
MATERIALS

Teacher: Minicomputer set
Student: Pencil; Worksheets N25*, **, ***, and ****

DESCRIPTION OF LESSON

Exercise 1: Multiplying by 10

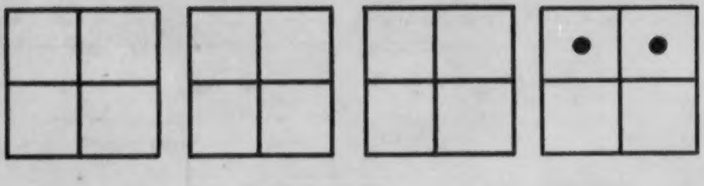
Put this configuration on the Minicomputer.



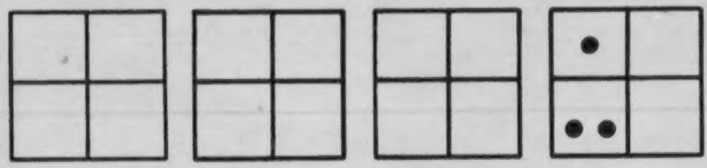
T: What number is this? (12)
How do you know?

Record the number stories of several students on the board. Be sure to include both "4 + 4 + 4 = 12" and "3 x 4 = 12". If the latter is not suggested, point to the "4 + 4 + 4 = 12" number story and ask, "How many times did we add 4? Do you know another way to write this calculation?" You may need to suggest "3 x 4 = 12" yourself.

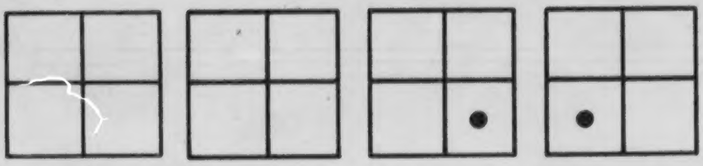
T: Some of you think the number on the Minicomputer is 12. Who can make a play so that this number will be easier to read?



S: 4 + 4 = 8.



S: 4 = 2 + 2.



S: 8 + 2 = 10.

Erase the board and return the three checkers to the 4-square.

T: We have three 4's on the Minicomputer. What number is 3 x 4?

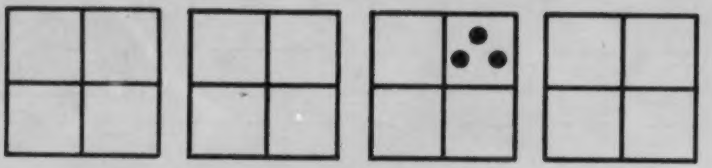
S: 12.

Write "3 x 4 = 12" on the board.

T: Watch what I am going to do.

N 25

Move the three checkers from the 4-square to the 40-square.



T: What number is this? (120)

How do you know?

Record several number stories on the board. If necessary, use parentheses in recording some of the stories without calling the class' attention to them. Number stories suggested by one second grade class included:

$$40 + 40 + 40 = 120$$

$$40 + 40 + 20 + 20 = 120$$

$$3 \times 40 = 120$$

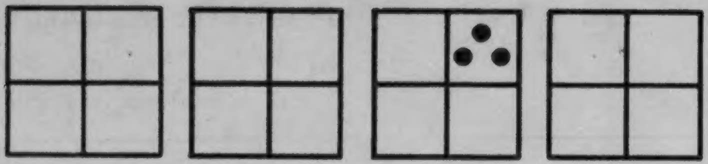
$$2 \times (40 + 20) = 120$$

If no one suggests "3 × 40 = 120", suggest it yourself.

T: Some of you think the number on the Minicomputer is 120. Who can make a play so that this number will be easier to read?

Involve many students in making the plays. The plays will be analagous to those used to calculate "3 × 4", but this time on the tens' board.

When standard configuration for 120 is obtained, ask a student to write the number below the Minicomputer. Erase the board except for the number story "3 × 4 = 12" and return the three checkers to the 40-square.



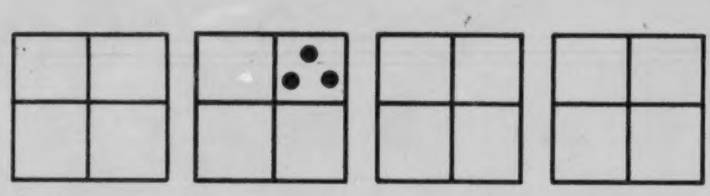
T: What number is 3 x 40?

S: 120.

Write "3 x 40 = 120" on the board under "3 x 4 = 12".

T: Watch me again as I move the checkers.

Move the three checkers from the 40-square to the 400-square.



T: What number is this?

S: 1,200.

Ask students who give the correct answer how they did the calculation. It is possible that a student put another "0" on "120" to get the answer, but very likely many students will need to make plays before they are certain that 3 x 400 = 1,200.

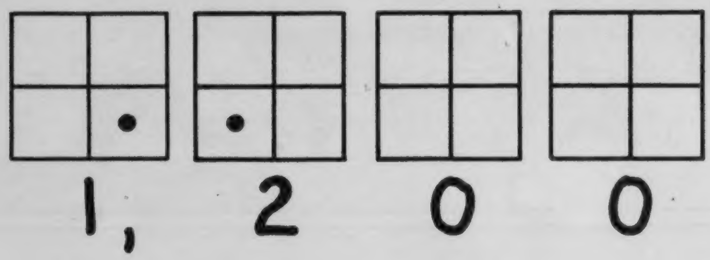
If some students suggest incorrect answers, write them on the board and appeal to their sense of pattern. For example, if a student suggests 1,012, point to the calculations on the board as you read them, "3 x 4 = 12 and

3 x 40 = 120." Write the suggested calculation on the board under the other two as you ask, "Does it look right to have 1,012 here?"

T: Who can play so that this number will be easier to read?

Help students who have trouble verbalizing the plays; for example, if a student says "40 = 20 + 20", you should say "400 = 200 + 200" but do not criticize the error.

Continue until standard configuration for 1,200 is obtained. Ask someone to write the number under the Minicomputer.



T: What number is 3 x 400?

S: 1,200.

Write "3 x 400 = 1,200" under "3 x 40 = 120".

You should have this list on the board.

3 x 4 = 12
3 x 40 = 120
3 x 400 = 1200

T: Does anyone see a pattern?

S: There is a "12" in each answer.

S: There is one more "0" each time.

Do not expect answers to this question to be well formed.

If your class is doing well with the activity, ask them to calculate "3 x 4,000". Erase the board before going on to Exercise 2.

__Exercise 2:__ Addition Problems

T: Today we are going to add the odd numbers between 0 and 40. Does anyone know how many odd numbers there are between 0 and 40?

There are twenty odd numbers between 0 and 40 but do not stress this; the question is intended to challenge your better students. If no one knows the answer, simply continue with the lesson.

T: Let's write all the odd numbers between 0 and 40 on the board. Tell me which numbers to write.

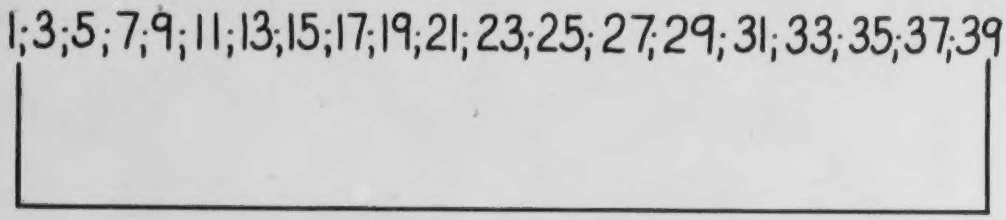
1; 3; 5; 7; 9; 11; 13; 15; 17; 19; 21; 23; 25; 27; 29; 31; 33; 35; 37; 39

T: Does anyone see an easy way to add these numbers?

Allow the students to react to this problem. Perhaps someone will suggest pairing numbers whose sum is 40.

T: Maybe it would be easier to solve this problem if we paired numbers whose sum is 40. For example, 39 + 1 = 40.

Indicate pairing 39 and 1 on your list.

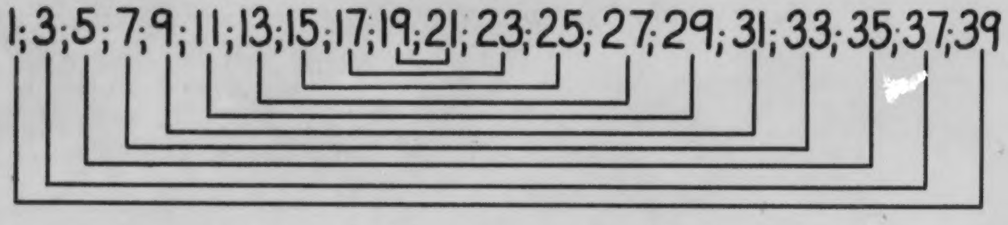


1; 3; 5; 7; 9; 11; 13; 15; 17; 19; 21; 23; 25; 27; 29; 31; 33; 35; 37; 39

N 25

T: Does anyone see other pairs of numbers whose sum is 40?

On your list, indicate correct pairs as they are suggested.



T: How many 40's are there? (Point to the list of numbers on the board.)

S: 10.

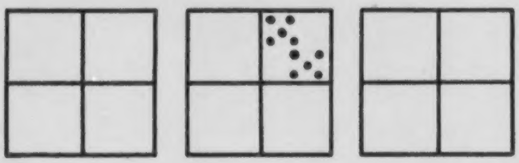
T: What number is 10 × 40?

Ask students to explain their answers, but be prepared for no one's knowing that 10 × 40 = 400. Perhaps someone will suggest using the Minicomputer.

T: Let's solve this problem on the Minicomputer. How can we show 10 × 40 on the Minicomputer?

Guide the discussion until it is clear that 40 should be put on the Minicomputer ten times. You may need to review briefly from previous lessons multiplication by 2 and by 3.

Invite someone to put ten checkers on the 40-square.



T: We put 10 × 40 on the Minicomputer. What can we do to make the number easier to read?

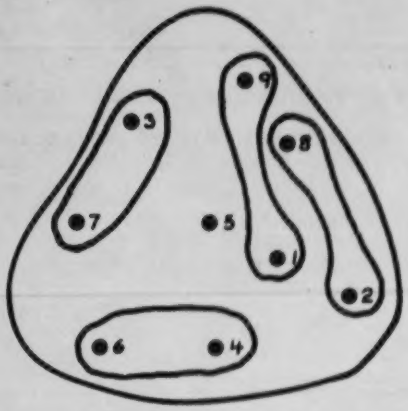
S: Make some plays.

Invite students to make plays. Since this exercise requires plays similar to those in Exercise 1, it should move quickly. Continue until the standard configuration for 400 is obtained.

Worksheets N25*, **, ***, and **** are available for independent work. Read the directions with the class and emphasize that the number story at the bottom of each page is about the numbers in the string picture on that page.

Name ____________________ **N 25 ***

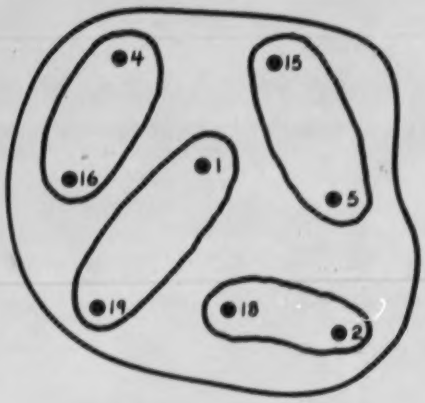
Draw a string around each pair of numbers whose sum is 10.



10 + 10 + 10 + 10 + 5 = 45

Name ____________________ **N 25 ****

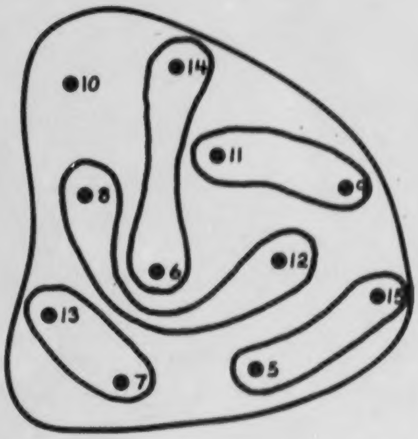
Draw a string around each pair of numbers whose sum is 20.


4
15
1
16
5
19
18
2

20 + 20 + 20 + 20 = 80

Name ____________________ **N 25 *****

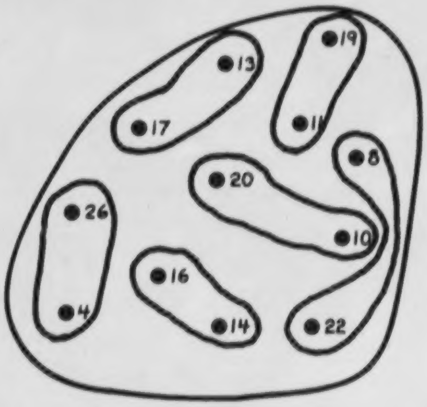
Draw a string around each pair of numbers whose sum is 20.


10
14
11
8
9
12
6
13
15
5
7

20 + 20 + 20 + 20 + 20 + 10 = 90

Name ____________________ **N 25 ******

Draw a string around each pair of numbers whose sum is 30.


19
13
17
11
8
20
26
10
16
4
14
22

Write a number story for this string picture.

30 + 30 + 30 + 30 + 30 + 30 = 180

N26 The Functions 2x and 1/2x

CAPSULE LESSON SUMMARY

Relate the functions 2x and $\frac{1}{2}$x through the use of a simple arrow diagram: the arrow for one is the return arrow for the other. Label the dots in a 2x arrow picture with the help of the Minicomputer, and then draw the return arrows.

MATERIALS

Teacher: Colored chalk; Minicomputer set
Student: None

DESCRIPTION OF LESSON

Exercise 1

Begin the lesson with some mental arithmetic.

T: What number is 10 + 10? (20)
Do you know another name for 10 + 10? ("2 × 10")
What number is 2 × 10? (20)
What number is $\frac{1}{2}$ × 20? (10)

What number is 50 + 50? (100)
What number is 2 × 50? (100)
What number is $\frac{1}{2}$ × 100? (50)

What number is 300 + 300? (600)
What number is 2 × 300? (600)
What number is $\frac{1}{2}$ × 600? (300)

NOTE: "$\frac{1}{2}$x" should be read "one half of"; for example, "$\frac{1}{2}$ × 20" is read "one half of twenty."

N 26

Continue the exercise with these calculations.

2×30 (60)

$\frac{1}{2} \times 60$ (30)

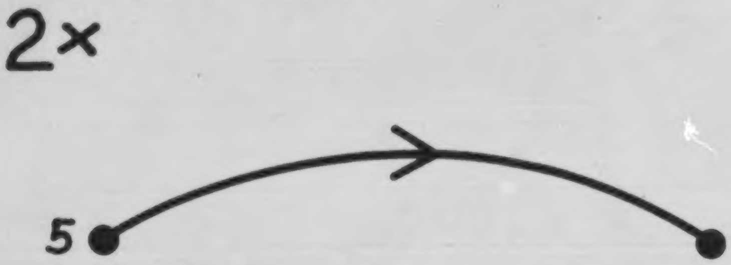
2×25 (50)

$\frac{1}{2} \times 30$ (15)

2×16 (32)

$\frac{1}{2} \times 32$ (16)

Draw this arrow picture on the board.



Point to the dot on the right.

T: What number is this?

S: 10.

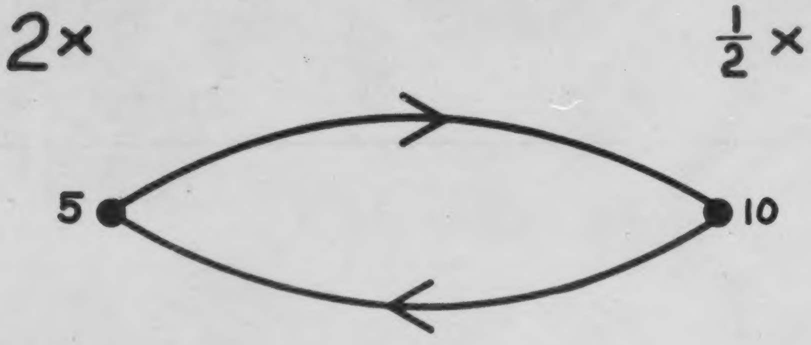
Label the dot "10" and draw the return arrow from 10 to 5 in blue.

T: What could this blue arrow be for?

S: $\frac{1}{2}$x.

If a student suggests that the blue arrow could be for − 5 (or + $\widehat{5}$), agree that 10 − 5 = 5 (or 10 + $\widehat{5}$ = 5), but then ask what else the blue arrow could be for.

When someone suggests $\frac{1}{2}\times$, write "$\frac{1}{2}\times$" near the arrow picture.



T: The blue arrow is for $\frac{1}{2}\times$. What number is $\frac{1}{2}\times 10$?

S: 5.

Erase "5" and "10". Point to the dot on the left.

T: If this number were 3, what number would the other dot be for?

S: 6.

T (tracing the red arrow): Yes, $2 \times 3 = 6$.

Continue in this manner with several other starting numbers at the left dot such as: 7; 20; 500; and 1,000.

Point to the dot on the right.

T: If this number were 8, what number would the other dot be for?

S: 4.

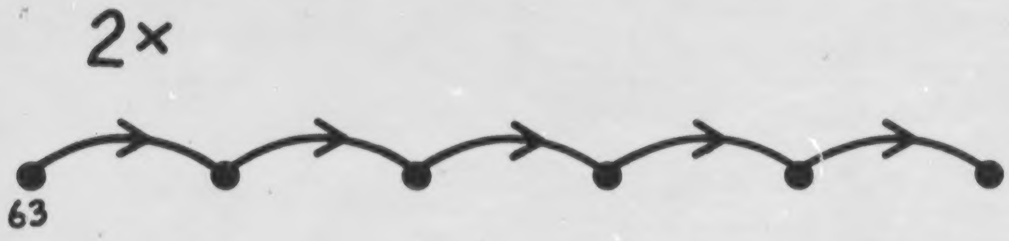
T (tracing the blue arrow): Yes, $\frac{1}{2} \times 8 = 4$.

Continue with several other starting numbers at the right dot such as: 10; 20; 80; 16; and 100.

Alternating dots, assign a number to one of the dots and ask what the other number in the picture would be. Choose numbers appropriate to the abilities of your students.

Exercise 2

Draw this arrow picture on the board.



T: What are the red arrows for?

S: 2x.

Point to the second dot from the left.

T: Tell me with whispers what number is here.

Accept whispers from several students and then ask someone to give the answer (126) aloud.

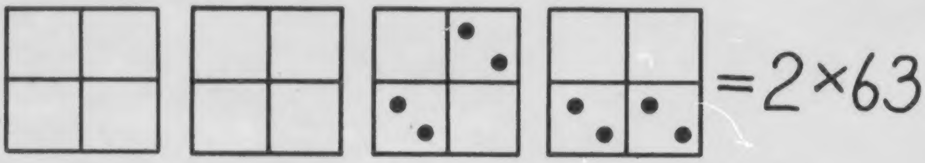
T: How can we calculate "2 x 63"?

S: Use the Minicomputer.

Display four Minicomputer boards.

T: How can we show 2 × 63 on the Minicomputer?

Guide the discussion until it is clear that 63 should be put on the Minicomputer twice, and then ask two students to each put 63 on the Minicomputer.

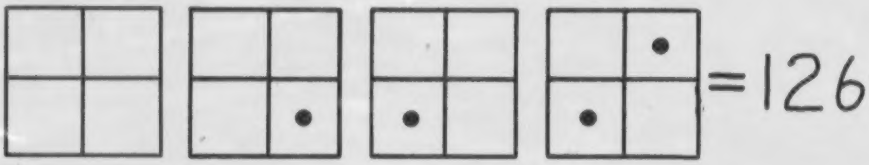


T: What could we do to make the number easier to read?

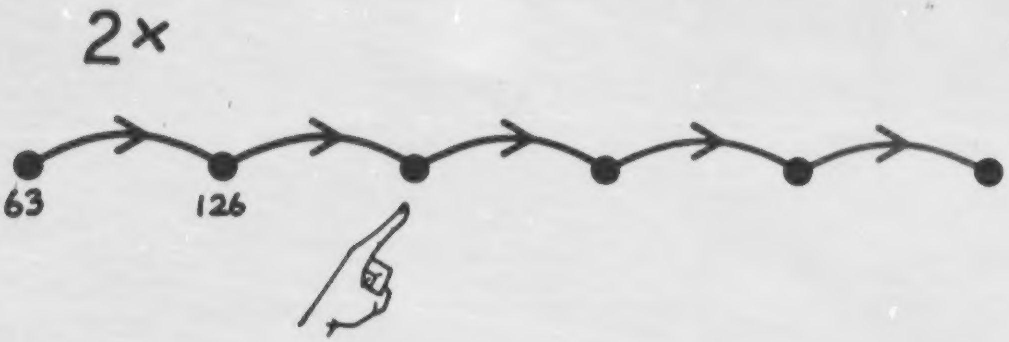
S: Make some plays.

Although the "20 + 20 = 40" play is not the most efficient, it is a correct play and should be accepted. If this play is made, later someone will need to make the "40 = 20 + 20" play in order to set up the "80 + 20 = 100" play. It may be necessary for you to suggest a backward play.

Invite students to make plays until the standard configuration for 126 is obtained.



Label the second dot "126" and point to the third dot.



N 26

T: Tell me with whispers what number is here.

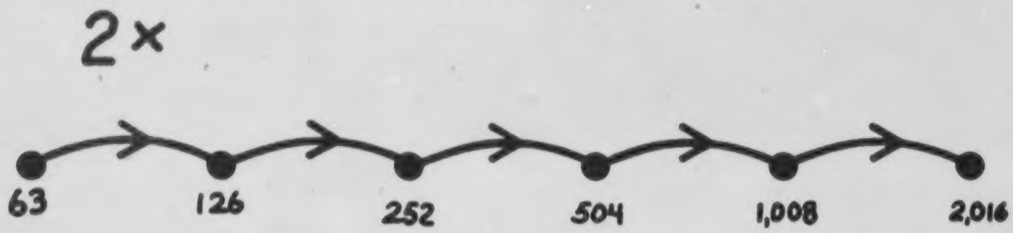
This may be a very difficult problem. Accept many whispers, but do not be surprised if none of your students answers correctly.

T: How can we calculate "2 x 126"?

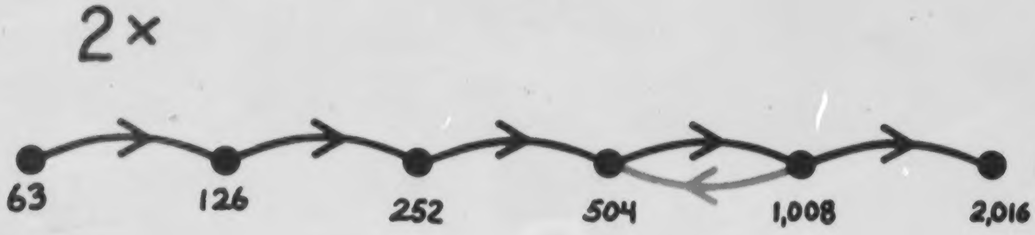
S: On the Minicomputer; put on 126 two times.

Ask two students to each put 126 on the Minicomputer. Then invite students to make plays until the standard configuration for 252 is obtained. Ask students who had whispered the correct answer to you earlier how they had done the calculation.

Continue in this manner until all the dots are labeled.



Draw a blue return arrow from 1,008 to 504.



T: What could this blue arrow be for?

S: $\frac{1}{2}$×.

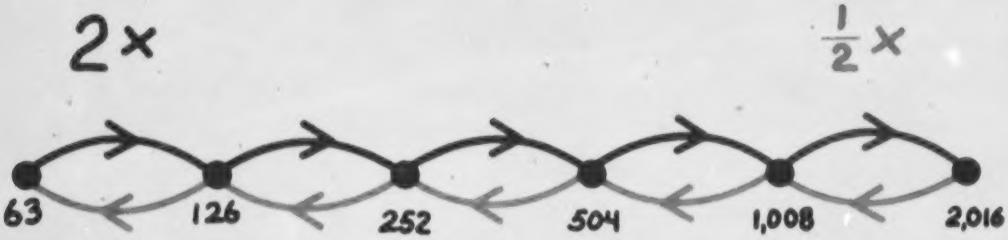
Write "$\frac{1}{2}$×" near the arrow picture.

T (tracing the blue arrow): $\frac{1}{2} \times 1{,}008 = 504$.

Write "$\frac{1}{2} \times 1{,}008 = 504$" on the board.

T: Could we draw some more blue arrows?

Invite students to draw the remaining $\frac{1}{2}$× arrows and for each one record the appropriate number story on the board.



$\frac{1}{2} \times 2{,}016 = 1{,}008$

$\frac{1}{2} \times 1{,}008 = 504$

$\frac{1}{2} \times 504 = 252$

$\frac{1}{2} \times 252 = 126$

$\frac{1}{2} \times 126 = 63$

N 27

N27 Multiplication and Subtraction #5

CAPSULE LESSON SUMMARY

Draw a picture with 4 x 9 dots and one with 9 x 4 dots. Conclude 4 x 9 = 9 x 4. Calculate "2 x 75" in a variety of ways by looking at 75 as a sum of smaller numbers. Check this calculation on the Minicomputer. Do several sequences of subtraction problems on the Minicomputer.

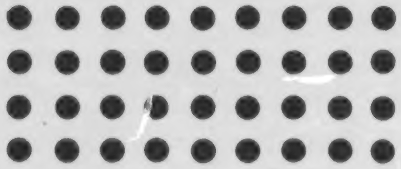
MATERIALS

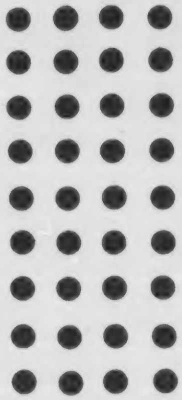
Teacher: Minicomputer set
Student: Paper; pencil; Minicomputer set

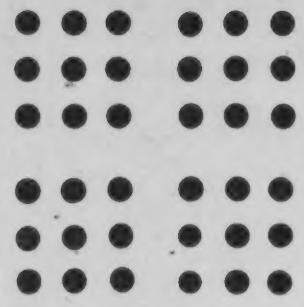
DESCRIPTION OF LESSON

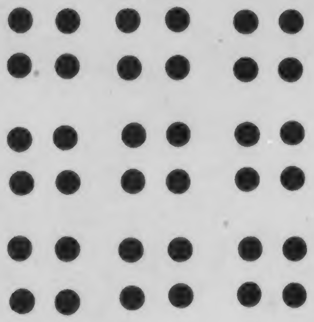
Exercise 1: Dot Pictures for Multiplication

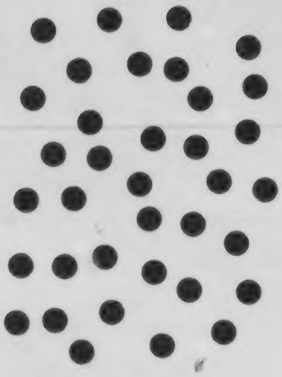
Ask the students to draw a picture with 4 x 9 dots and a picture with 9 x 4 dots. Let them solve this problem on their own, but watch what they draw and select a few of their pictures to show to the class. Choose pictures in which the dots are clearly visible. Some possibilities are shown below and on the next page.

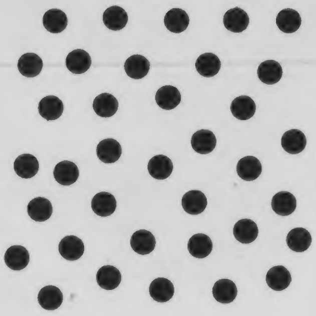












It is possible that some of your students will only draw one dot picture for both 4 x 9 and 9 x 4. Do not insist that these students draw a second picture, but ask them to explain why they drew only one picture. For example, a student might say, "4 x 9 = 36 and 9 x 4 = 36, so any picture with 36 dots is correct."

Ask the students which pictures they like the best and why and if there are any pictures which are clearer than others.

Do not criticize pictures in which the dots are not drawn in rows. They may be interesting from an aesthetic point of view. Be happy if there are 36 dots.

T: How many dots are in your picture with 4 x 9 dots?

S: 36.

T: How many dots are in your picture with 9 x 4 dots?

S: 36.

T: Are there any pictures where we could see that 9 x 4 = 4 x 9 without counting all the dots?

Any picture which is organized in rows and columns (4 rows and 9 columns, or 9 rows and 4 columns) nicely shows that 4 x 9 = 9 x 4, but actually any picture with 36 dots could be organized to show this.

Exercise 2: Multiplication on the Minicomputer

Display four Minicomputer boards. Distribute individual Minicomputers. Each student should have four Minicomputer boards, some checkers, and paper.

T: Put 2 x 75 on your (individual) Minicomputer. Remember that 2 x 75 is the same as 75 + 75. Do not make any plays now.

Give individual help as necessary to be sure everyone has put on 2 x 75. Ask someone to put 2 x 75 on the demonstration Minicomputer.

When students respond to these estimation questions, ask them to explain their answers.

T: Is this number more than 10? (Yes)
Is this number more than 100? (Yes)
Is this number more than 200? (No)

This last question may cause some confusion, but it is not essential that your class agree that the number is less than 200. These questions will direct the class towards a first estimation of the number.

T: How can we calculate "2 x 75" without making any plays on the Minicomputer?

Many suggestions are possible; for example,

$$\begin{aligned} 2 \times 75 &= (2 \times 50) + (2 \times 20) + (2 \times 5) \\ &= 100 + 40 + 10 \\ &= 150 \end{aligned}$$

or

$$\begin{aligned} 2 \times 75 &= (2 \times 40) + (2 \times 20) + (2 \times 10) + (2 \times 5) \\ &= 80 + 40 + 20 + 10 \\ &= 150 \end{aligned}$$

Encourage students to choose their own methods of calculating and let them share their ideas with the class. Indicate on the board some of their approaches to the calculations. Afterward, let the students calculate "2 x 75" by making plays on their individual Minicomputers.

Repeat this activity with a few of the following calculations. When the students have put a number on their Minicomputers, ask them to try to calculate the number before they make any plays. Let students share their methods with the class.

2 x 125
2 x 243
2 x 364
2 x 575

3 x 35
3 x 56
3 x 223
3 x 247

4 x 216
4 x 132
4 x 251
4 x 263

5 x 22
5 x 220
5 x 222
5 x 1,223

Be patient! The goal of these exercises is not to find results quickly, but to think of many ways to solve the problems mentally.

N27

Exercise 3: Subtraction Problems

T: Put 397 on your Minicomputer.

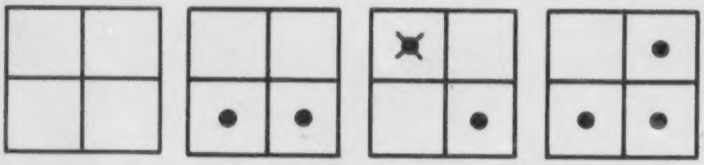
Give individual help so all students have 397 on their Minicomputers. Invite someone to put 397 on the demonstration Minicomputer.

To one side of the Minicomputer, write this problem on the board.

$$397 - 80 = ?$$

T: Calculate "397 - 80" on your Minicomputer.

Ask a student to take 80 off the demonstration Minicomputer and complete the number story on the board.



397-80=317

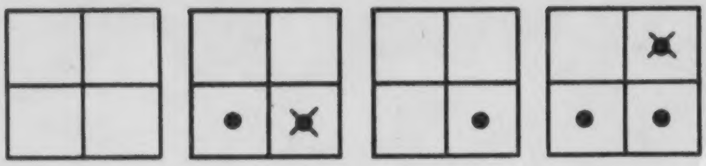
Check to make sure all your students have 317 on their Minicomputers.

Write this problem on the board under "397 - 80 = 317".

$$317 - 104 = ?$$

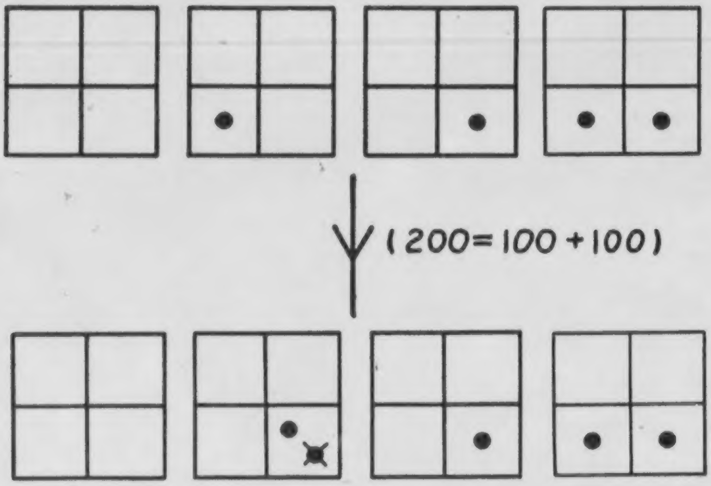
T: Calculate "317 - 104" on your Minicomputer.

Invite someone to do the subtraction on the demonstration Minicomputer and complete the number story on the board.



397-80=317
317-104=213

Write "213 - 100 = ? " on the board under the first two number stories and proceed as before. This calculation requires a backward play.



397-80=317
317-104=213
213-100= 113

Continue the sequence with these calculations:

113 - 101 = 12

12 - 11 = 1

1 - 1 = 0

N 27

Repeat this activity with other sequences of subtraction problems appropriate to the numerical ability of your class. Two possibilities are suggested below.

569 - 10 = 559
559 - 18 = 541
541 - 200 = 341
341 - 240 = 101
101 - 1 = 100
100 - 80 = 20

875 - 400 = 475
475 - 50 = 425
425 - 200 = 225
225 - 201 = 24
24 - 12 = 12
12 - 4 = 8

N28 The Functions + 10 and - 10

CAPSULE LESSON SUMMARY

Count by tens starting at 0 and then at 3. Relate the functions + 10 and - 10 through the use of a simple arrow diagram: the arrow for one is the return arrow for the other. Use return arrows to label the dots in a - 10 arrow picture. The worksheet has a Christmas theme.

MATERIAL

Teacher: Colored chalk

Student: Colored pencils; pencil; Worksheet N28

DESCRIPTION OF LESSON

Exercise 1: Mental Arithmetic

Indicate to your class an order for this activity which is natural to their seating arrangement, i.e., indicate which person is first, who is second, and so on.

T: Today we are going to count by tens. I will say "0", the first student will say "10"; the second will say "20" because 10 + 10 = 20; the next "30" because 20 + 10 = 30; and so on.

T: How many students are here today?

Determine how many students are present.

T: If everyone has exactly one turn, what number will the last person say?

Record some of the students' predictions on the board.

T: 0.

N 28

Walk around the room as students count by tens until each student has said a number. Make corrections if necessary. Compare the students' predictions with the last number said, commending those who correctly predicted which number would be said last. Repeat this activity again but start with a different student and this time ask the students to remember their numbers. Then go on to another counting by tens activity as described in the dialogue which follows.

T: We are going to count by tens again, but this time I will start by saying the number 3. Each person will say the number which is ten more than the last number. Remember which number you say.

Say "three" and then point to the student who was first during the last counting. If the student hesitates, ask, "What number is 3 + 10?" Continue in exactly the same order as in the last counting. If a student has difficulty, hold up ten fingers and ask the student to count (by ones) from the previous number.

T: Why did I ask you to remember the number you said before?

S: The number I said this time was three more than the number I said last time.

If this response is not made or is not clear, ask several students, "What was your number last time we counted? . . . this time?", until the relationship becomes clear.

T: Let's write some of these numbers on the board. I said "3."

Write "3" on the board.

T: What number came after 3? (13)

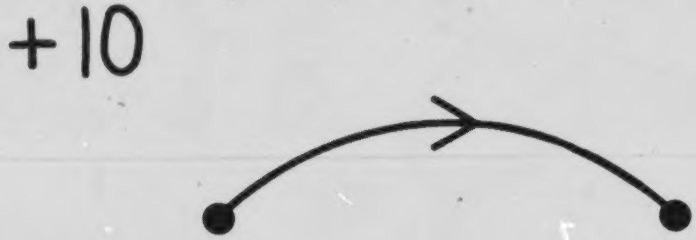
Record "13" below "3". Continue in this way until "113" is written on the board.

T: Does anyone see a pattern?

Very likely students will observe that all the numbers end in 3 and if the last digit from each numeral were removed, the new numbers would be increasing by 1.

Exercise 2

Draw this arrow picture on the board.



Point to each dot.

T: Where is the smaller number in this picture?

The students should indicate the dot on the left. Point to this dot.

T: If this number were 40, what number would the other dot be for? (50)
If this number were 100, . . . ? (110)
If this number were 5, . . . ? (15)
If this number were 15, . . . ? (25)
If this number were 25, . . . ? (35)

Point to the dot on the right.

T: If this number were 30, what number would the other dot be for?

S: 20, because 30 - 10 = 20.

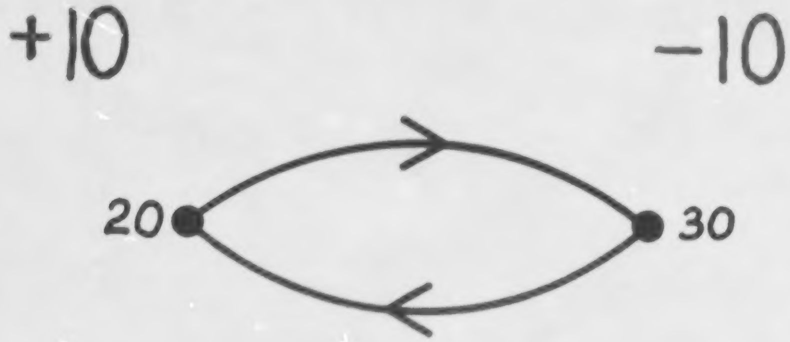
N28

If a student says "20, because 20 + 10 = 30," ask if someone did it another way. Label the dots "20" and "30". Draw a blue return arrow from 30 to 20.

T: What could this blue arrow be for?

S: -10.

If a student suggests $+\overset{\frown}{10}$, record it on the board but ask if there is another name we could give this arrow. When a student suggests -10, write "-10" on the board near the arrow picture.



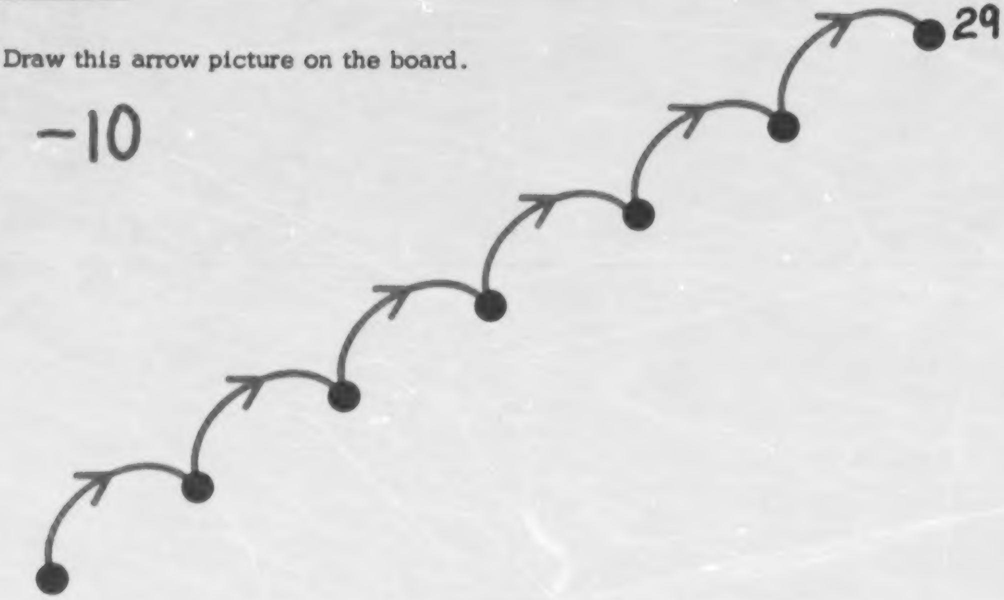
Erase "20" and "30". Point to the dot on the right.

T: If this number were 100, what would the other number be? (90)
If it were 14, . . . ? (4)
If it were 38, . . . ? (28)
If it were 10, . . . ? (0)
If it were 500, . . . ? (490)

Continue this activity with other numbers appropriate for the abilities of your students. Alternating dots, assign a number to one of the dots and ask what the other number in the picture is.

Exercise 3

Draw this arrow picture on the board.



T: Where is the smallest number in this arrow picture? (At the upper right)

What is the smallest number in this arrow picture? (29)

Where is the largest number in this arrow picture? (At the lower left)

Tell me with whispers what the largest number is.

This last question may challenge your best students. Accept several whispers, but do not announce the correct answer now. Point to the second dot from the right.

T: Who can label this dot?

Let a student label the dot "39".

T: How do you know this number is 39?

S: 29 + 10 = 39.

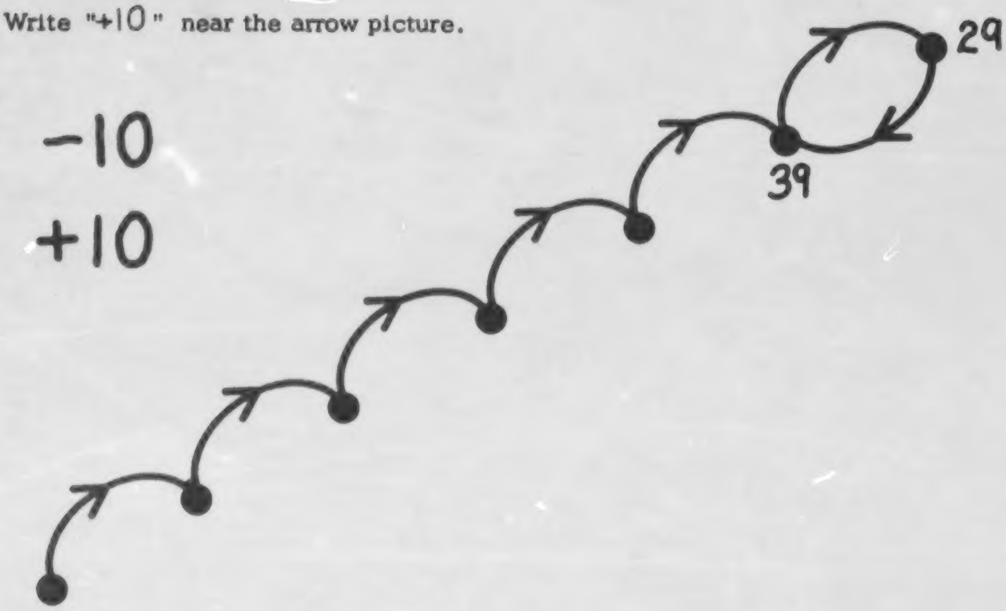
If "39 - 10 = 29" is suggested, ask if someone did it another way. Draw a red arrow from 29 to 39.

N28

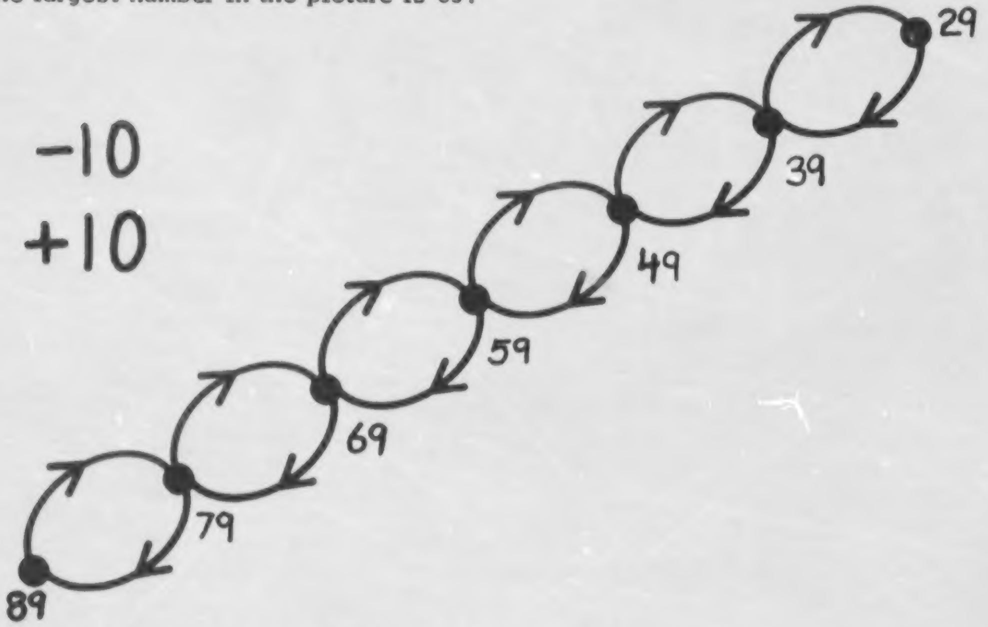
T: What could this arrow be for?

S: + 10.

Write "+10" near the arrow picture.



Continue by asking students to either label a dot or to draw another arrow until the arrow picture is complete. Acknowledge students who predicted that the largest number in the picture is 89.


29
-10
39
+10
49
59
69
79
89

T: Does anyone see a pattern to the numbers in this picture?

S: The numbers in this picture all end in 9.

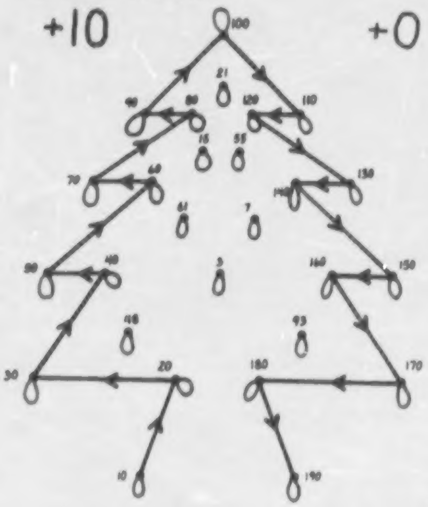
S: 89, 79, 69, . . . , 29. The first number (digit) is getting smaller by one each time.

Worksheet N28 is available for independent work. Some students may need help recognizing that a + 0 arrow is always a loop. Allow the students to draw the arrows as they wish; gracefully curved arrows might be aesthetically more pleasing to some students.

Name ____________________

N 28 *

Draw all the green and red arrows.


+10
+0
100
21
90
80
120
110
15
55
70
60
140
130
61
7
50
40
5
160
150
48
93
30
20
180
170
10
190

N29 Multiplication and Subtraction #6

CAPSULE LESSON SUMMARY

Calculate "2 × 65" in a variety of ways by looking at 65 as the sum of smaller numbers. Check the calculation on the Minicomputer. Use "2 × 65 = 130" to calculate "2 × 265" and "2 × 565". Do some sequences of subtraction problems on the Minicomputer.

MATERIALS

Teacher: Minicomputer set

Student: Minicomputer set; pencil; Worksheets N29*, **, ***, and ****

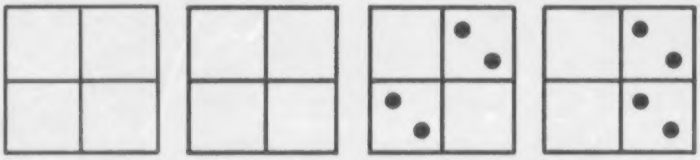
DESCRIPTION OF LESSON

Exercise 1: Multiplication Problems

Display four Minicomputer boards and distribute individual Minicomputers. Each student will need four Minicomputer boards (two individual Minicomputer sheets), some checkers, and paper.

T: Put 2 × 65 on your (individual) Minicomputer.

Invite a student to put 2 × 65 on the demonstration Minicomputer.



T: How can we calculate this number without making any plays on the Minicomputer?

Let the students explain their methods of doing the calculation; two of the many possibilities are indicated on the next page.

$$\begin{aligned} 2 \times 65 &= (2 \times 60) + (2 \times 5) \\ &= 120 + 10 \\ &= 130 \end{aligned}$$

and

$$\begin{aligned} 2 \times 65 &= (2 \times 40) + (2 \times 20) + (2 \times 5) \\ &= 80 + 40 + 10 \\ &= 130 \end{aligned}$$

Afterward, ask the students to calculate "2 x 65" by making plays on their Minicomputers. Record this number story on the board.

$$2 \times 65 = 130$$

T: Put 2 x 265 on your Minicomputer. Try to calculate "2 x 265" without making plays on the Minicomputer.

Invite a student to put 2 x 265 on the demonstration Minicomputer.

T: We already know that 2 x 65 = 130. Does this help us solve the new problem?

S: 2 x 65 = 130, and 2 x 200 = 400, so 2 x 265 = 530.

To emphasize this method, point to the checkers on the Minicomputer which show 2 x 65 and say, "2 x 65 = 130." Then point to the checkers which show 2 x 200 and say, "2 x 200 = 400." Conclude that 2 x 265 = 400 + 130 = 530.

Let the students check these calculations by making plays on their Minicomputers.

T: Put 2 x 565 on your Minicomputer. Try to calculate "2 x 565" without making any plays.

Invite someone to put 2 x 565 on the demonstration Minicomputer.

T: We already know that 2 x 65 = 130. Does this help us to calculate "2 x 565"?

S: 2 x 65 = 130 and 2 x 500 = 1,000, so 2 x 565 = 1,130.

To emphasize this method, point to the checkers on the Minicomputer which show 2 x 65 and say, "2 x 65 = 130." Then point to the checkers which show 2 x 500 and say, "2 x 500 = 1,000." Conclude that 2 x 565 = 1,000 + 130 = 1,130.

Repeat this activity with the following sequence of calculations or a similar one appropriate for your class.

3 x 25
3 x 125
3 x 325
3 x 425

Exercise 2: Subtraction Problems

T: Put 689 on your Minicomputer.

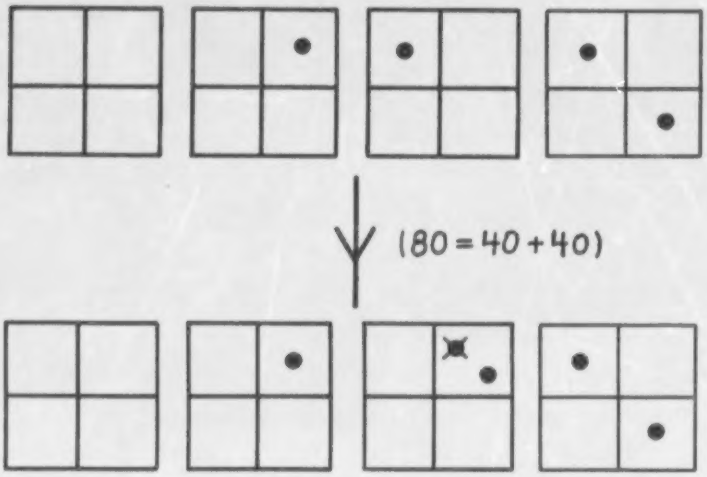
Invite someone to put 689 on the demonstration Minicomputer. To one side of the Minicomputer, write this problem.

689-40=?

T: Calculate "689 - 40" on your Minicomputer.

N 29

A backward play will be needed before 40 can be taken off the Minicomputer. Direct the students to make the play and then to do the calculation. Complete the number story on the board.



689-40=649

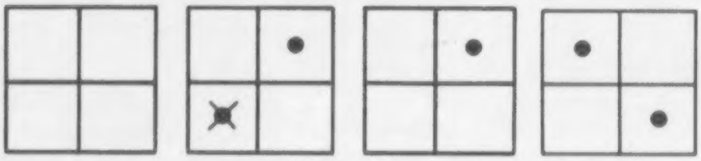
T: Everyone should have 649 on their Minicomputers.

Occasionally check that the students have the correct number on their Minicomputers. Write this problem under "689 - 40 = 649".

649-200=?

T: Calculate "649 - 200" on your Minicomputer.

Let a student do the calculation on the demonstration Minicomputer, in this case, removing the checker from a 200-square. Complete the number story on the board.

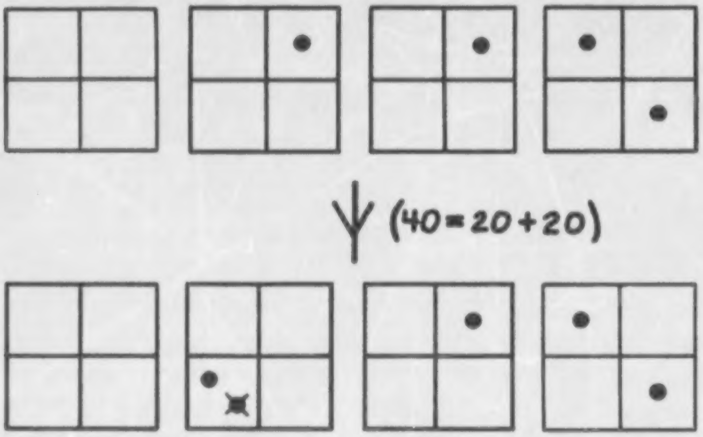


689-40=649
649-200=449

Write this problem under "649 - 200 = 449".

449-200=?

Ask the students to take 200 away from the 449 on their Minicomputers. A backward play is needed before 200 can be taken off.



689-40=649
649-200=449
449-200=249

Continue the lesson with the following sequences of subtraction problems or similar ones appropriate to the numerical ability of your students.

249 - 41 = 208
208 - 4 = 204
204 - 100 = 104
104 - 104 = 0

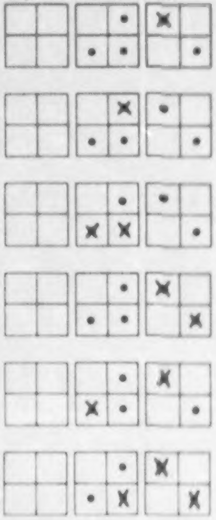
567 - 41 = 526
526 - 10 = 516
516 - 402 = 114
114 - 80 = 34

Worksheets N29*, **, ***, and **** are available for independent work.

Name

N 29 *

Complete these number stories.



79 - 8 = 71

79 - 40 = 39

79 - 30 = 49

79 - 9 = 70

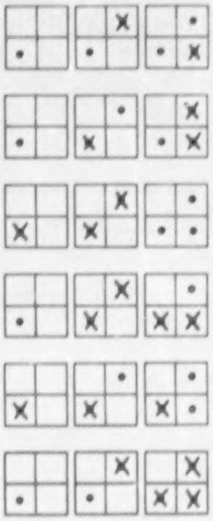
79 - 28 = 51

79 - 19 = 60

Name

N 29 **

Complete these number stories.



267 - 41 = 226

267 - 25 = 242

267 - 260 = 7

267 - 63 = 204

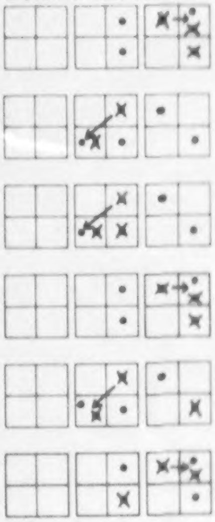
267 - 222 = 45

267 - 47 = 220

Name

N 29 ***

Complete these number stories.



59 - 4 = 55

59 - 20 = 39

59 - 30 = 29

59 - 5 = 54

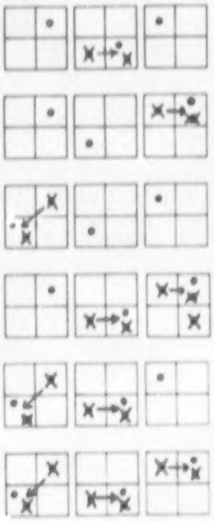
59 - 21 = 38

59 - 14 = 45

Name

N 29 ****

Complete these number stories.



428 - 10 = 418

428 - 4 = 424

428 - 200 = 228

428 - 14 = 414

428 - 210 = 218

428 - 214 = 214

N30 Roads #2

CAPSULE LESSON SUMMARY

Build a road from 0 to 14, collectively, using + 5 and - 3 arrows.

Build a road from 10 to 25, individually, using + 3 and - 2 arrows.

MATERIALS

Teacher: Colored chalk
Student : Colored pencils ; pencil ; unlined paper ;
Worksheets N30 * and **

DESCRIPTION OF LESSON

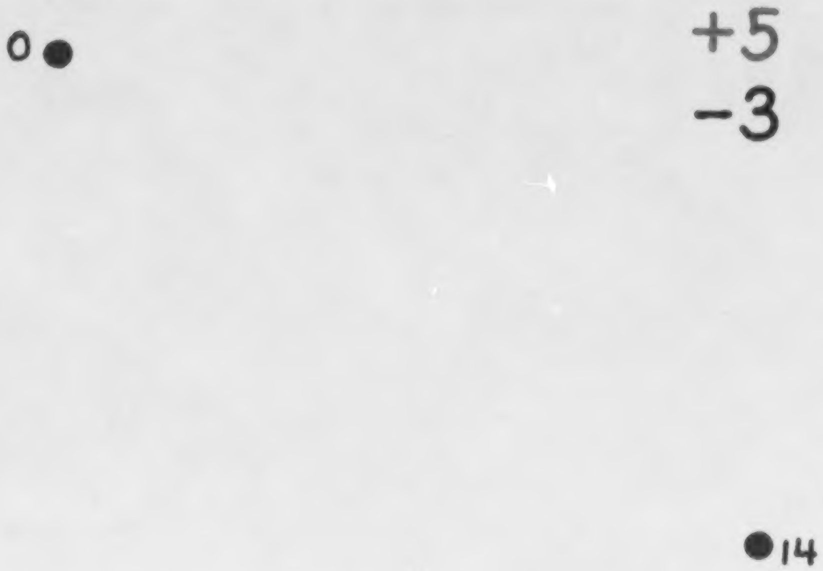
Exercise 1

Draw and label two dots on the board.

N 30

T: Today we are going to build a road from 0 to 14. Before we start, we need to know what kind of arrows we can use to build our road.

Write "+5" and "−3" near the dots.

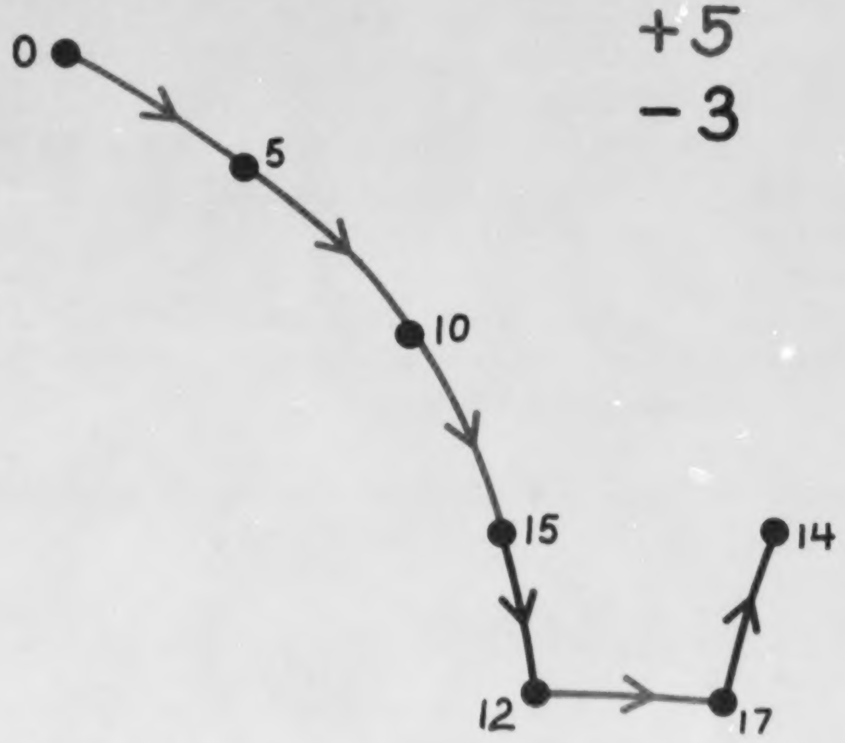


T: We can use blue and red arrows. The blue arrows are for + 5 and the red arrows are for − 3. With which kind of arrow should we start the road?

Take whatever suggestion is made and draw the arrow yourself. Point to the ending dot of this arrow.

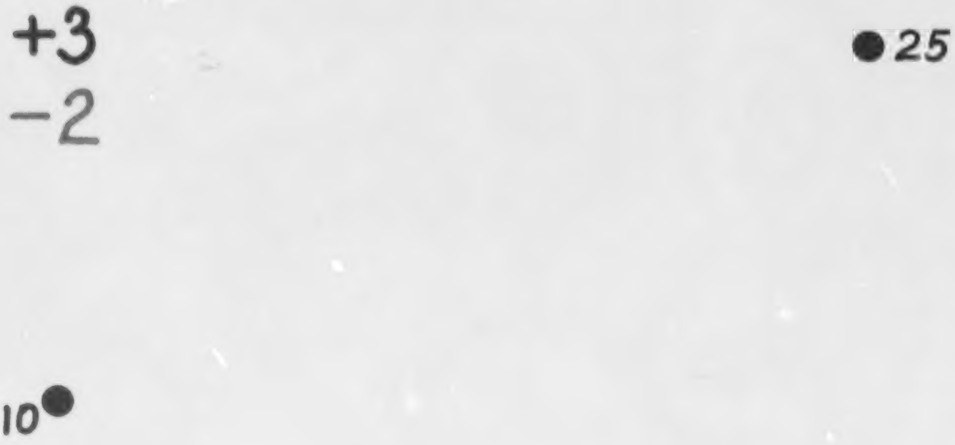
T: What number is here? (5, if a blue arrow was drawn; $\widehat{3}$, if a red arrow was drawn)

When someone gives the correct number, label the dot. Ask for volunteers to complete the road, one arrow at a time. This is one possible road from 0 to 14.



Exercise 2

Each student should have unlined paper, a red pencil, and a blue pencil. Put this information on the board.



N 30

Ask the students to draw a road from 10 to 25 on their papers, using + 3 and - 2 arrows. Encourage them to use the entire sheet of paper. Walk around as the students work, giving assistance to those who need it. As the rest of the class continues to work, ask two or three students with different numbers of arrows in their roads to copy their pictures on the board. When most of the students have finished, call the class' attention to the examples on the board. Check each picture on the board with the class to see that it is correct. Point out that there are many correct solutions to this problem.

Ask which road on the board has the fewest arrows and whether anyone has a road with even fewer arrows. Allow some discussion of how to draw the shorttest (fewest number of arrows) road.

Worksheet N30* and ** are available for independent work. Students who finish quickly can build all or some of the following roads on unlined paper.

1) Build a red-blue road from 0 back to 0 using − 5 and + 2 arrows.

2) Build a red-blue road from 0 back to 0 using − 4 and + 3 arrows.

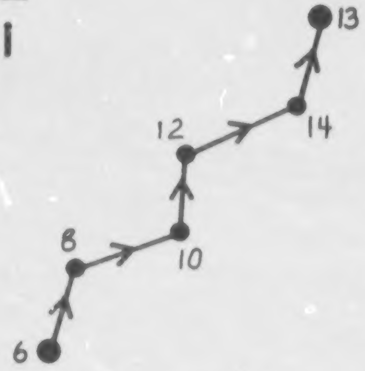
3) Build a red-blue road from 61 back to 61 using − 4 and + 3 arrows.

Name______________________ **N 30 ***

Build a red-blue road from 6 to 13.

+2

−1


13
12
14
8
10
6

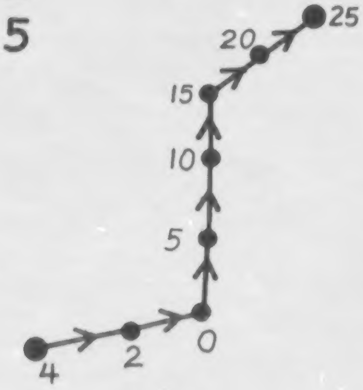
(Many solutions are possible.)

Name______________________ **N 30 ****

Build a red-blue road from 4 to 25.

−2

+5


25
20
15
10
5
0
2
4

(Many solutions are possible.)

N31

N31 Addition Algorithm #1

CAPSULE LESSON SUMMARY

Introduce a play which replaces ten checkers on a square with one checker on the square of the same color of the next board to the left. Present an addition algorithm.

MATERIALS

Teacher: Minicomputer set

Student: Paper; pencil; Addition Problems booklet.

DESCRIPTION OF LESSON

Exercise 1: Multiplying by 10

Beginning in the first weeks of first grade, CSMP students were introduced to the three binary rules of the Minicomputer.

$1 + 1 = 2 \qquad 2 + 2 = 4 \qquad 4 + 4 = 8$

Very soon they learned to jump from the first board to the second by applying the decimal rule.

$8 + 2 = 10$

Gradually they realized that these four rules can be extended to all of the Minicomputer boards.

$10 + 10 = 20 \qquad 100 + 100 = 200$

$20 + 20 = 40 \qquad 200 + 200 = 400$

$40 + 40 = 80 \qquad 400 + 400 = 800$ and so on.

$80 + 20 = 100 \qquad 800 + 200 = 1,000$

In the language of the Minicomputer, these basic facts can be easily verbalized.

On each board:

two checkers on the white square
is the same as
one checker on the red square

two checkers on the red square
is the same as
one checker on the purple square

two checkers on the purple square
is the same as
one checker on the brown square

From board to board:

one checker on the brown square and one checker on the red square
is the same as
one checker on the white square of the next board to the left.

Right from the beginning, the children's numerical thinking has been supported by the binary structure of the Minicomputer boards.

We are ready now to introduce a new basic fact which is a consequence of the four Minicomputer rules and which is the key to the decimal positional system.

ten checkers on a square is the same as one checker on the square of the same color of the next board to the left

Developing this new basic fact is the goal of several lessons in the World of Numbers strand. It has been our experience that some children find the new idea difficult to grasp. Therefore, you should be patient; you should not expect all the students to assimilate this idea at the same moment.

About 20 minutes of several lessons are devoted to this topic in order to allow the child's thinking to mature in a spontaneous and natural way.

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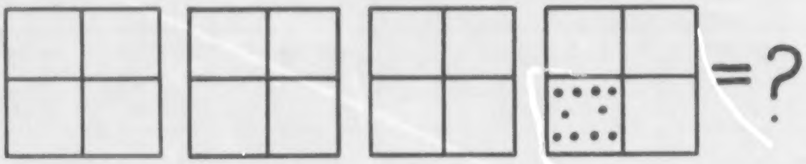
Exercise 1

Put this configuration on the Minicomputer.

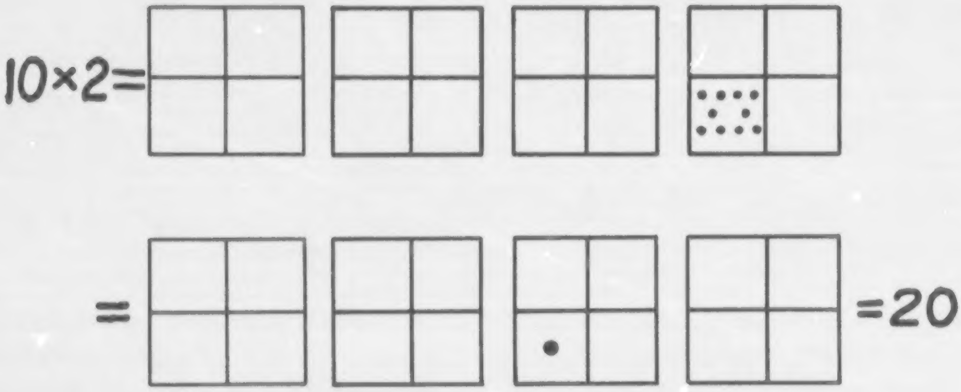


T: What number is this?

What multiplication story could we write about this number?

Do we need to make some plays before we know which number this is?

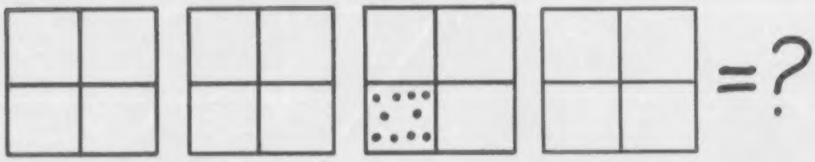
Let the students react freely to these questions. Probably many of them already know that 10 x 2 = 20 and they do not need to make plays on the Minicomputer in order to find the answer. Nevertheless, encourage them to make the usual plays until they obtain the standard configuration for 20.



Record this number sentence on the board.

$$10 \times 2 = 20$$

Put this configuration on the Minicomputer and ask the students to think about which number this is.



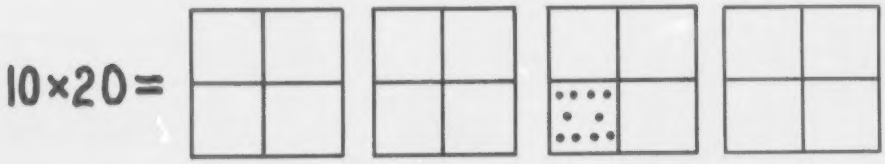
T: What do you think about this new problem? Is it very different from the first one?

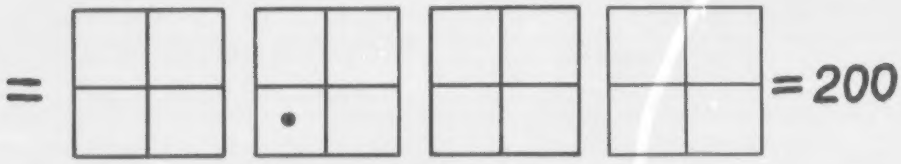
S: We still have ten checkers on the same square, but this time they are all on the 20-square instead of the 2-square.

T: If we were to make plays on the Minicomputer, how would they be different from the moves we made when the checkers were on the 2-square?

S: We'd make plays the same way, except starting on the tens' board. The ones' board is empty.

Let students make plays until they obtain the standard configuration for 200.





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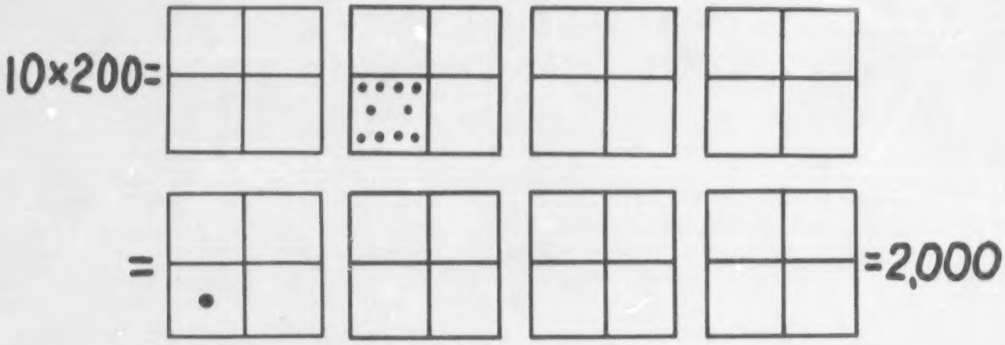
Record this number story under "10 × 2 = 20".

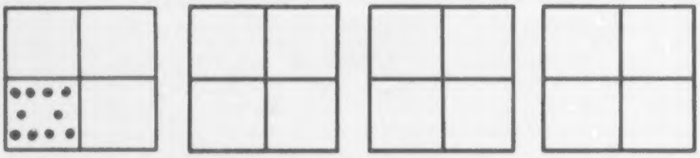
$$10 \times 20 = 200$$

Repeat this activity with ten checkers on the 200-square.



$$10 \times 2 = 20$$
$$10 \times 20 = 200$$
$$10 \times 200 = 2{,}000$$

Put this configuration on the Minicomputer.



T: Let's see if we can figure out what number 10 × 2,000 is without making plays on the Minicomputer.

Write this problem in your list on the board.

$$10 \times 2 = 20$$
$$10 \times 20 = 200$$
$$10 \times 200 = 2{,}000$$
$$10 \times 2{,}000 = ?$$

T: Do you see a pattern?

Allow the students to express themselves.

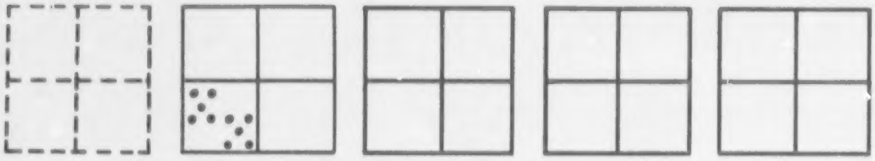
T: Can anyone tell us what number we should write to continue the pattern?

S: 20,000.

T: Who can write this number?

NOTE: A student may be able to write the next number without knowing how to read it.

If no one is able to write "20,000", draw a fifth Minicomputer board directly on the chalkboard.



Allow the students to make plays (if your chalkboard is not magnetic, they will have to draw checkers on the fifth Minicomputer board). When the standard configuration for 20,000 is obtained, ask a student to write the number below the Minicomputer and another to complete the number story on the board.

Students may need help placing the comma in a number larger than 999. If necessary, insert the comma yourself. Explain that the comma helps us to read large numbers and that it goes between the thousands' place (board) and the hundreds' place (board).

Exercise 2

Write this problem on the board.

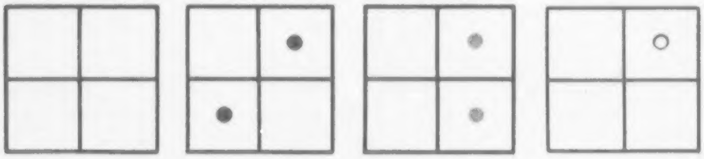
$$654+135=?$$

T: What number is 654 + 135?

Record students' predictions on the board.

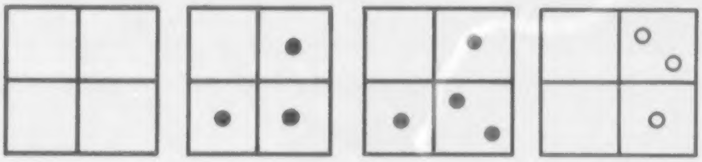
T: Let's add 654 and 135 on the Minicomputer.

Put 654 on the Minicomputer using red checkers on the hundreds' board, blue checkers on the tens' board, and yellow checkers on the ones' board.

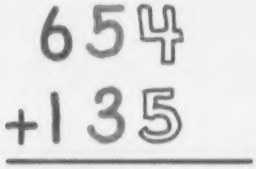


In the illustrations in this lesson, "○" indicates a yellow checker.

Add 135 to the 654 on the Minicomputer, again using red checkers on the hundreds' board, blue checkers on the tens' board, and yellow checkers on the ones' board.



Write this calculation on the board with colors corresponding to the checkers.



In the illustrations in this lesson, an outlined numeral, e.g. "5", indicates a numeral in yellow.

Stand in front of the tens' and hundreds' boards and direct the class to look only at the ones' board.

T: What number is on the ones' board?

S: 9.

T: Can we make a play?

S: Yes, we can make a "4 + 4 = 8" play.

Let a student make this play and then ask another student to write "9" (in yellow) below the ones' board.

Point to the ones' column of the addition problem written on the board. Cover the other two columns and ask:

T: What do you see in the ones' column of this addition problem?

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S: "4" and "5".

T: 4 + 5 = . . . ?

S: 9.

$$\begin{array}{r} 654 \\ +135 \\ \hline 9 \end{array}$$

T: So we have 9 on the ones' board and below the ones' column.

Stand in front of the hundreds' board and direct the class to look at the tens' board.

T: How many tens are on this board? Can we make a play?

Let students make appropriate plays on the tens' board and then ask a student to write "8" (in blue) below the tens' board.

Point to the tens' column of the addition problem written on the board.

T: What do you see in the tens' column of this addition problem?

S: "5" and "3".

T: Five tens plus three tens is . . . ?

S: Eight tens.

$$\begin{array}{r} 654 \\ +135 \\ \hline 89 \end{array}$$

T: So we have 8 on the tens' board and below the tens' column.

Direct the class to look at the hundreds' board.

T: How many hundreds are on this board?

S: Seven.

Ask a student to write "7" (in red) below the hundreds' board. Point to the hundreds' column of the addition problem written on the board.

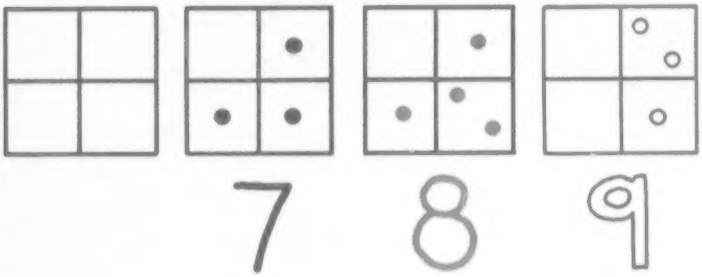
T: What do you see in the hundreds' column?

S: "6" and "1".

T: Six hundreds plus one hundred is . . . ?

S: Seven hundreds.

654+135=?



```
 654
+135
 789
```

T: What number is 654 + 135?

S: 789.

Complete the number story on the board. Compare the result with the students' predictions, commending those who correctly predicted the answer. Keep this calculation on the board and write these problems on another part of the board.

$$\begin{array}{r} 325 \\ +263 \\ \hline \end{array} \qquad \begin{array}{r} 417 \\ +482 \\ \hline \end{array} \qquad \begin{array}{r} 614 \\ +272 \\ \hline \end{array}$$

Divide the class into three groups and assign one of these addition problems to each group. Allow the students to work independently. Students who finish quickly should be encouraged to solve the problems assigned to other groups. When most of the students have attempted to solve at least one problem, ask a student from each group to present its problem on the board. Do not insist that students work from right to left, although you should do so when demonstrating an addition calculation or when clarifying a student's solution.

Exercise 3

Erase the board and remove the checkers from the Minicomputer. Write this problem on the board.

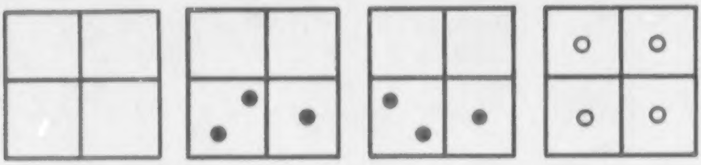
$$239+326=?$$

T: What number is 239 + 326?

Record some guesses on the board.

T: Let's add 239 and 326 on the Minicomputer.

As in Exercise 2, put checkers of three colors on the Minicomputer.



Write this calculation on the board with colors corresponding to the checkers.

$$\begin{array}{r} 239 \\ +\,326 \\ \hline \end{array}$$

Stand in front of the tens' and hundreds' board and direct the class to look only at the ones' board.

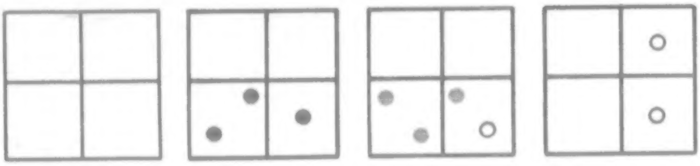
T: What number do you see on the ones' board?

S: 15.

T: Can we make a play?

S: Yes, a "8 + 2 = 10" play.

Ask a student to make the "8 + 2 = 10" play.



T: What number is on the ones' board now?

S: 5.

Ask a student to write "5" (in yellow) below the ones' board.

T: Before we had 15 on the ones' board and now there is only 5. What happened?

Be sure the students understand that ten ones were moved to the tens' board. Point to the addition problem written on the board.

T: How could we show in this calculation (point to where the problem is written on the board) that we now have another checker on the tens' board?

Allow the students to discuss this situation.

T: This is the way that many people would show this. How many more tens do we have?

S: One.

T: I will write "1" above the tens' column. This shows that we have another ten.

NOTE: The "1" should be written in yellow to correspond to the yellow checker on the tens' board.

$$\begin{array}{r} 1 \\ 239 \\ +\,326 \\ \hline \end{array}$$

T: What should we write below the ones' column?

S: "5".

T: Yes, there is 5 left on the ones' board.

$$\begin{array}{r} 1 \\ 239 \\ +\,326 \\ \hline 5 \end{array}$$

T: Now we have one more ten on the tens' board (point to the yellow checker on the tens' board) and in the tens' column, and we have 5 on the ones' board and below the ones' column.

Direct the class to look at the tens' board of the Minicomputer.

T: How many tens are on this board? Can we make a play?

Invite students to make plays on the tens' board, and then ask a student to write "6" (in blue) below the tens' board.

T: Did we move any more checkers to the other boards?

S: No.

Point to the tens' column of the addition problem written on the board.

T: What do you see in the tens' column?

S: "1", "3", and "2".

T: One ten plus three tens plus two tens is . . . ?

S: Six tens.

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$$\begin{array}{r} 1 \\ 239 \\ +\,326 \\ \hline 65 \end{array}$$

T: Did we need to write anything above the hundreds' column?

S: No, because we didn't make any plays which put more checkers on the hundreds' board.

T: What do you see in the hundreds' column?

S: "2" and "3".

T: What is two hundreds plus three hundreds?

S: Five hundreds.

T: How many hundreds are on this board (point to the hundreds' board)? Can we make a play?

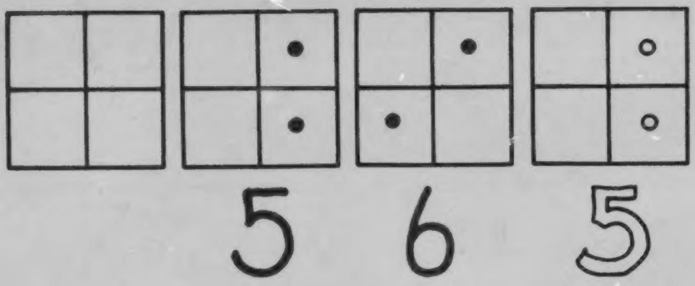
Invite a student to make the "200 + 200 = 400" play and another student to write "5" in red below the hundreds' board and below the hundreds' column.

T: What number is 239 + 326?

S: 565.

Complete the number story on the board.

239+326=565



$$\begin{array}{r} \overset{1}{2}39 \\ +326 \\ \hline 565 \end{array}$$

The worksheets for the entire sequence of lessons involving the addition algorithm for both Part 1 and Part 2 of CSMP Mathematics for the Upper Primary Grades are contained in a 16 page booklet, "Addition Problems". The students should begin on page 2 today and progress at their own pace. Allow about 10 minutes today for independent work in this problem book. After the lesson, collect the booklets, check them, and store them for future use in the remaining lessons on the addition algorithm, i.e., UPG I: N33 and UPG II: N7, N9, and N25.

Addition Problems

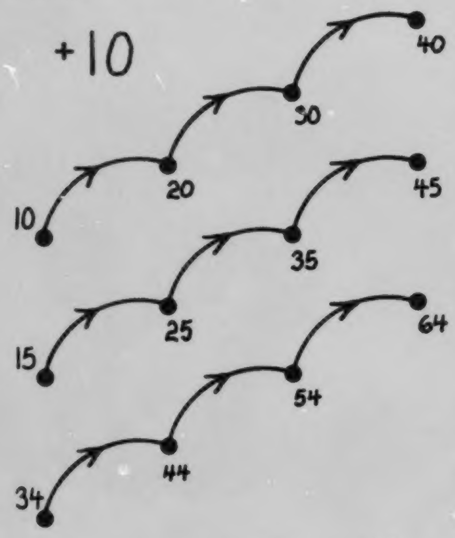
Calculate:

22 +43 65	53 +21 74	62 +15 77
47 +12 59	36 +52 88	30 +38 68
43 +46 89	42 +26 68	53 +44 97

Calculate:

42 +31 73	47 +51 98	231 +423 654
51 +16 67	68 +21 89	143 +222 365
59 +30 89	56 +23 79	528 + 21 549

Label the dots.


+10
10
20
30
40
15
25
35
45
34
44
54
64

Calculate:

50 + 14 = 64

25 + 24 = 49

65 + 13 = 78

17 + 50 = 67

12 + 72 = 84

27 + 52 = 79

42 + 36 = 78

35 + 24 = 59

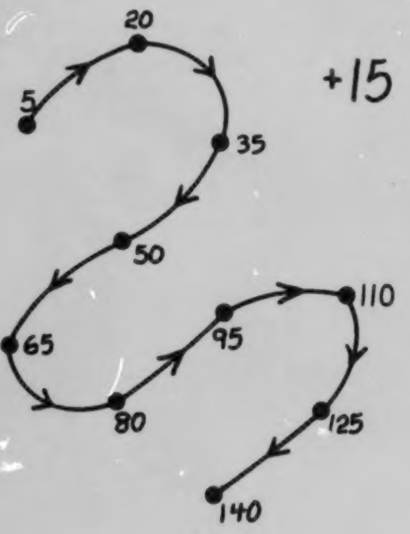
Calculate:

$\begin{array}{r}63\\+17\\\hline 80\end{array}$	$\begin{array}{r}42\\+20\\\hline 62\end{array}$	$\begin{array}{r}350\\+240\\\hline 590\end{array}$
$\begin{array}{r}68\\+24\\\hline 92\end{array}$	$\begin{array}{r}65\\+31\\\hline 96\end{array}$	$\begin{array}{r}132\\+67\\\hline 199\end{array}$
$\begin{array}{r}35\\+45\\\hline 80\end{array}$	$\begin{array}{r}48\\+43\\\hline 91\end{array}$	$\begin{array}{r}207\\+177\\\hline 384\end{array}$

Calculate:

$\begin{array}{r}20\\+36\\\hline 56\end{array}$	$\begin{array}{r}42\\+29\\\hline 71\end{array}$	$\begin{array}{r}416\\+322\\\hline 738\end{array}$
$\begin{array}{r}19\\+57\\\hline 76\end{array}$	$\begin{array}{r}52\\+27\\\hline 79\end{array}$	$\begin{array}{r}321\\+272\\\hline 593\end{array}$
$\begin{array}{r}48\\+32\\\hline 80\end{array}$	$\begin{array}{r}66\\+15\\\hline 81\end{array}$	$\begin{array}{r}23\\+356\\\hline 379\end{array}$

Label the dots.



Calculate:

$\begin{array}{r}25\\+49\\\hline 74\end{array}$	$\begin{array}{r}42\\+25\\\hline 67\end{array}$	$\begin{array}{r}437\\+28\\\hline 465\end{array}$
$\begin{array}{r}63\\+12\\\hline 75\end{array}$	$\begin{array}{r}27\\+44\\\hline 71\end{array}$	$\begin{array}{r}145\\+333\\\hline 478\end{array}$
$\begin{array}{r}24\\+52\\\hline 76\end{array}$	$\begin{array}{r}65\\+19\\\hline 84\end{array}$	$\begin{array}{r}364\\+121\\\hline 485\end{array}$

Calculate:

35 + 45 = 80

263 + 121 = 384

74 + 26 = 100

343 + 434 = 777

26 + 46 = 72

34 + 27 = 61

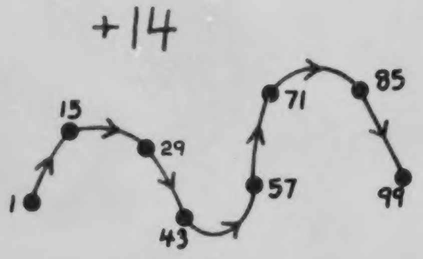
Calculate:

25 +46 71	62 +25 87	37 +28 65
63 +43 106	45 +53 98	47 +44 91
32 +56 88	55 +17 72	20 +76 96
48 +12 60	84 +50 134	59 +23 82

Calculate:

37 +17 54	58 +35 93	424 +355 779	746 + 45 791
37 +36 73	58 +60 118	123 +717 840	328 +533 861

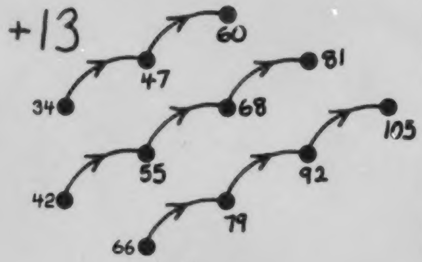
Label the dots.


+14
1
15
29
43
57
71
85
99

Calculate:

450 +139 589	263 +165 428	88 +25 113
356 +244 600	751 + 59 810	89 +88 177

Label the dots.


+13
34
47
60
42
55
68
81
66
79
92
105

Calculate:

271
+ 68
339

532
+516
1,048

258
+742
1,000

777
+213
990

74
+643
717

199
+144
343

520
+299
819

426
+574
1,000

355
+ 97
452

14

Calculate:

234
345
+213
792

422
255
+125
802

372
338
+355
1,065

2,323
56
+ 341
2,720

17
455
+1,255
1,727

8
218
+ 88
314

125 + 355 + 465 = 945

45 + 125 + 75 = 245

7 + 77 + 223 = 307

15

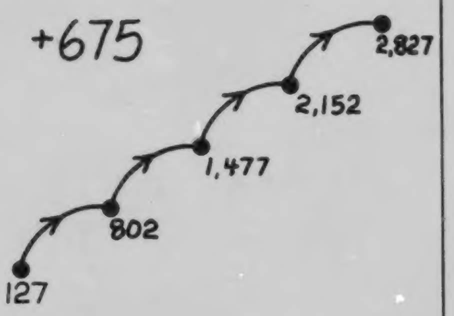
Calculate:

5,432
+ 3,076
8,508

4,327
+ 2,498
6,825

6,566
+5,354
11,920

Label the dots.



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N32

N32 Patterns Generated by the Functions +5 and -5

CAPSULE LESSON SUMMARY

Use return arrows to label the dots in two arrow pictures: one with +5 arrows and one with -5 arrows. Observe a pattern to the numbers which are in these arrow pictures.

MATERIALS

Teacher: Colored chalk

Student: Colored pencils; pencil; Worksheets N32* and **

DESCRIPTION OF LESSON

Exercise 1: Mental Arithmetic

Indicate to your class an order for this activity which is natural to their seating arrangement, i.e., indicate which person is first, who is second, and so on.

T: Today we are going to count by fives. I will say "0"; the first student will say "5"; the second will say "10" because 5 + 5 = 10; the next "15" because 10 + 5 = 15; and so on.

Determine how many students are present.

T: If everyone has exactly one turn, what number will the last person say?

Record some of the students' predictions on the board.

T: 0.

Walk around the room as students count by fives until each has said a number. Make corrections if necessary. Compare the predictions to the last number said, commending those students who had correctly predicted which number would be said last.

T: Now we are going to count by fives again, but this time I am going to start with 1 . Each person will say the number which is five more than the last number. What number do you think the last person will say?

Record some of the students' predictions on the board.

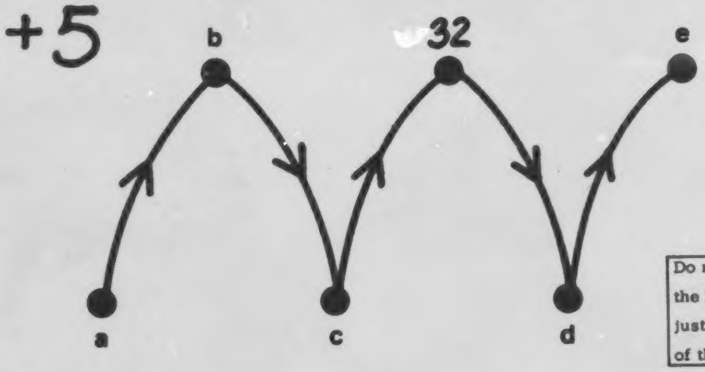
Say " one " and then go around the room counting by fives until each has said a number. To help a student who is having difficulty, hold up five fingers and help the student count (by ones) from the previous number. Compare the predictions with the last number said, commending those students who correctly predicted the number which would be said last.

T: Did anyone notice a pattern when we started at 1 ?

S: The numbers we said always ended in 1 or 6.

Exercise 2

Draw this arrow picture on the board.



Do not write the letters on the board. They are here just to make the description of the lesson easier to follow.

T: Where is the largest number in this arrow picture? (At e)
Tell me with whispers which is the largest number. (42)
Where is the smallest number in this arrow picture? (At a)

T: Tell me with whispers which is the smallest number. (17)

Do not announce the smallest and largest numbers now. Later when all the dots are labeled you could check which are these numbers.

Point to a , b , and c .

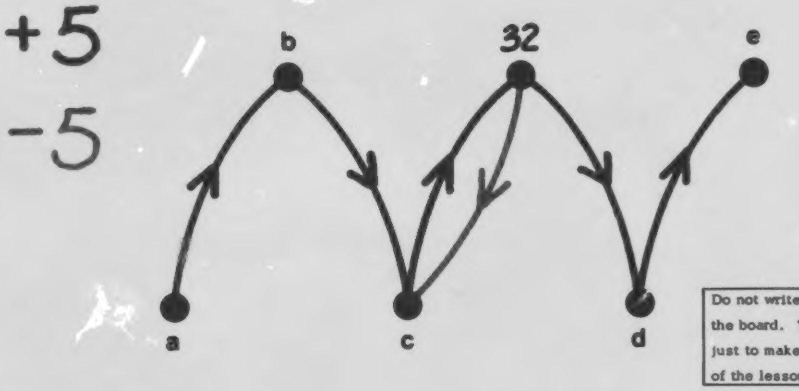
T: How can we label these dots?

Guide the discussion so that it is clear that you cannot follow the +5 arrows from 32 to any of those three dots. Students may suggest drawing -5 arrows, or if necessary, make this suggestion yourself.

Point to c .

T: Who can draw a -5 arrow which will help us to label this dot?

Let a student first trace an arrow from 32 to c and then draw the arrow in blue. Write " -5 " near the arrow picture.



Do not write the letters on the board. They are here just to make the description of the lesson easier to follow.

Point to "32" and trace the -5 arrow.

T: 32 - 5 = . . . ?

S: 27 .

Label the dot "27". Trace the red arrow from 27 to 32 as you ask:

T: 27 + 5 = . . . ?

S: 32.

T: Who can show us a way to go from 32 back to 32, always following arrows?

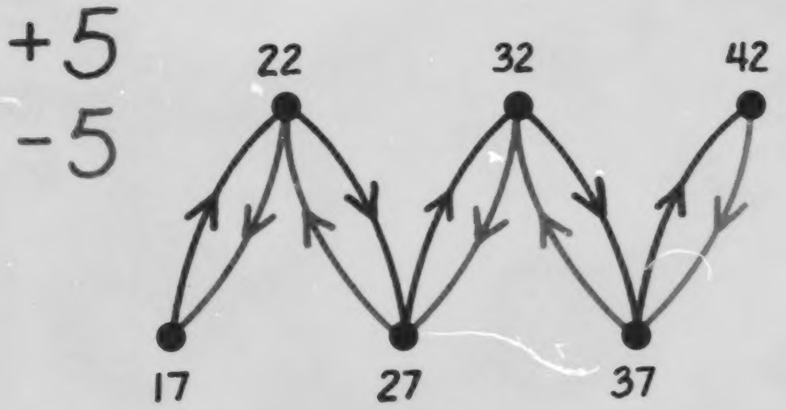
Whoever volunteers should start at 32, trace the blue arrow to 27, and then trace a red arrow back to 32. Repeat the tracing yourself as you say:

T: (32 - 5) + 5 is 32. Do you see a way we could follow the arrows from 27 back to 27?

Call on a student to trace the arrows and then repeat the tracing yourself.

T: (27 + 5) - 5 is 27. A blue arrow followed by a red arrow or a red arrow followed by a blue arrow takes us back to the number we started at.

Invite students to either label dots or draw -5 arrows until the arrow picture is complete.



300

N32

T: Does anyone see a pattern to the numbers in the arrow picture?

S: The numbers at the top of the picture end with 2, and the numbers at the bottom end with 7.

T: If we were to draw more +5 arrows, going on from 42, what are some numbers we would meet?

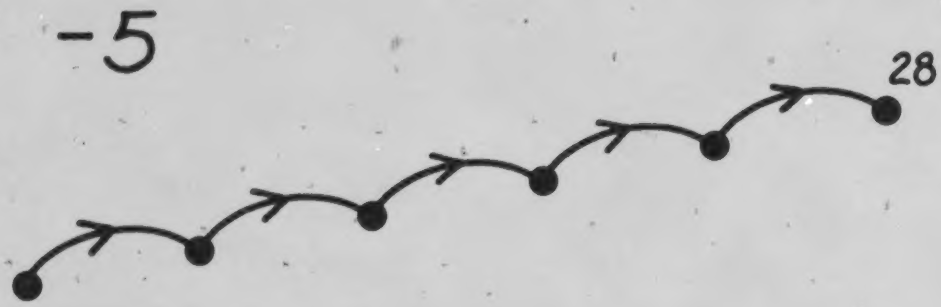
S: 57.

S: 92.

Any whole number (larger than 42) which ends with 2 or 7 is a correct answer. Encourage students to suggest a wide range of numbers.

Exercise 3

Draw this arrow picture on the board.



T: Where is the smallest number in this arrow picture? (At the upper right)
Which is the smallest number? (28)

T: Where is the largest number in this arrow picture? (At the lower left) Tell me with whispers which is the largest number.

The last question may challenge your best students. Do not announce yet which number is the largest.

T: If we follow these arrows, will we be able to label the other dots?

S: No, they are pointing in the wrong direction.

T: What could we do to help us label the other dots?

S: Draw return arrows.

Draw the return arrow which starts at 28 in red.

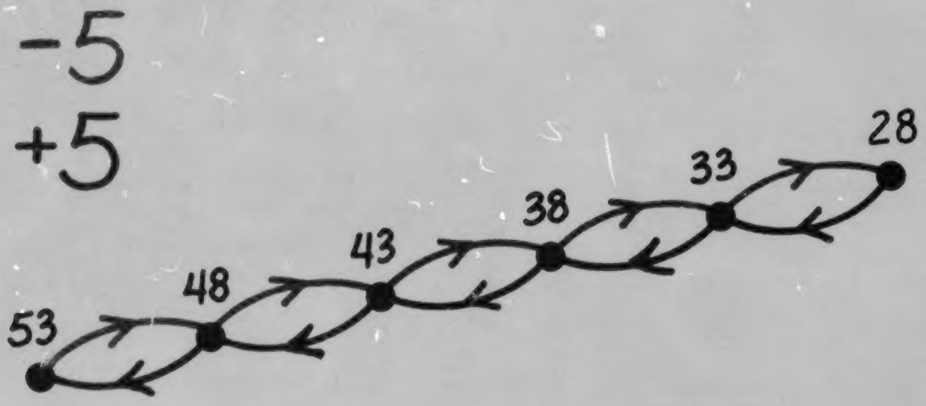
T: What could this arrow be for?

S: +5.

Write "+5" near the arrow picture.

Ask for volunteers to either label a dot or to draw another +5 arrow until the arrow picture is completed. Observe that 53 is the largest number in this arrow picture and ask students who predicted this how they found the answer. Accept any reasonable explanation.

N32



T: Do you notice a pattern to the numbers in the picture?

S: All the numbers end in 3 or in 8 .

T: If we were to draw more +5 arrows, going on from 53 , what are some other numbers we would meet?

S: 58 .

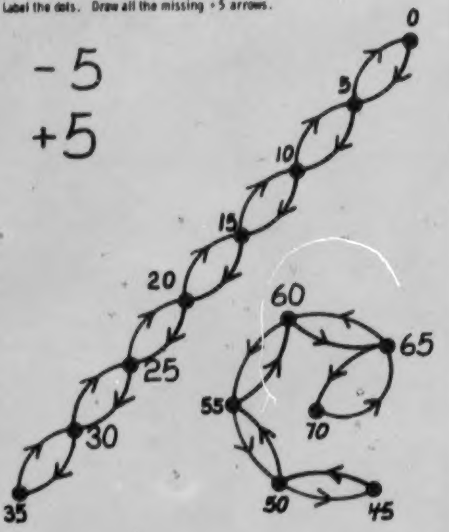
S: 73 .

Any whole number (larger than 53) which ends in 3 or 8 is correct. Encourage students to suggest a wide range of numbers.

Worksheets N32* and ** are available for independent work. The students should draw all the return arrows in these arrow pictures.

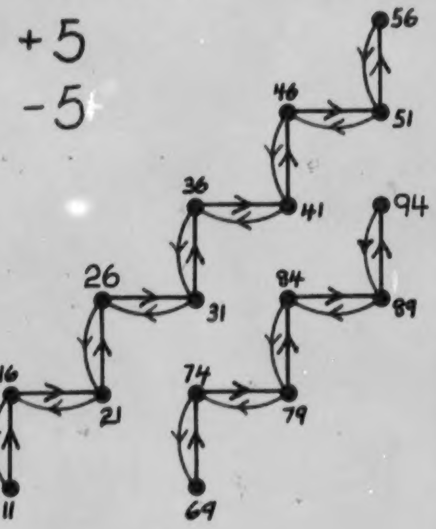
Name ____________________ **N 32 ***

Label the dots. Draw all the missing +5 arrows.


-5
+5
0
5
10
15
20
25
30
35
60
65
55
70
50
45

Name ____________________ **N 32 ****

Label the dots. Draw all the missing -5 arrows.


+5
-5
56
46
51
36
41
94
26
31
84
89
16
21
74
79
11
69

N 33

N33 The Addition Algorithm #2

CAPSULE LESSON SUMMARY

Explore the effect of moving, removing, or adding some checkers to a configuration on the Minicomputer - in each case, does the numerical value increase, decrease, or stay the same? Estimate a number on the Minicomputer and then make plays, stopping periodically to get better estimates, until the standard configuration is obtained. Practice using the addition algorithm previously introduced to do addition calculations, such as "453 + 276".

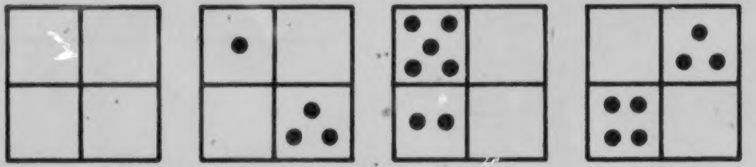
MATERIALS

Teacher: Minicomputer set; colored chalk
Student: Pencil; Addition Problems booklet[*]

DESCRIPTION OF LESSON

Exercise 1: Transforming a Number

Put this configuration on the Minicomputer.



T: I put a number on the Minicomputer. Is the number larger than 100? How do you know?

S: Yes, there are three checkers on the 100-square and more checkers on other squares.

T: Is the number larger than 1,000? How do you know?

---

[*] The Addition Problems booklet was used previously in Lesson N31 The Addition Algorithm #1. The answer key is at the end of that lesson, pages 228 to 231.

S: Yes, there is a checker on the 800-square and more than two checkers on the 100-square.

T: We do not know exactly which number is on the Minicomputer, but we do know that it is larger than 1,000. We do not need to figure out this number exactly to compare it to other numbers.

Write these words on the board close to the Minicomputer.

Larger
Smaller
Same

T: I am going to move, remove, or add some checkers. Each time tell me if the number on the Minicomputer is larger, smaller, or the same.

Replace any checker by a checker of another color. Point to the three words in turn.

T: If you think the number is larger, hold up your hand.
If you think the number is smaller, hold up your hand.
If you think the same number is on the Minicomputer, hold up your hand.

You might need to repeat the move very obviously if many students do not know that the same number is on the Minicomputer.

T: Why is it the same number on the Minicomputer?

S: All you did was exchange a checker for one of a different color.

Continue this activity by making the moves listed on the next page or similar ones.

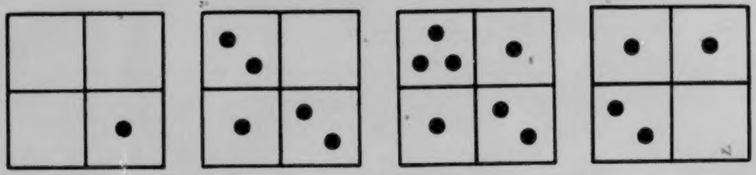
- Move a checker from the 2-square to the 200-square. (Larger)
- Move a checker from the 80-square to the 8-square. (Smaller)
- Make a "100 + 100 = 200" play. (Same)
- Remove two checkers from the 80-square and place one checker on the 100-square. (Smaller)
- Make a "80 = 40 + 40" play. (Same)

After you have made four or five changes yourself, invite students to transform the numbers on the Minicomputer.

At the end of the activity, leave the checkers on the Minicomputer for Exercise 2.

Exercise 2: Estimation

There will be some number displayed on the Minicomputer from Exercise 1. For the lesson description, we will assume this configuration is on the Minicomputer.



T: Is the number on the Minicomputer more than 1,000? How do you know?

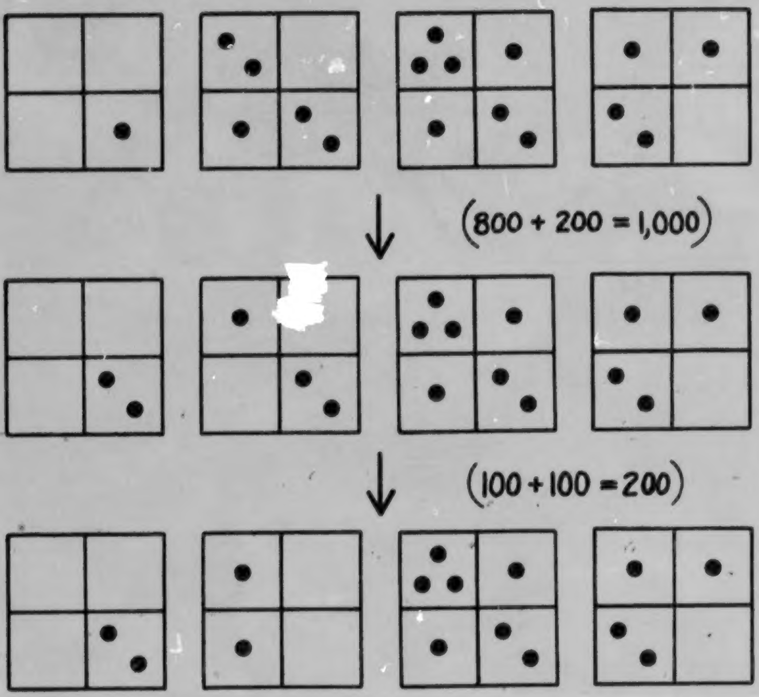
S: Yes, there is a checker on the 1,000-square and checkers on other squares.

T: Is this number more than 2,000? How do you know?

S: Yes, there is a checker on the 1,000-square, one on the 800-square, one on the 200-square, and still more checkers.

T: Is it more than 3,000 ? (Disagreement)
We know this number is more than 2,000. Let's make some plays which will help us make a closer estimate. Who can make a play?

These are some plays which might be made.



T: Is the number on the Minicomputer more than 3,000 ? How do you know know?

S: Yes, there are two checkers on the 1,000-square, one on the 800-square, one on the 200-square, and still more checkers.

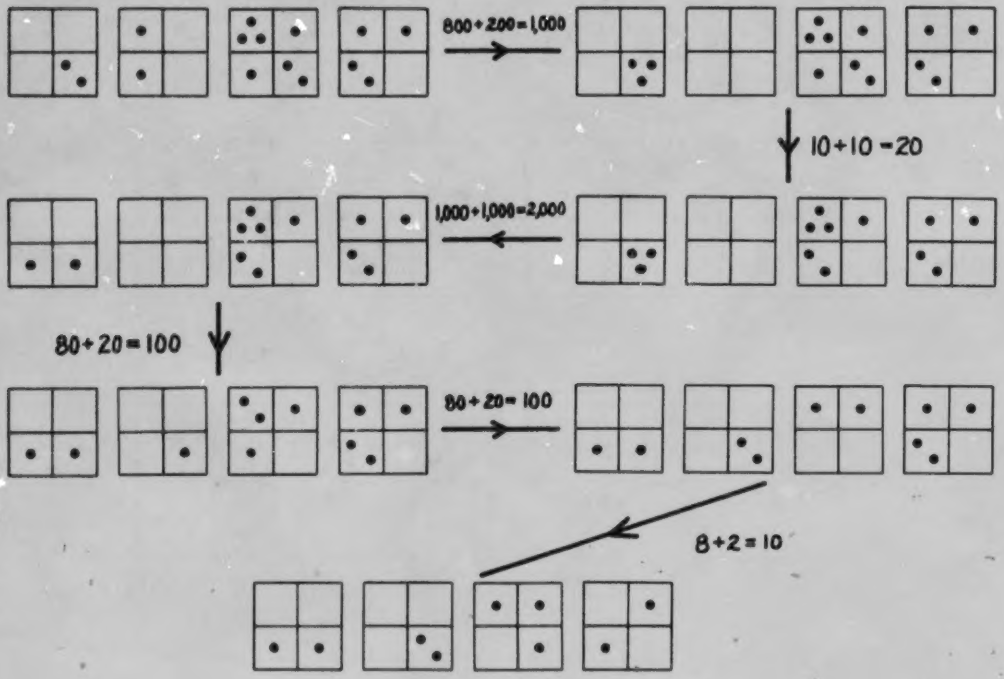
T: Is it more than 4,000 ?

S: No.

T: We now know that the number is between 3,000 and 4,000 .

N 33

Again invite students to make some plays to get a closer estimate. For example,



T: We know this number is more than 3,000.
Is it more than 3,100? (Yes)
Is it more than 3,200? (Yes)
Is it more than 3,300? (Yes)
Is it more than 3,400? (Disagreement).

T: We know this number is between 3,300 and 4,000. Which number do you think it is?

Record some of the students' guesses. In this example, numbers which are less than 3,300 or more than 4,000 should not be recorded.

Afterward, invite students to make plays until standard configuration is obtained. Let the class decide which guess was the closest to the number.

Erase the board and remove the checkers from the Minicomputer.

Exercise 3: Addition Practice

Write this problem on the board.

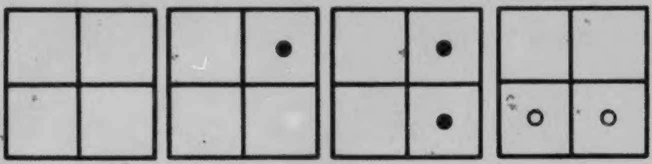
$$453 + 276 = ?$$

T: What number is 453 + 276 ?

Record some of the students' guesses on the board.

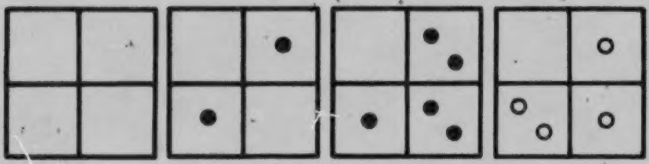
T: Let's add 453 and 276 on the Minicomputer.

Put 453 on the Minicomputer. Use red checkers on the hundreds' board, blue checkers on the tens' board, and yellow checkers on the ones' board.



In the illustrations in this lesson, "○" indicates a yellow checker.

Add 276 to the 453 on the Minicomputer, using the same color scheme.



Write the problem on the board with colored numerals.

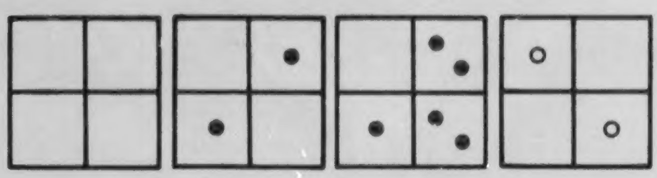
$$\begin{array}{r} 453 \\ +\ 276 \\ \hline \end{array}$$

In the illustrations in this lesson, an outlined numeral, e.g. "5", indicates a numeral in yellow.

N33

T: What number do you see on the ones' board? (9)
Can we make a play? (Yes)

Let students make plays on the ones' board only.



T: How many ones are on this board (point to the ones' board)?

S: Nine.

Ask a student to write "9" in yellow below the ones' board. Point to the ones' column of the addition problem.

T: What do you see in the ones' column?

S: "3" and "6".

T: 3 + 6 = . . . ?

S: 9.

$$\begin{array}{r} 453 \\ +\,276 \\ \hline 9 \end{array}$$

T: What do you see in the tens' column?

S: "5" and "7".

T: Five tens plus seven tens is . . . ?

S: Twelve tens.

T: What should we do now?

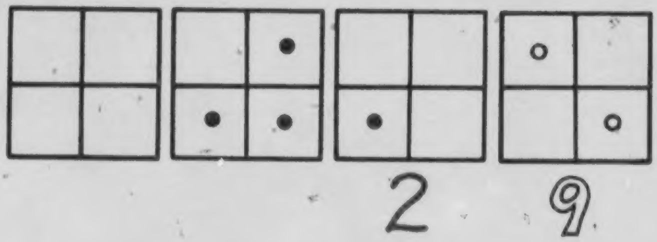
Allow the students to make suggestions, but encourage them to look at the tens' board of the Minicomputer. If a student says there is another hundred, pursue the idea.

T: Let's see what happens on the Minicomputer.
How many tens are on this board (point to the tens' board)?

S: Twelve.

T: Can we make a play? (Yes)

Ask students to make the appropriate plays and then to write "2" in blue below the tens' board.



T: How many tens are on this board now?

S: Two.

T: Before we had twelve tens and now there are only two tens. What happened?

S: 10 tens is 100, and there is another checker on the hundreds' board now.

N 33

Be sure the students understand that the other 10 tens are now on the hundreds' board. Point to the addition problem written on the board.

T: How could we show that we have another hundred on the hundreds' board?

S: Write "1" above "4" in the hundreds' column.

T: This blue checker shows us that we have one more hundred on the hundreds' board. I will write a "1" above the hundreds' column. This shows that we have one more hundred. We write "2" below the tens' column because there are two tens left on the tens' board.

$$\begin{array}{r} {\scriptstyle 1} \\ 453 \\ +\,276 \\ \hline 29 \end{array}$$

T: What do you see in the hundreds' column?

S: "1", "4", and "2".

T: 100 + 400 + 200 = . . . ?

S: 700.

Point to the hundreds' board of the Minicomputer.

T: How many hundreds are on this board?

S: Seven.

Write "7" in red below the hundreds' board and below the hundreds' column of the addition problem.

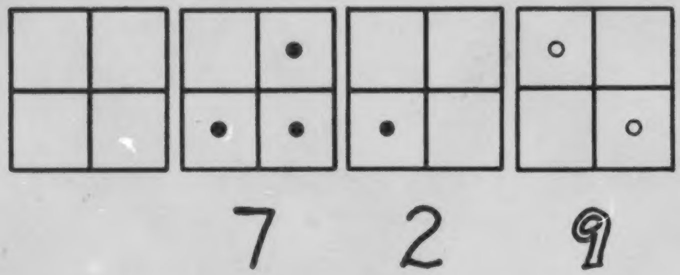
3.3

T: What number is 453 + 276 ?

S: 729 .

Complete the number story on the board. Compare the students' guesses with the answer and determine which guess is the closest.

$453 + 276 = 729$



$$\begin{array}{r} 1 \\ 453 \\ +276 \\ \hline 729 \end{array}$$

Distribute the Addition Problems booklets and give about 10 to 15 minutes for independent work. Encourage students to correct any errors they made the previous week. Allow students to use individual Minicomputers if they wish. After the lesson, collect the booklets, check them, and keep them ready for the remaining lessons on the addition algorithm.

N34 Less Than - More Than

CAPSULE LESSON SUMMARY

Compare 87 , 87 - 29 , 87 + 38 , and 2 x 87 in an arrow picture where the arrows are for the less than and more than relations.

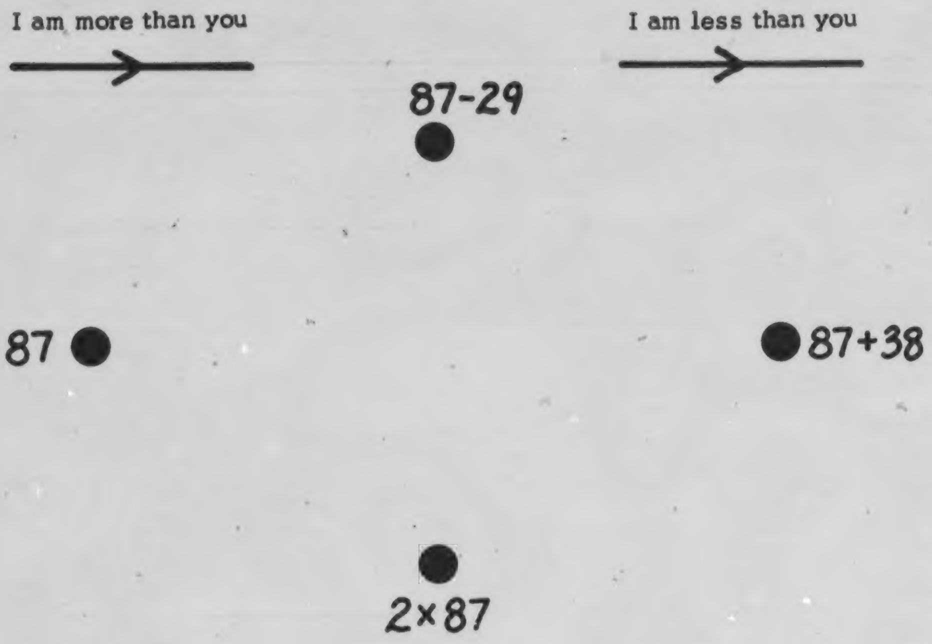
MATERIALS

Teacher: Colored chalk

Student: None

DESCRIPTION OF LESSON

Draw this picture on the board. Label the dots as illustrated or choose other calculations more appropriate to the ability of your students.



T: For the moment do not try to calculate these numbers.
What are the red arrows for? (I am more than you)
What are the blue arrows for? (I am less than you)
Where can we draw an arrow?

Ask each student who volunteers to trace an arrow, to identify what the arrow is for, and to explain why such an arrow can be drawn between the two numbers involved. Discourage the students from doing any calculations. If a correct arrow is traced and the class agrees on it, draw the red arrow yourself. If students cannot agree that an arrow is correct, do not draw it at this time in the lesson.

Suppose a student has indicated than an arrow can be drawn from 2 x 87 to 87 + 38 .

T: What could this arrow be for?

S: I am more than you.

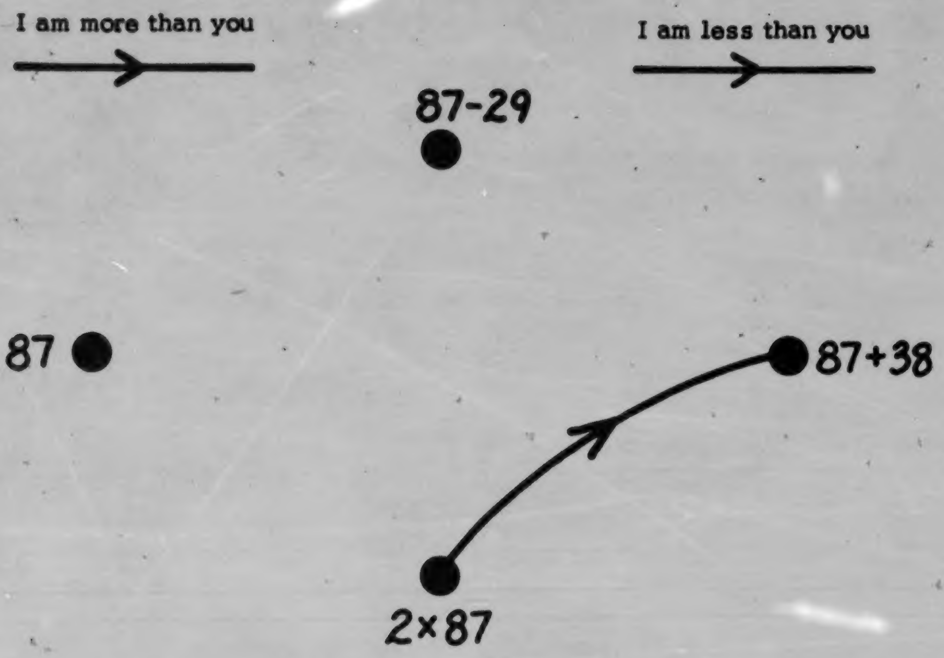
T: 2 x 87 is more than 87 + 38 . How do you know?

S: 2 x 87 is 87 + 87 . 87 + 87 is more than 87 + 38 .

T (looking at the class): Do you agree? (Yes)

Draw a red arrow from 2 x 87 to 87 + 38 yourself (see next illustration).

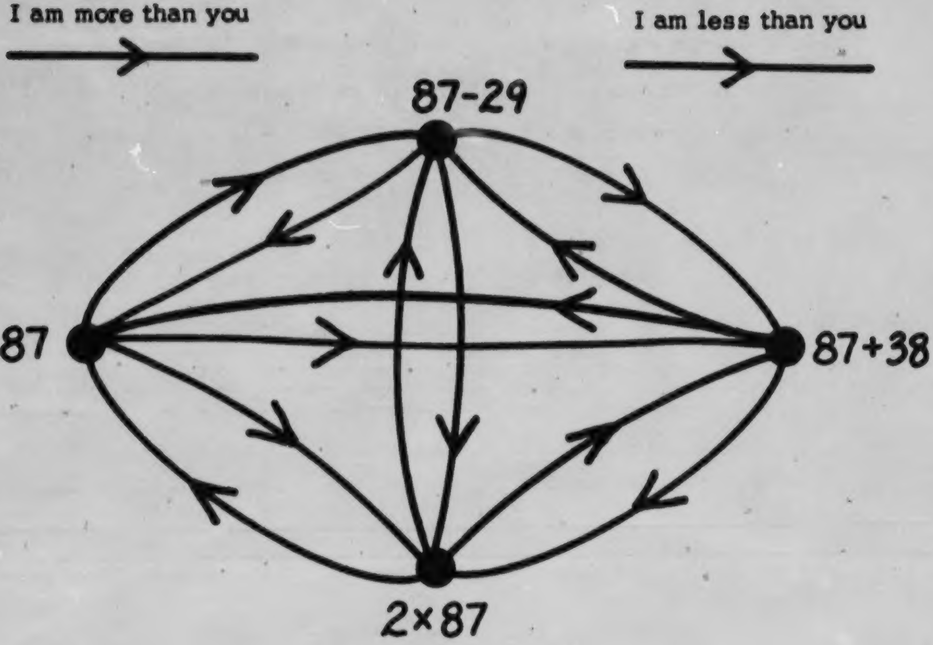
N 34


I am more than you
I am less than you
87-29
87
87+38
2×87

Continue this activity until the students indicate they can not draw any more arrows or can not agree on any more arrows. If an arrow has not been discovered, tell the class that an arrow can be drawn between, for example, 87 - 29 and 2 x 87. When there is general uncertainty as to which number is larger, you might use the Minicomputer and do the calculations.

The completed arrow picture is shown on the next page.

N 34


I am more than you
I am less than you
87-29
87
87+38
2×87

N35

N35 Multiplying by Ten

CAPSULE LESSON SUMMARY

Review the play which replaces ten checkers on a square with one checker on the square of the same color of the next board to the left. Investigate ways of putting 100 and 25 on the Minicomputer using exactly ten checkers.

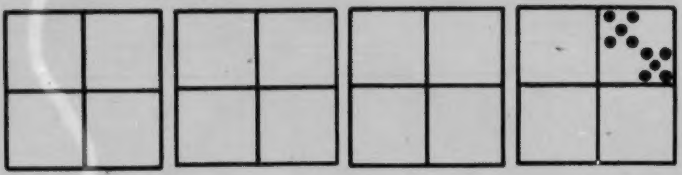
MATERIALS

Teacher: Minicomputer set
Student: Minicomputer set

DESCRIPTION OF LESSON

Exercise 1: Multiplying by Ten

Put ten checkers on the 4-square of the Minicomputer.

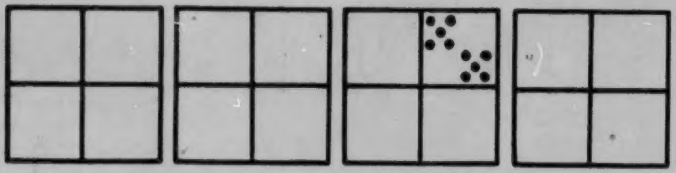


T: What number is this?
What multiplication story could we write about this number?
Do we need to make some plays before we know which number this is?

Let the students react freely to these questions. Many of them already know that 10 x 4 = 40 and no longer need to make plays on the Minicomputer in order to find the answer. Nevertheless, invite students to make plays until you obtain the standard configuration for 40. Try to keep this part of the lesson moving quickly and involve many students in making plays.

T: Ten checkers on this purple square (4-square) are the same as one checker on this purple square (40-square).

Record "10 x 4 = 40" on the board. Put ten checkers on the 40-square of the Minicomputer.



T: What multiplication story could we write about this number?

S: "10 x 40 = ?".

T: Is this very different from the previous multiplication story?

S: No, because we still have ten checkers on a purple square. The only difference is they are on the tens' board instead of the ones' board.

T: After we make certain plays on the Minicomputer, we could end with only one checker on the Minicomputer. Where would this checker be?

S: On the 400-square.

Invite students to make the appropriate plays until they obtain the standard configuration for 400.

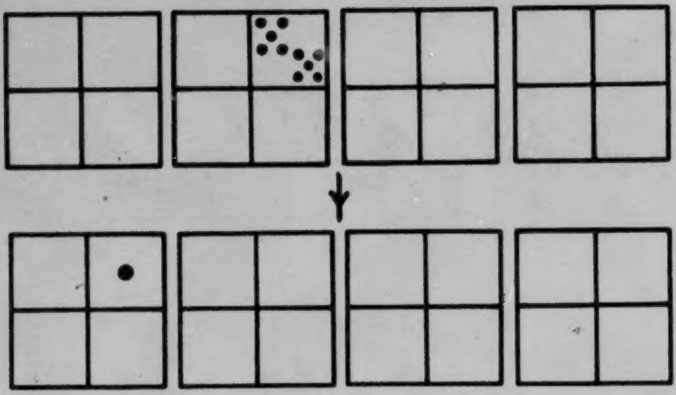
T: Ten checkers on this purple square (40-square) are the same as one checker on this purple square (400-square).

Record "10 x 40 = 400" on the board.

$$10 \times 4 = 40$$
$$10 \times 40 = 400$$

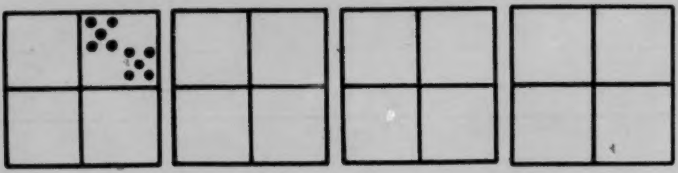
N 35

Similarly, let the class predict and discover that 10 x 400 = 4,000 .



$10 \times 4 = 40$
$10 \times 40 = 400$
$10 \times 400 = 4{,}000$

Put ten checkers on the 4,000 -square of the Minicomputer.



T: What multiplication story could we write about this number?

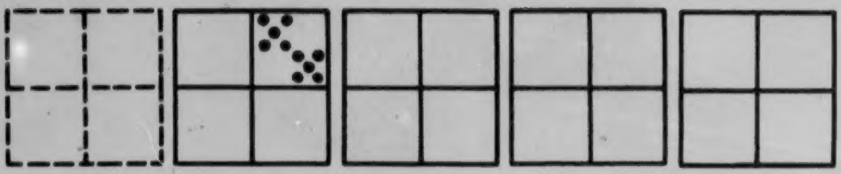
S: " 10 x 4,000 = ? ".

T: What number is this ?

Allow the students to whisper the answer to you.

T: Could we find out what this number is without making any plays ? If we were to make plays until there is only one checker remaining on the Minicomputer, where would this checker be ?

Let the students react freely to these questions. The students will probably tell you that they do not need to make any plays but that they do need another Minicomputer board. When the students ask for a fifth Minicomputer board, draw it on the chalkboard.



Call on a student to point to the appropriate square (40,000-square).

T: Ten checkers on this square (4,000-square) are the same as one checker on which square?

Take all the checkers off the 4,000-square and draw one checker on the 40,000-square.

T: What number is this?

S: 40,000.

It is possible that no one will know how to read this number, but some students may know how to write it.

T: Who can write this number below the Minicomputer?

Students may be able to write the numeral correctly but misplace the comma. Perhaps another student will be able to place the comma correctly or, if necessary, insert the comma yourself. Explain that the comma helps us to read large numbers and that it goes between the hundreds' place (board) and the thousands' place (board).

T: Who can read this number?
What number story should we write on the board?

Help the students to read "40,000" if necessary. Record the number story in your list.

$$10 \times 4 = 40$$
$$10 \times 40 = 400$$
$$10 \times 400 = 4{,}000$$
$$10 \times 4{,}000 = 40{,}000$$

T: Do you notice a pattern?

S: Each time there is another "0".

Exercise 2: Mental Arithmetic

This activity should move quickly. Do not expect all of your students to be able to participate. If no one is able to respond correctly to a question, put the calculation on the Minicomputer and ask students to make plays until a student answers correctly. Then record the appropriate number story on the board.

T: What number is 10 x 8 ? (80)

Record "10 x 8 = 80" on the board. Continue this activity with these or similar calculations which are appropriate for your students.

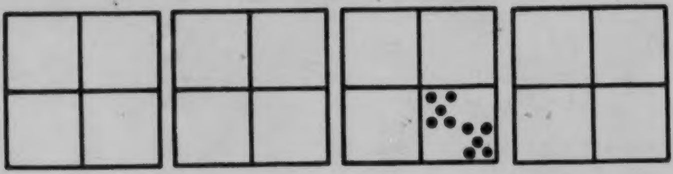
T: What number is 10 x 7 ? (70)
10 x 2 ? (20)
10 x 5 ? (50)
10 x 10 ? (100)
10 x 9 ? (90)
10 x 0 ? (0)
10 x 3 ? (30)

Exercise 3: Individual Minicomputers

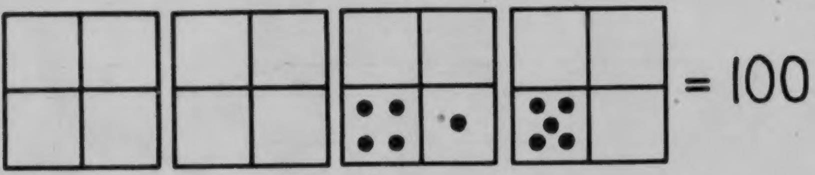
Each student will need two individual Minicomputer sheets and at least ten positive checkers.

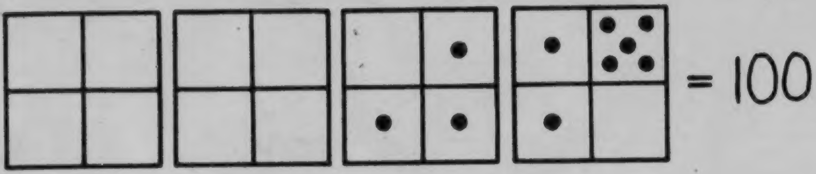
T: Can you put 100 on your Minicomputer using exactly ten checkers?

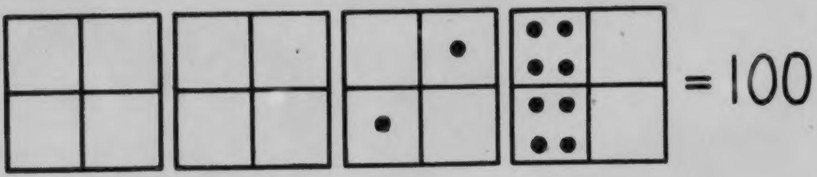
Many students will probably put this configuration on their Minicomputers.



This is the most obvious and an important solution, but there are many other possibilities. Encourage students to find others. A few solutions are given below.





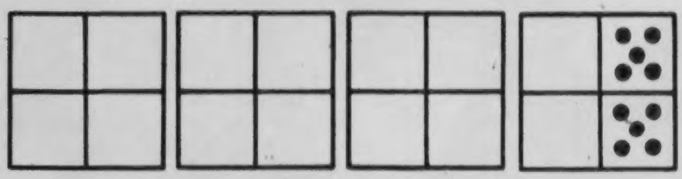


As different solutions are found, ask a student to put them on the demonstration Minicomputer. You may wish to discuss some of the solutions with the entire class.

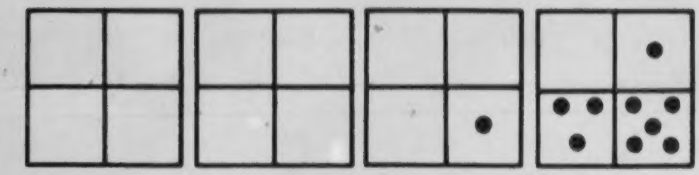
N35

T: Can you put 25 on your Minicomputer with exactly ten checkers?

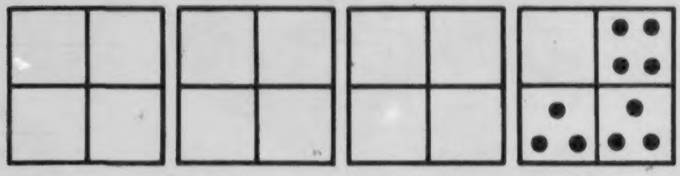
Let the students work on their own for several minutes. As different solutions are found, ask a student to put them on the demonstration Minicomputer. You may wish to discuss some of the solutions with the entire class. Several solutions are given below.



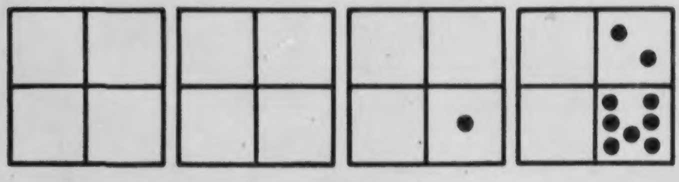
= 25



= 25



= 25



= 25

N36 Composition of Functions #3

CAPSULE LESSON SUMMARY

Start the lesson with a mental arithmetic exercise involving +10. Explore the composition of +10 and -1 functions through the use of an arrow picture. Use knowledge of the +10 function to gain familiarity with the +9 function; for example, $(7 + 10) - 1 = 7 + 9$.

MATERIALS

Teacher: Colored chalk
Student: None

LESSON DESCRIPTION

Exercise 1: Mental Arithmetic

T: I will say a number and you will tell me the number which is ten more than the number I say.

Choose numbers which fit the abilities of your students, alternating problems which are difficult with problems which are easy. When a student answers correctly, you should say the appropriate addition number story; for example,

T: 8.

S: 18.

T: $8 + 10 = 18$.
18.

S: 28.

T: $18 + 10 = 28$.
28.

N 36

S: 38.

T: 28 + 10 = 38.
38.

S: 48.

Continue in this way for a few minutes.

7. (17)

7 + 10 = 17.

25. (35)

25 + 10 = 35.

90. (100)

90 + 10 = 100.

92. (102)

92 + 10 = 102.

97. (107)

97 + 10 = 107.

140. (150)

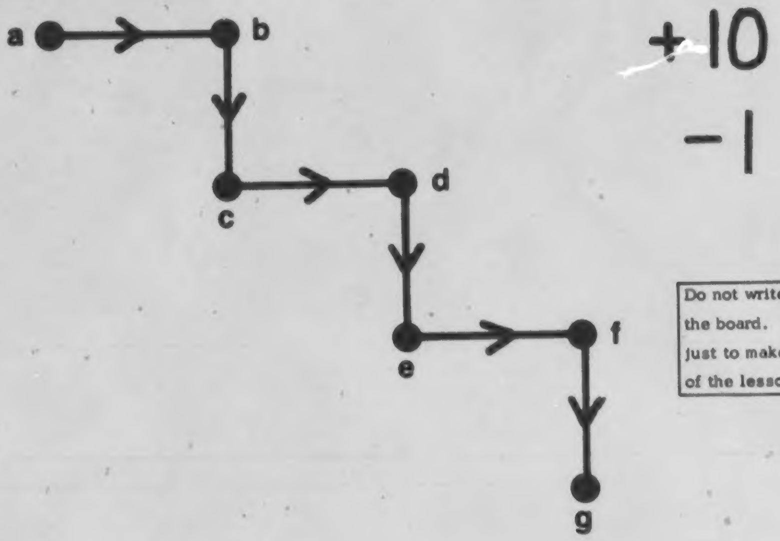
140 + 10 = 150.

163. (173)

163 + 10 = 173.

Exercise 2

Draw this arrow picture on the board.



Do not write the letters on the board. They are here just to make the description of the lesson easier to follow.

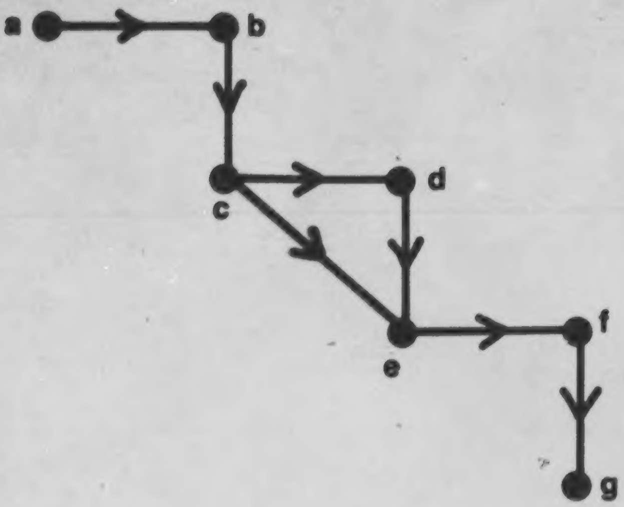
T: Where is the smallest number in this arrow picture? (At a)
How do you know where the smallest number in this arrow picture is?

This may be very hard for students to verbalize, so do not expect well formed explanations. If the students appear to be focusing only on the +10 arrows, remind them of the -1 arrows. Perhaps your class will conclude that they do not know where the smallest number is.

If your class is fairly sure that the smallest number is at the upper left, ask them where is the largest number in the arrow picture. (At f) Students might suggest that the largest number is at the lower right. Ask how they know that this number is the largest. Again, do not expect well formed answers. Someone might see that the number at the extreme lower right can not be the largest because the number directly above it is one more.

Draw an arrow from c to e (see illustration on the next page).

N 36

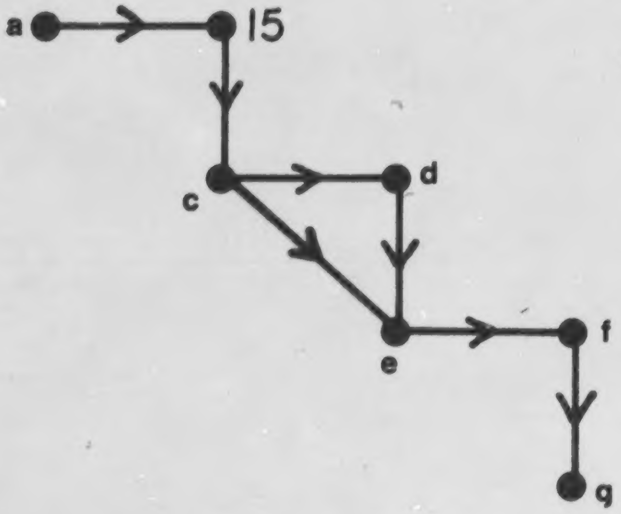


Do not write the letters on the board. They are here just to make the description of the lesson easier to follow.

T: What could this green arrow be for? (+9)
How do you know?

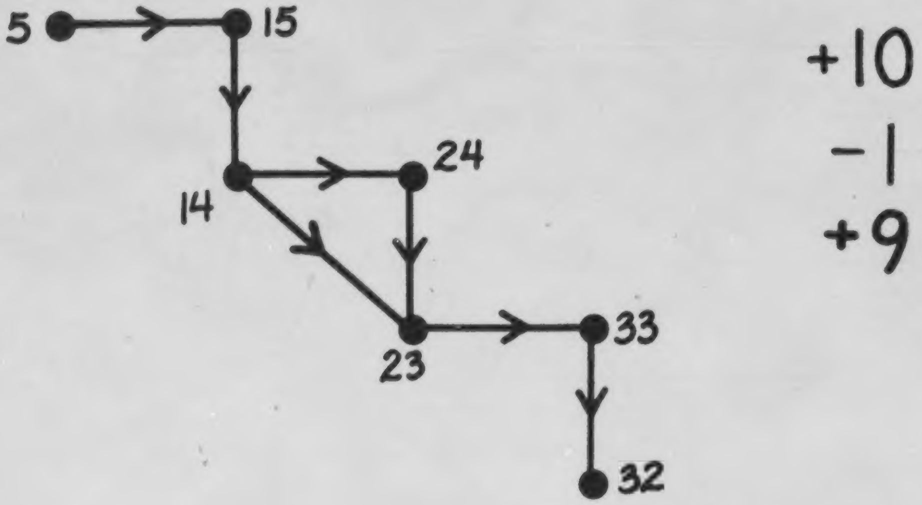
This question may challenge even your best students. Commend any student who says that adding ten and then taking away one is the same as adding nine. If none of the students know the green arrow could be for +9, do not reveal this yourself but continue with the lesson.

T: Let's label the dots. I will tell you that one of these numbers is 15.



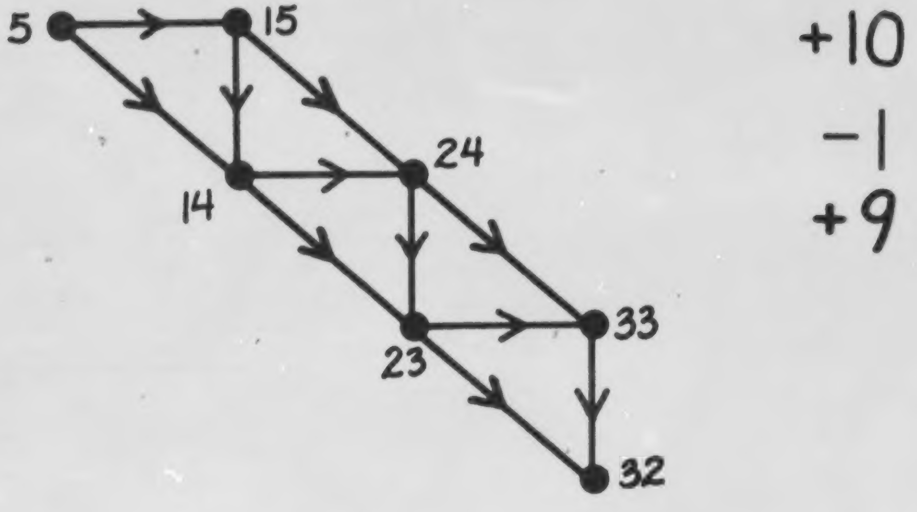
Do not write the letters on the board. They are here just to make the description of the lesson easier to follow.

Call on students to label the other dots. Each time a dot is labeled ask for an explanation. When your class is confident that the green arrow is for +9, write "+9" near the arrow picture. Continue until all the dots are labeled.



T: Who can draw another green +9 arrow?

Let volunteers draw the missing green arrows.



T: Which is the smallest number in this arrow picture? (5)

Which is the largest number in this arrow picture? (33)

If your class had thought that the number on the extreme lower right would be the largest, you may wish to ask why 33 is the largest number in this arrow picture and what would happen if you started with another number instead of 15 . Pursue a suggestion that the number on the extreme lower right will always be less than the number directly above it in this arrow picture.

Exercise 3: Mental Arithmetic

T: What number is 5 + 10 ?

S: 15 .

T: Yes, 5 + 10 = 15. So what number is 5 + 9? How do you know?

S: 14 , because 5 + 9 is one less than 5 + 10 .

Point to " 5 " in the arrow picture on the board from the previous exercise and trace the appropriate arrows as you say:

T: 5 + 9 is the same as (5 + 10) - 1 .
What number is 7 + 10 ?

S: 17 .

T: Yes, 7 + 10 = 17. So what number is 7 + 9? How do you know?

S: 16, because 17 - 1 = 16 .

T: 7 + 10 = 17 , so 7 + 9 = 16 .

S: What number is 24 + 10 ?

S: 34 .

T: Yes, 24 + 10 = 34. So what number is 24 + 9? How do you know?

S: 33 , because 34 - 1 = 33 .

T: What number is 38 + 10 ?

S: 48 .

T: If 38 + 10 = 48 , what number is 38 + 9 ?

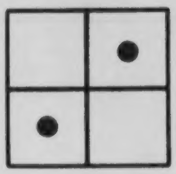
S: 47 .

Continue this activity for a few minutes, occasionally asking for an explanation. Choose problems which are appropriate for the abilities of your students.

APPENDIX A: STANDARD CONFIGURATION AND STANDARD MINICOMPUTER MOVES

A number can be represented by putting checkers[*] on the squares of the Minicomputer. Each square has a numerical (positional) value, so that the number represented by a checker depends on which square the checker is placed. If several checkers are put on the Minicomputer, the number shown is the sum of the numbers represented by the checkers. A number can be put on the Minicomputer in a variety of ways, but the representation which uses at most one checker on each square and uses checkers to represent a digit 9 or less on each board is usually the easiest to read. In this case, we say that the number is in standard configuration. Standard configurations for the numbers 1 - 9 soon become as familiar to the students as the usual numerals so that they no longer need to do any mental calculation for such configurations.

For example,



. . . is seen as a name for 6 without doing the calculation "4 + 2 = 6".

Doing calculations on the Minicomputer involves putting one or more checkers on the various squares and making "plays" to get a specific configuration. In this context, a play is an exchange of checkers in a configuration which does not change the number represented. Some plays are natural to the design of the Minicomputer; we call these standard or usual plays (these are described in detail in this section). There are advantages to establishing such a set of plays:

---

[*] In this appendix we refer only to whole numbers and, therefore, use only regular (positive) checkers.

- They become routine, and a large group of students with varying abilities can follow a calculation without needing to verify every play which is made.
- They lead to standard configurations of numbers.

These are the standard Minicomputer moves.

1 + 1 = 2
10 + 10 = 20
100 + 100 = 200
1,000 + 1,000 = 2,000

two checkers on the white square
of a board
=
one checker on the red square
of the same board

2 = 1 + 1
20 = 10 + 10
200 = 100 + 100
2,000 = 1,000 + 1,000

one checker on the red square
of a board
=
two checkers on the white square
of the same board

2 + 2 = 4
20 + 20 = 40
200 + 200 = 400
2,000 + 2,000 = 4,000

two checkers on the red square
of a board
=
one checker on the purple square
of the same board

4 = 2 + 2
40 = 20 + 20
400 = 200 + 200
4,000 = 2,000 + 2,000

one checker on the purple square
of a board
=
two checkers on the red square
of the same board

$$4 + 4 = 8$$
$$40 + 40 = 80$$
$$400 + 400 = 800$$
$$4{,}000 + 4{,}000 = 8{,}000$$

two checkers on the purple square of a board

=

one checker on the brown square of the same board

$$8 = 4 + 4$$
$$80 = 40 + 40$$
$$800 = 400 + 400$$
$$8{,}000 = 4{,}000 + 4{,}000$$

one checker on the brown square of a board

=

two checkers on the purple square of the same board

$$8 + 2 = 10$$
$$80 + 20 = 100$$
$$800 + 200 = 1{,}000$$
$$8{,}000 + 2{,}000 = 10{,}000$$

one checker on each of the brown and red squares of a board

=

one checker on the white square of the next board to the left

$$10 = 8 + 2$$
$$100 = 80 + 20$$
$$1{,}000 = 800 + 200$$
$$10{,}000 = 8{,}000 + 2{,}000$$

one checker on the white square of a board

=

one checker on each of the brown and red squares of the next board to the right[*]

[*] This provides the link that motivates the introduction of the decimal numbers. In particular, we have that one checker on the white square of the ones' board equals one checker on each of the brown and red squares of a special board (the tenths' board - or as it is often referred to in the lessons, the dimes' board) located to the right of the ones' board and separated from it by a special barrier.

After many experiences with the Minicomputer, some students might suggest "non-standard" plays, e.g. "8 + 4 = 10 + 2". When students suggest such plays, ask them to convince the class that the plays are indeed correct. There will be a natural acceptance or rejection of non-standard plays by the students, sometimes depending on the situation.

Some "non-standard" plays will be introduced for specific purposes; for example, ten checkers on a square can be exchanged for one checker on the same color square of the next board to the left. Such plays serve to reinforce the support that the Minicomputer gives to decimal place value or to provide insight into a certain kind of problem. They are inviting in particular situations, but do not play the same "routine" role that the usual plays do.

APPENDIX B: MENTAL ARITHMETIC

Mental arithmetic activities are short, fast-moving question and answer sessions where students are asked to calculate mentally, preferably without the aid of paper and pencil. Often a mental arithmetic session consists of several sequences in which the answers to one or more questions lead to the answer of another, more difficult question; for example,

$$2 \times 3 = ? \quad (6)$$
$$2 \times 10 = ? \quad (20)$$
$$2 \times 13 = ? \quad (26)$$

There are specific lessons which call for mental arithmetic; however, we suggest you try to include such activities for five to ten minutes at least twice a week. Use mental arithmetic as a warm-up activity to a lesson, as a transition between two exercises in a lesson, as a conclusion to a lesson, or as a quick change of pace at any time during the day.

Why is Mental Arithmetic Important?

1) It develops mental computational skills.
2) It provides an opportunity to involve and help students at all ability levels.
3) It creates an awareness of patterns and mathematical relationships.
4) It helps students to be able eventually to recall arithmetic facts easily.
5) It provides an opportunity to keep alive and to reinforce concepts previously introduced.
6) It helps the students to see that there is often more than one way to solve a problem.

Hints for Mental Arithmetic Activities

1) Vary the level of difficulty of questions throughout a session.

2) Involve many students.

3) Follow a pattern for awhile and then start a new pattern.

4) Keep a brisk pace, although you should allow students time for thought.

5) Give extra attention to a student who shows signs of improvement.

6) Illustrate, whenever possible, the value of estimation in making calculations.

7) Occasionally ask a student to explain an answer.

8) Occasionally use the chalkboard to emphasize a pattern.

9) Occasionally ask students to whisper an answer to you so that many students will have an opportunity to answer.

SUGGESTED MENTAL ARITHMETIC ACTIVITIES

This section contains specific examples that illustrate patterns in arithmetic. You should elaborate and expand upon ideas depending upon the abilities and interests of your students. As you work with your class on the regularly scheduled mathematics lessons, you might make a note of computational skills you would like to improve; then plan a short mental arithmetic activity which develops those skills. Do not expect everyone of your students to be good at all of these suggested activities, even at the end of the year.

Counting Activities

1) Count by twos, threes, fives, or tens collectively; everyone counting together. Refer to the 0-109 chart as the class counts if you wish.

2) Count by twos, threes, fives, or tens as a group activity with each student saying one number. Use a natural seating order in your class (such as going up and down rows) and ask each student in turn to say the number which comes next. If you are counting by threes, the first student will say "3"; the second will say "6"; and so on. Repeat the counting but start with a different student.

3) Count by twos, threes, fives, or tens, starting at 1 or any number other than 0.

4) Count backward by twos, fives, or tens. Do this immediately following a similar forward count.

5) Play "BUZZ". The students count by ones starting at 0 but skip every fourth number by saying "buzz". This should be done rapidly. This can be done collectively or as a group activity with each student in turn saying the number that comes next or "buzz".

More challenging variations of this activity involve:

- skipping every sixth number rather than every fourth number;
- starting the counting at 2 and skipping every fourth number;
- counting by twos or fives and skipping every fourth number.

Addition and Subtraction

The first nine examples are lists of calculations to be done mentally. Each list suggests a pattern in arithmetic which students should begin to use in doing such calculations.

1) $3 + 4$
$3 + 5$
$3 + 6$
$3 + 7$

$8 + 2$
$8 + 3$
$8 + 4$
$8 + 5$

$20 + 20$
$20 + 21$
$20 + 22$
$20 + 23$

$2 + 2$
$3 + 3$
$4 + 4$
$5 + 5$

2) 4 + 4
40 + 40
400 + 400
4,000 + 4,000
4,000,000 + 4,000,000

2 + 5
20 + 50
200 + 500
2,000 + 5,000
2,000,000 + 5,000,000

3) 10 + 5
5 + 10
10 + 7
7 + 10

10 + 6
20 + 6
30 + 6
40 + 6

30 + 6
31 + 6
40 + 5
43 + 5

4) 5 + 3
8 - 3
7 + 6
13 - 6

10 + 20
30 - 20
90 + 40
130 - 40

106 + 105
211 - 105
150 + 150
300 - 150

5) 10 - 4
10 - 5
10 - 6
10 - 7

40 - 1
40 - 2
40 - 3
40 - 4

20 - 10
20 - 11
20 - 12
20 - 13

6) 4 + 10
14 + 10
24 + 10
44 + 10

7 + 10
17 + 10
117 + 10
1,017 + 10

6 + 10
18 + 10
33 + 10
71 + 10

7) 7 - 5
17 - 5
27 - 5
37 - 5

10 - 3
20 - 3
30 - 3
40 - 3

100 - 2
200 - 2
300 - 2
400 - 2

8) 2 + 4
2 + 14
2 + 24
2 + 34

10 + 18
20 + 18
30 + 18
40 + 18

10 + 13
12 + 13
10 + 14
13 + 14

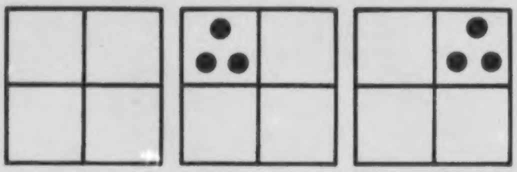
9) 50 + 40
7 + 2
57 + 42

30 + 20
5 + 7
35 + 27

40 + 60
9 + 4
49 + 64

10) Choose some numbers and ask the students to add 1 to each number. For example, if you say "108", a student responds "109". You can vary this activity by asking the students to add 2, 3, 5, or 10 to each of your numbers or to subtract 1, 2, 3, or 10 from each of your numbers.

11) Conduct an activity similar to an exercise entitled "Transforming a Number" in Lesson N33. Put four to ten checkers on the ones' and tens' boards. Move one checker from some square (it makes no difference which square you choose) to another square. Ask the students how much larger or smaller the new number is. For example, suppose this configuration is on the Minicomputer.



Move a checker from the 4-square to the 10-square. When the students tell you that the new number is larger, ask them how much larger this number is than the previous number. (6 larger)

12) Ask the students to give you various names for a particular number. Encourage them to suggest names involving subtraction and also names involving addition of three numbers. For example, names for 10 include: 6 + 4; 20 - 10; 4 + 4 + 2; $11 + \hat{1}$; 2 x 5; $\frac{1}{2} \times 20$.

Multiplication

The nine examples here are lists of calculations to be done mentally. Each list suggests a pattern in arithmetic which students should begin to use in doing such calculations.

1)	3 + 3	7 + 7	100 + 100
	2 × 3	2 × 7	2 × 100
	5 + 5	20 + 20	200 + 200
	2 × 5	2 × 20	2 × 200
2)	1 × 5	1 × 10	1 × 20
	2 × 5	2 × 10	2 × 20
	3 × 5	3 × 10	3 × 20
	4 × 5	4 × 10	4 × 20
3)	2 × 3	2 × 5	3 × 4
	2 × 30	2 × 50	3 × 40
	2 × 300	2 × 500	3 × 400
	2 × 3,000	2 × 5,000	3 × 4,000
	2 × 3,000,000	2 × 5,000,000	3 × 4,000,000
4)	2 × 3	3 × 3	2 × 5
	2 × 33	3 × 33	2 × 55
	2 × 333	3 × 333	2 × 555
	2 × 3,333	3 × 3,333	2 × 5,555
5)	6 + 6	10 + 10	50 + 50
	2 × 6	2 × 10	2 × 50
	$\frac{1}{2} \times 12$	$\frac{1}{2} \times 20$	$\frac{1}{2} \times 100$

6) 2 × 10
2 × 3
2 × 13

2 × 40
2 × 2
2 × 42

2 × 20
2 × 6
2 × 26

2 × 60
2 × 4
2 × 64

2 × 100
2 × 7
2 × 107

2 × 200
2 × 30
2 × 1
2 × 231

7) 10 × 2
10 × 20
10 × 200

10 × 5
10 × 50
10 × 500

10 × 8
10 × 80
10 × 800

8) $\frac{1}{2} \times 6$
$\frac{1}{2} \times 60$
$\frac{1}{2} \times 600$
$\frac{1}{2} \times 6{,}000$
$\frac{1}{2} \times 6{,}000{,}000$

$\frac{1}{2} \times 10$
$\frac{1}{2} \times 100$
$\frac{1}{2} \times 1{,}000$
$\frac{1}{2} \times 10{,}000$
$\frac{1}{2} \times 10{,}000{,}000$

$\frac{1}{2} \times 24$
$\frac{1}{2} \times 240$
$\frac{1}{2} \times 2{,}400$
$\frac{1}{2} \times 24{,}000$
$\frac{1}{2} \times 24{,}000{,}000$

9) 2 × 13
$\frac{1}{2} \times 26$

2 × 34
$\frac{1}{2} \times 68$

2 × 35
$\frac{1}{2} \times 70$

3 × 13
$\frac{1}{3} \times 39$

Estimation

Ask the students to estimate a calculation and to explain their answers. For example,

T: Is 218 + 157 more than 300? How do you know?

S: Yes, because 200 + 100 = 300.

T: Is it more than 500? How do you know?

S: No, because 300 + 200 = 500. 218 is less than 300 and 157 is less than 200, so 218 + 157 must be less than 500.

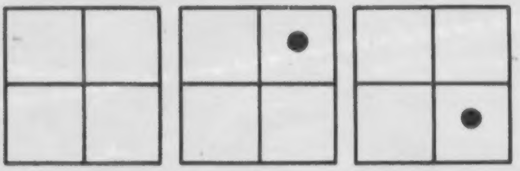
T: Is it more than 400?

S: No, because 18 + 57 is less than 100.

During the lessons, students are often asked to estimate a number displayed on the Minicomputer. See Lesson N33 for an example.

Other Mental Arithmetic Activities

1) Use the Minicomputer to give visual clues for addition and subtraction problems. For example, suppose this configuration is on the Minicomputer.



T: What number is on the Minicomputer? (41)

Hold a checker over the 8-square.

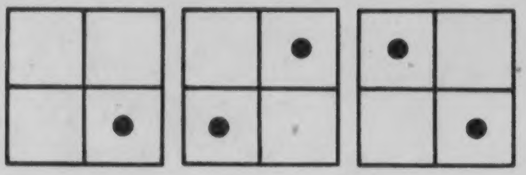
T: What number is 41 + 8? (49)

Hold a checker over the 10-square.

T: What number is 41 + 10? (51)

After a few such clues, hold a checker over the square only if the class has difficulty.

The next example involves subtraction.



T: What number is on the Minicomputer? (169)

Lift the checker temporarily from the 8-square.

T: What number is 169 - 8? (161)

Lift the checker temporarily from the 20-square.

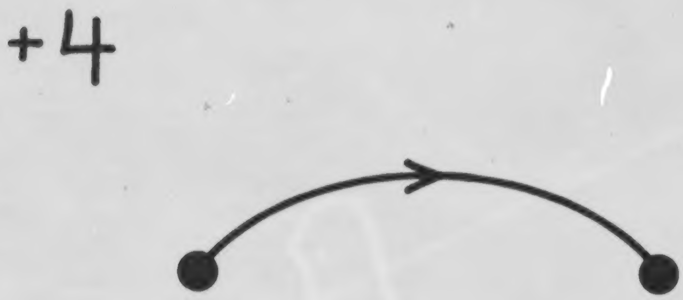
T: What number is 169 - 20? (149)

Lift the checkers temporarily from the 20-square and the 1-square.

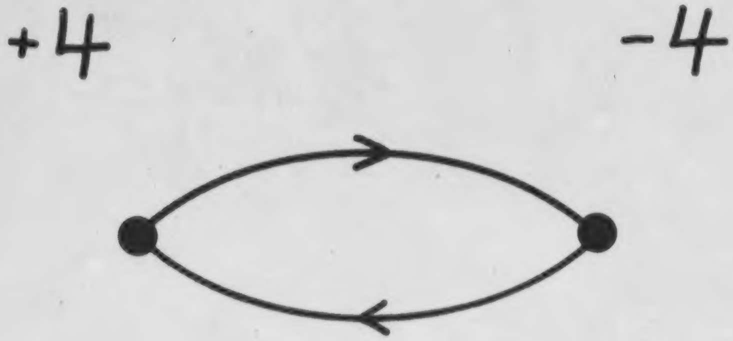
T: What number is 169 - 21? (148)

After a few such clues, lift the checkers only if the class has difficulty.

2) Use an arrow picture to pose mental arithmetic questions. For example, draw this arrow picture on the board.



Ask what number is at the right if the number at the left is, for example, 7. Repeat this activity several times and then draw the -4 return arrow.



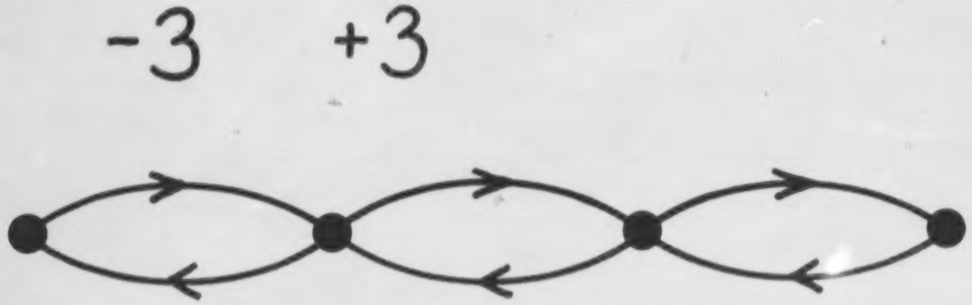
T : What could this blue return arrow be for?

S : -4.

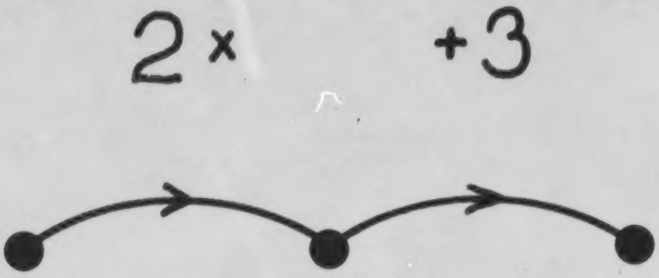
Ask the students what number is at the left if the number at the right is, for example, 10. Repeat this activity several times.

There are many variations of this activity; for example,

- use larger starting and ending numbers;
- use functions other than +4 and -4;
- use two or more arrows of the same function;



- use a composition of two functions.



NOTE: In this last variation, choose the ending number carefully when asking the class for the starting number. For example, if the ending number is 16, then 16 - 3 = 13 and "$\frac{1}{2}$ × 13" might be too difficult a calculation for most students, especially early in the year.

3) Play "Guess My Rule". Choose a secret rule such as +10. A student suggests a number to you and you answer with the number which is 10 more. This continues until the students discover your rule; for example,

S: 34.

T: 44.

S: 1.

T: 11.

S: 100.

T: 110.

S: 50.

T: 60. Does any one know my secret rule?

S: +10.

T: How do you know?

S: Each time you answered a number which was 10 more than the number we gave you.

Choose other secret rules such as +3, +5, -2, -1, and 2 x.

Hand Calculator Activities

Use the hand calculator (i) to develop and reinforce counting skills; (ii) to emphasize counting patterns; and (iii) to improve estimation skills.

1) Use one or more hand-calculators in the class to support counting activities in which patterns are generated. For example, such an activity might be similar to the first clue in "Detective Story #5", from Lesson W14. Start with any whole number on the display of your calculator and press $\boxed{+}$ $\boxed{5}$ $\boxed{=}$ $\boxed{=}$ $\boxed{=}$... . Record on the board the number that appears on the display of the hand calculator each time you press $\boxed{=}$. As soon as most of your students are able to predict the sequence quickly, you can abandon the calculator. Ask students to explain the pattern.

There are many variations of this activity, such as:

- press $\boxed{+}$ $\boxed{2}$, (or $\boxed{3}$ or $\boxed{4}$ or $\boxed{10}$) $\boxed{=}$ $\boxed{=}$ $\boxed{=}$... ;
- start with a large number and press $\boxed{-}$ $\boxed{5}$, (or $\boxed{2}$ or $\boxed{10}$ or $\boxed{100}$) $\boxed{=}$ $\boxed{=}$ $\boxed{=}$... .

2) Play "Let's Concentrate" using one or more hand-calculators in the class. Ask the students who have the calculators to put a number of your choice on the display and then to hide the display with one hand. Give a sequence of two operations for the students to do by pressing the appropriate keys and without looking at the display. Then ask what number is on the dis-

play and let the students check to see if they are correct. Continue with the new number and another sequence of two operations. For example,

Start with 4 on the display.

Press $\boxed{+}$ $\boxed{3}$ $\boxed{+}$ $\boxed{2}$ $\boxed{=}$. The resulting number is 9.
(Students check that 9 is on the display.)

Press $\boxed{+}$ $\boxed{1}$ $\boxed{\times}$ $\boxed{2}$ $\boxed{=}$. The resulting number is 20.
(Students check that 20 is on the display.)

Press $\boxed{-}$ $\boxed{3}$ $\boxed{+}$ $\boxed{1}$ $\boxed{=}$. The resulting number is 18.
(Students check that 18 is on the display.)

3) Play "Hand-Calculator Golf" with a small group of students, each one having a calculator. Ask the students to put a small whole number of your choice on the display of the calculators. Then let the students take turns suggesting some numbers to add or to subtract until a particular goal, such as 200, is reached.

A possible game where 200 is the goal and four students are playing is described below.

		Display
Start with 12 on the display.		[12]
First player:	+ 9	[21]
Second player:	+ 72	[93]
Third player:	+ 17	[110]
Fourth player:	+ 7	[117]
First player:	+ 93	[210]
Second player:	- 10	[200]

Some variations of this game include:

- change the starting number;
- make the goal a larger multiple of 100;
- allow the use of $\boxed{\times}$ $\boxed{\text{some number between 0 and 10}}$.

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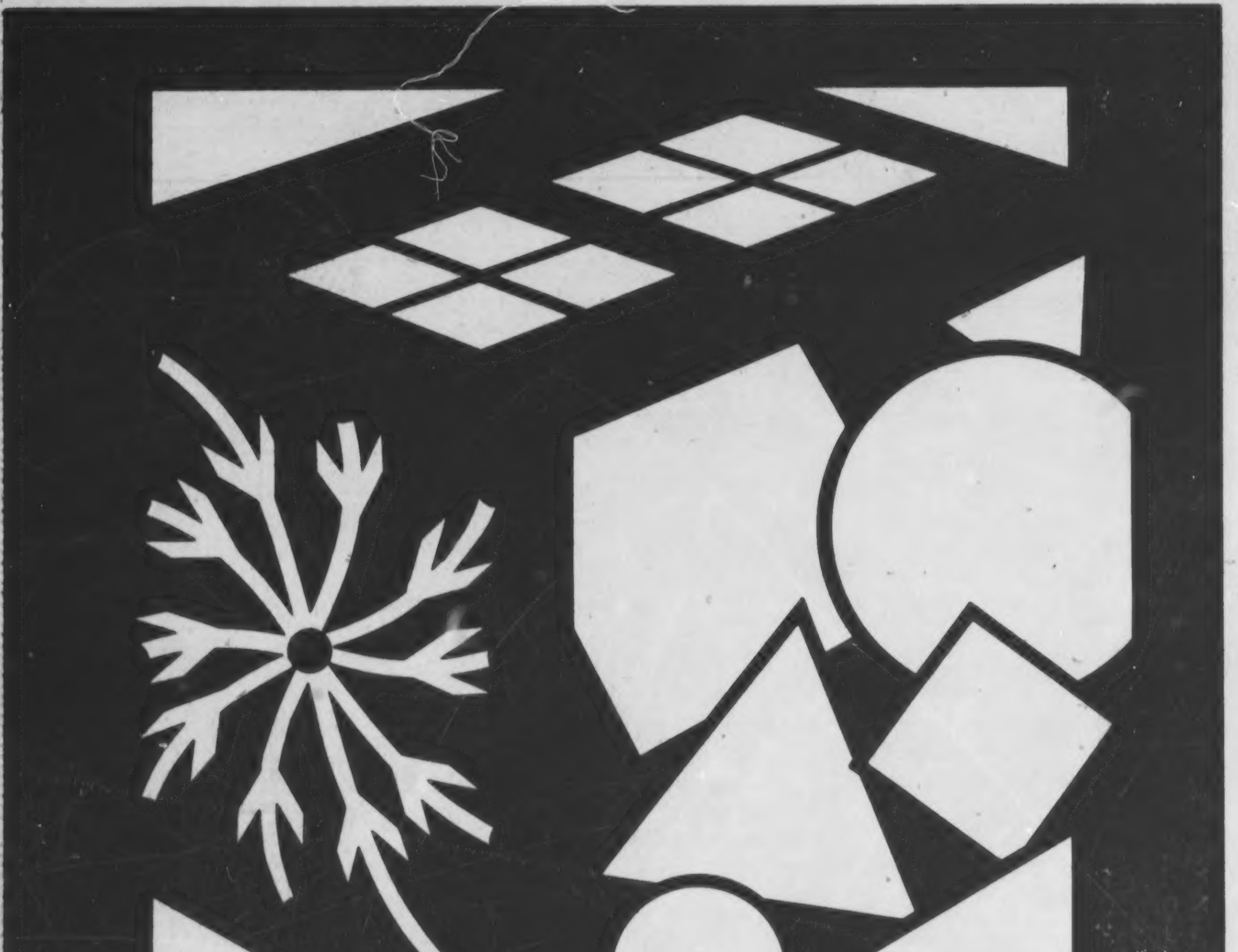
Name

Worksheets



WHAT'S IN THIS BOOK?

This book contains all the worksheets you will need for Part I of the CSMP Upper Primary Grades Program. Worksheets are labeled with the same letter and number as the lessons with which they are used. In this book, they are in the following order:

1) All the "N" worksheets: N1* through N32**
2) All the "L" worksheets: L3* through L8**
3) All the "G" worksheets: G1* through G13****
4) All the "W" worksheets: W1* through W8****

HOW DO YOU REMOVE PAGES?

Step 1: Find the worksheet you want.
The number of each worksheet is in the top right corner of the page.

Step 2: Fold and crease the page along the perforation line.

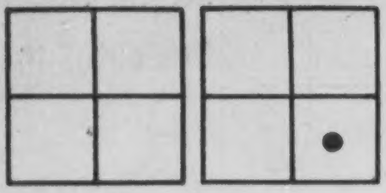
Step 3: With your left hand index finger and thumb, press book firmly down left of the perforation line and along center fold, starting at the top of the book.

Step 4: With your right hand, hold top right corner of page and tear along the perforation line. Follow the tear with your left hand holding down center fold of book.

Name ______________________

N 1 *

What number is on the Minicomputer?



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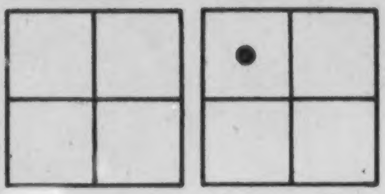

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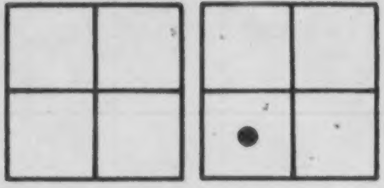

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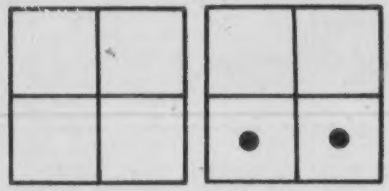

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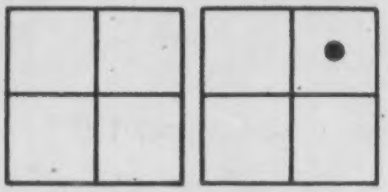

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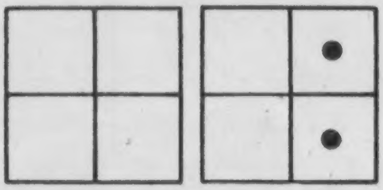

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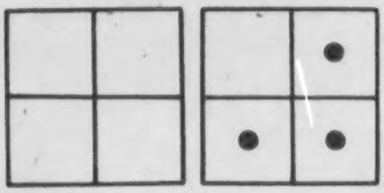

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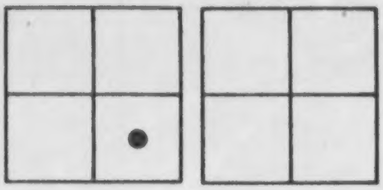

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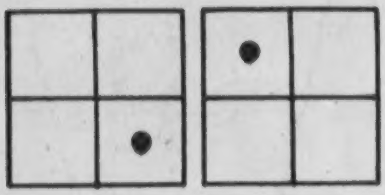
Name ______________________________

N 1 **

What number is on the Minicomputer?



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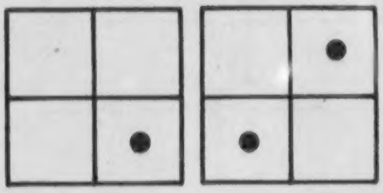

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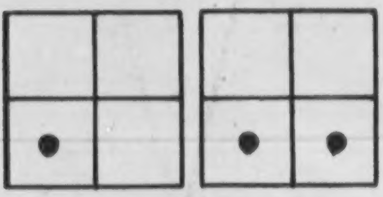

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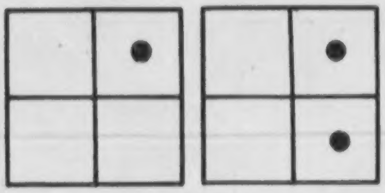

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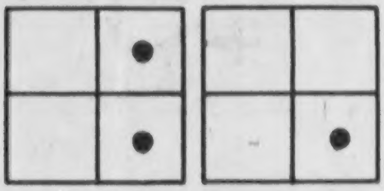

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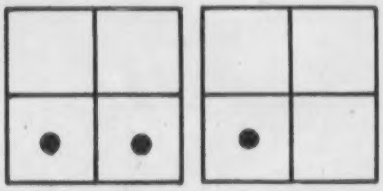

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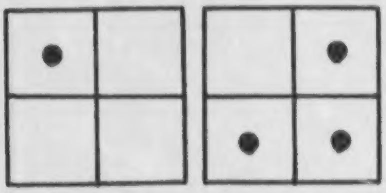

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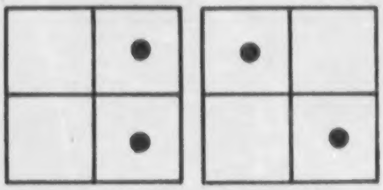

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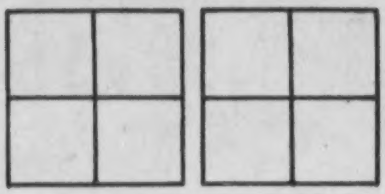
Name ______________________

N1 ***

Put these numbers on the Minicomputer.



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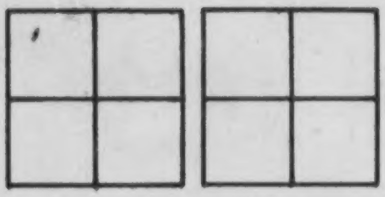

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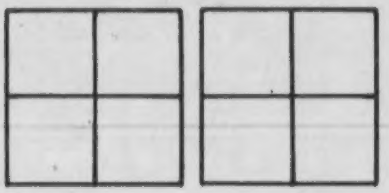

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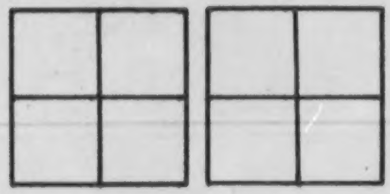

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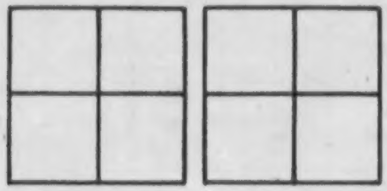

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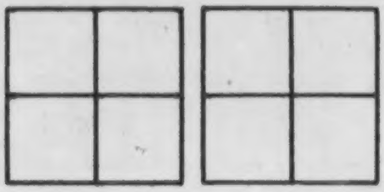

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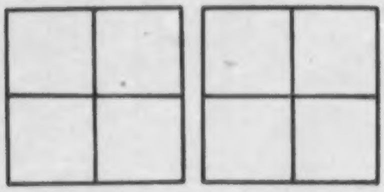

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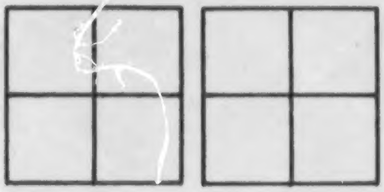
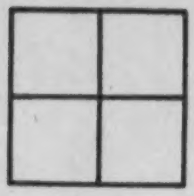


7 5

Name ____________________

N 1 ****

Put each number on the Minicomputer.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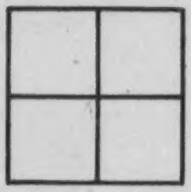
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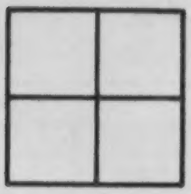
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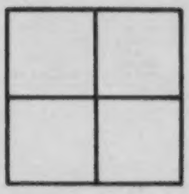
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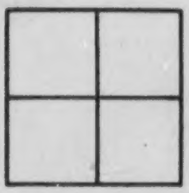
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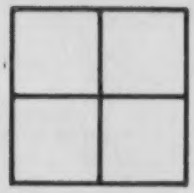
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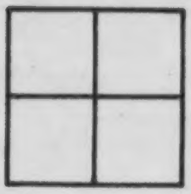
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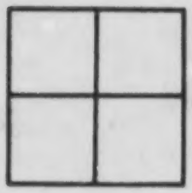
= 37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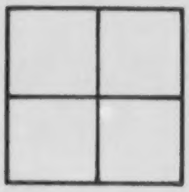
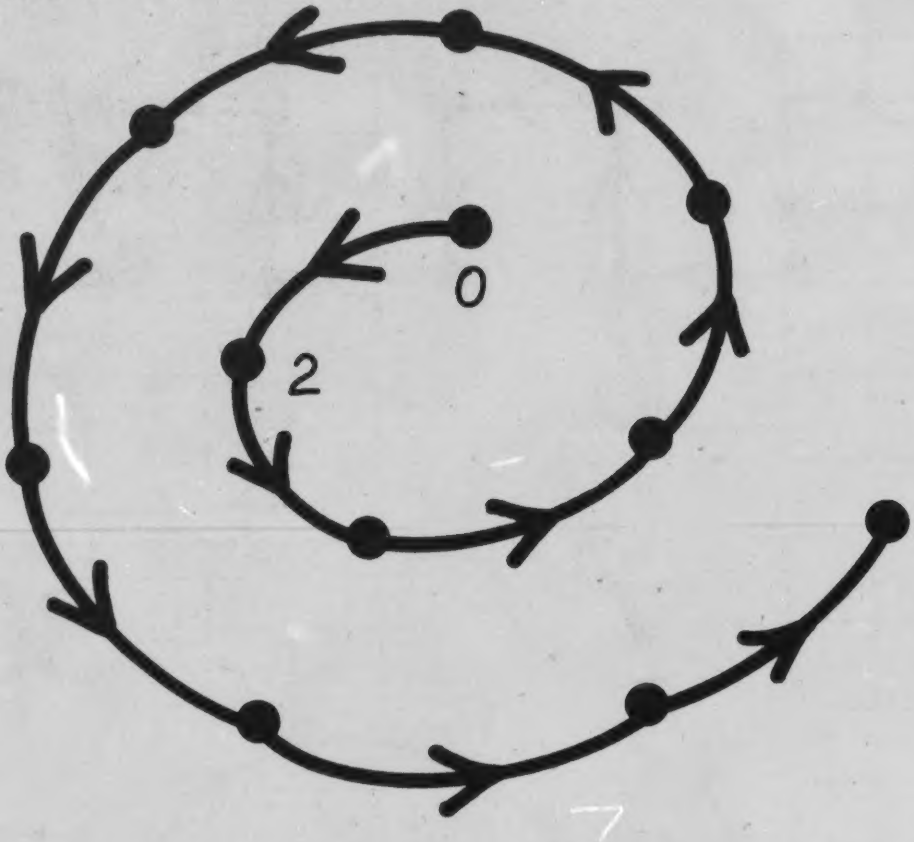
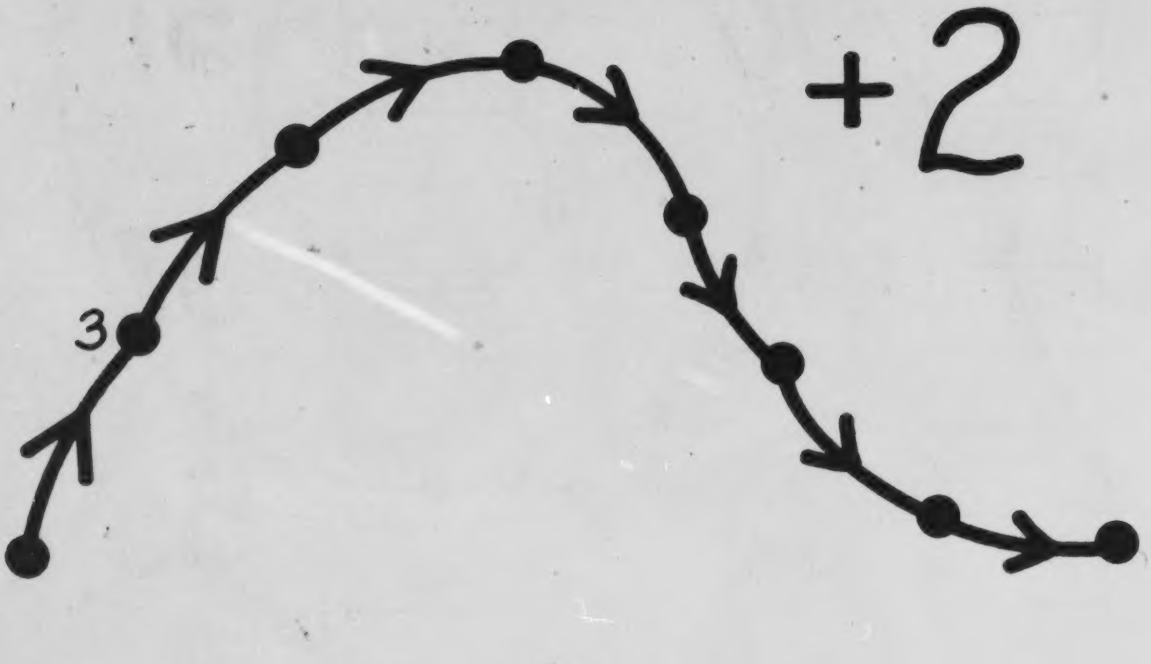

= 55

Name ____________________

N2 *

Label the dots.

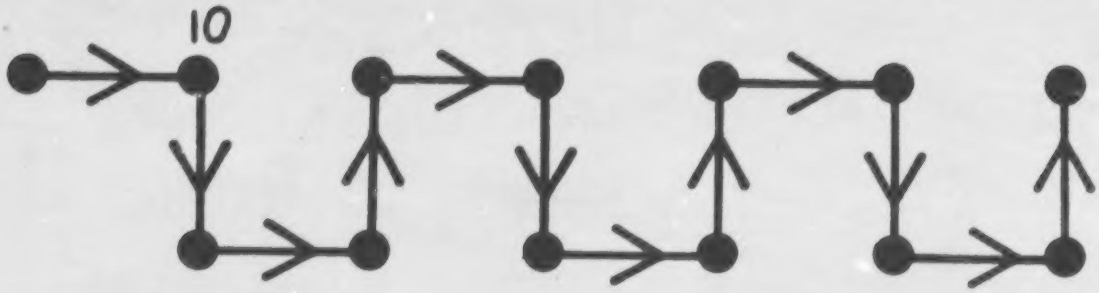


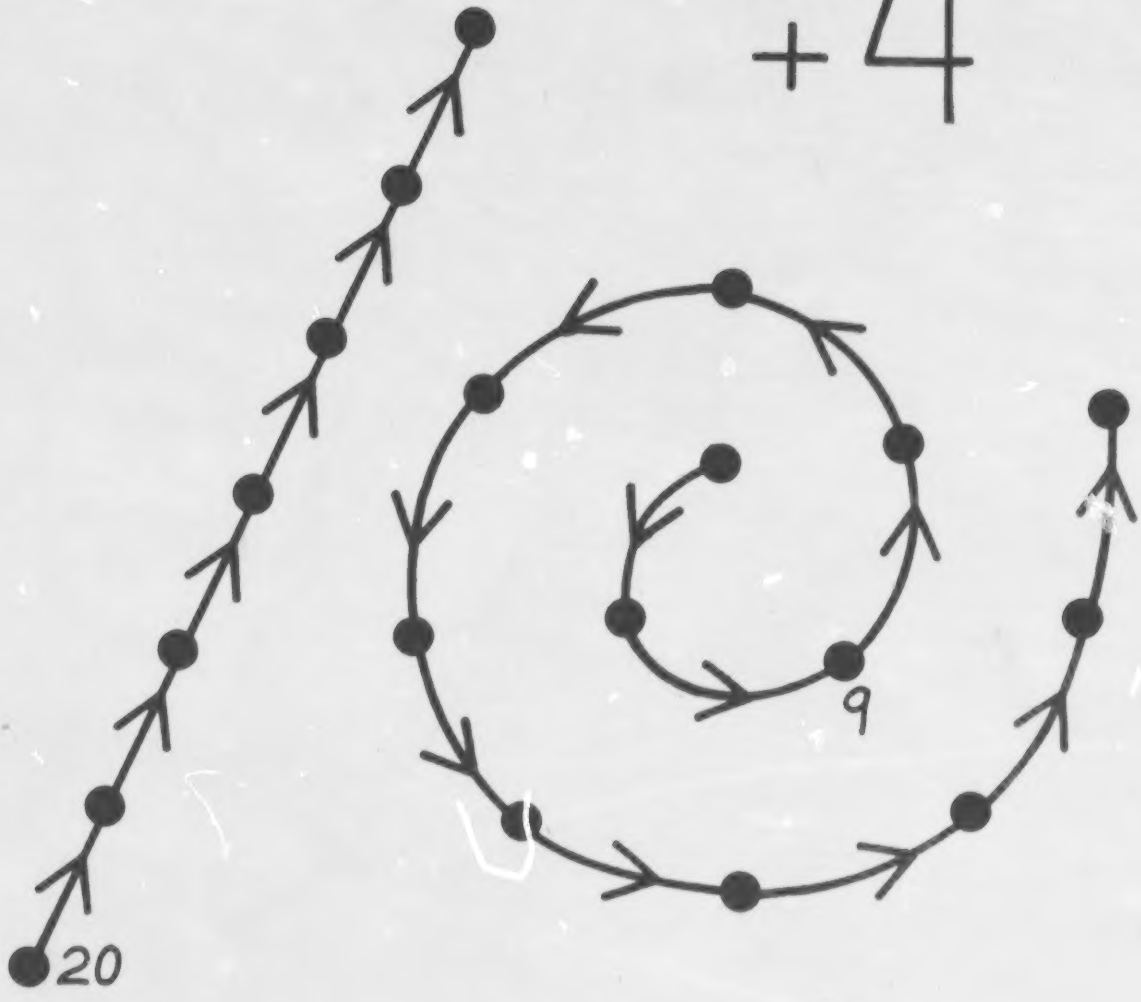


Name ________________________________

N 2 **

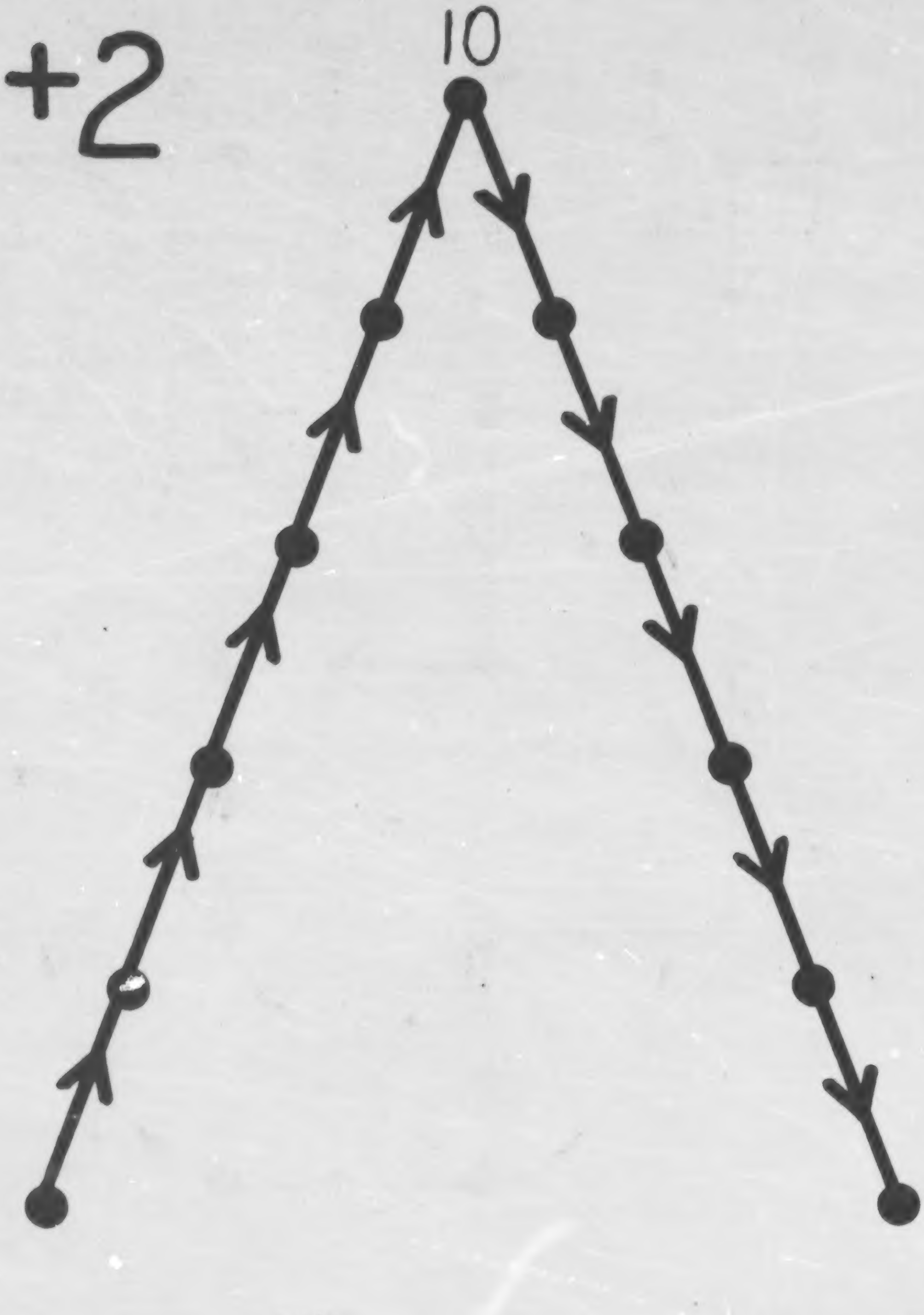
Label the dots.





Name ____________________ **N 8 ***

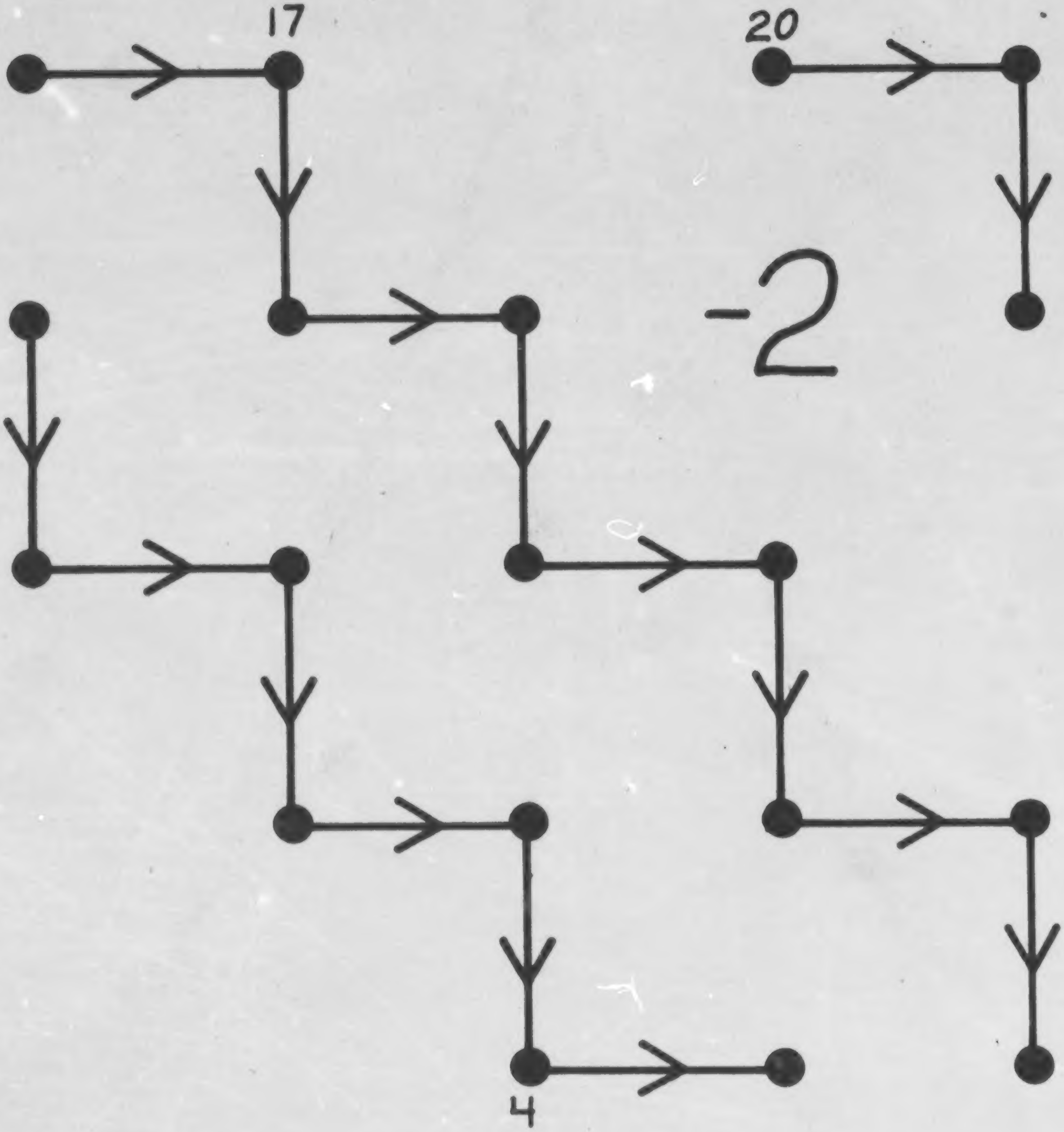
Label the dots.



Name ________________________________

N 8 **

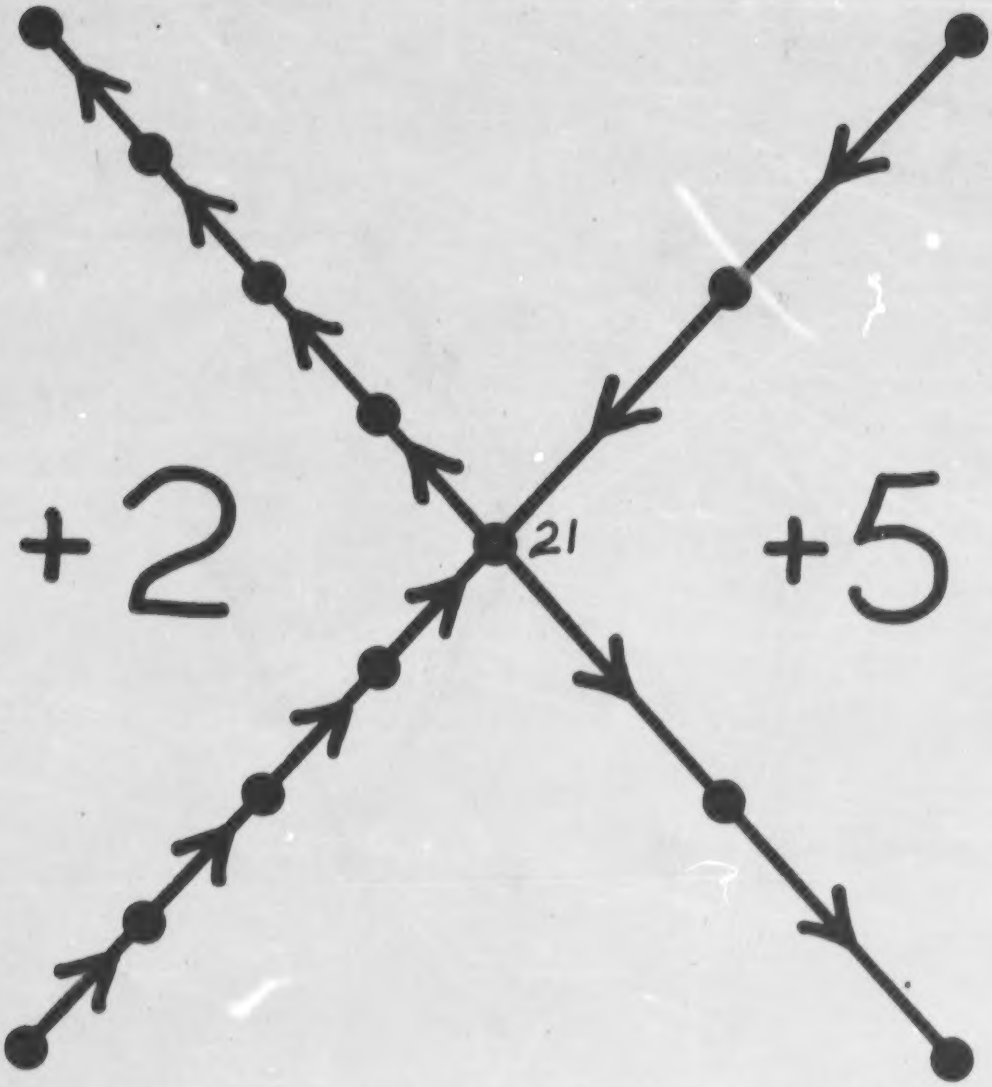
Label the dots.



Name

N8 ***

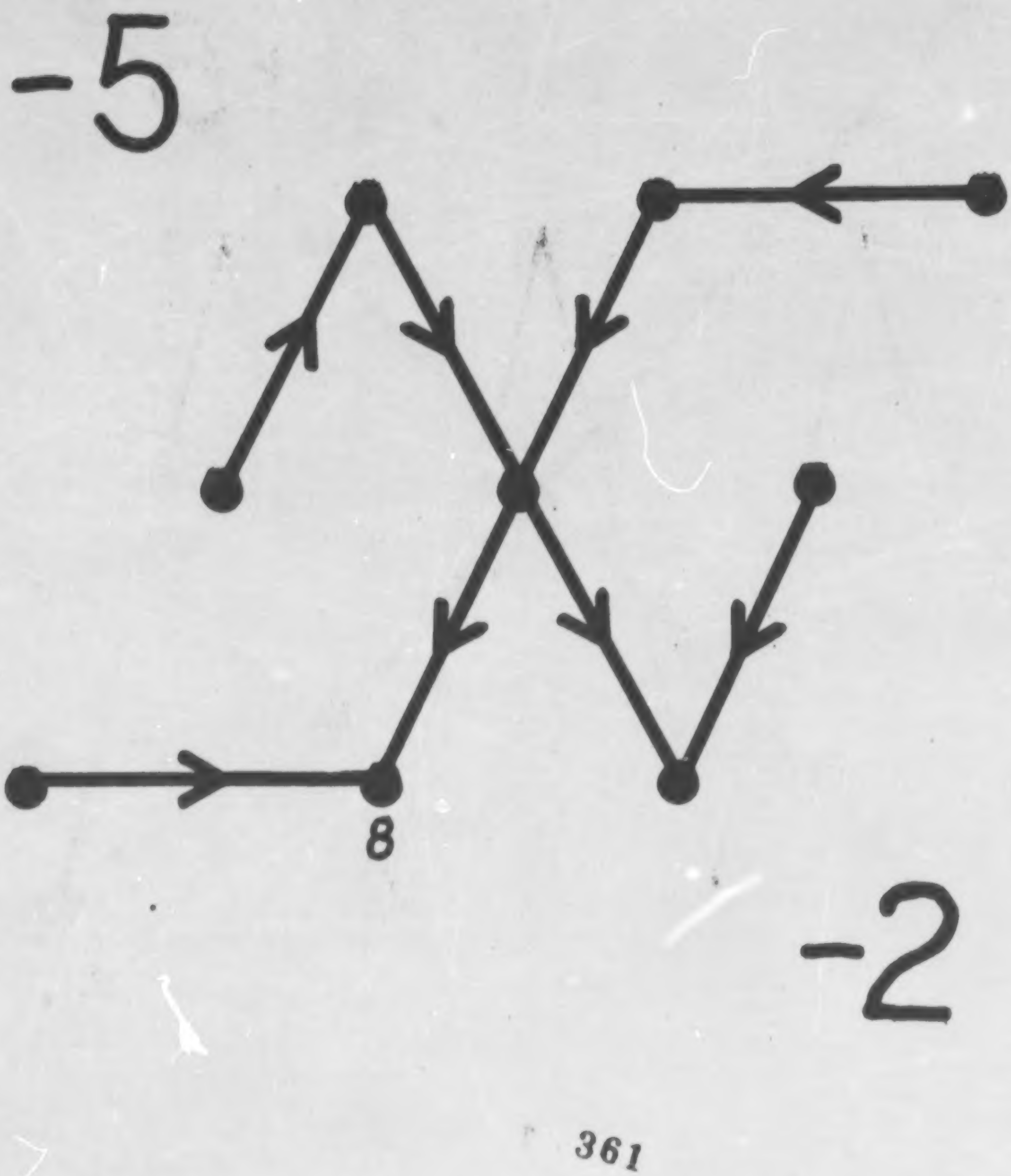
Label the dots.



Name

N8 ****

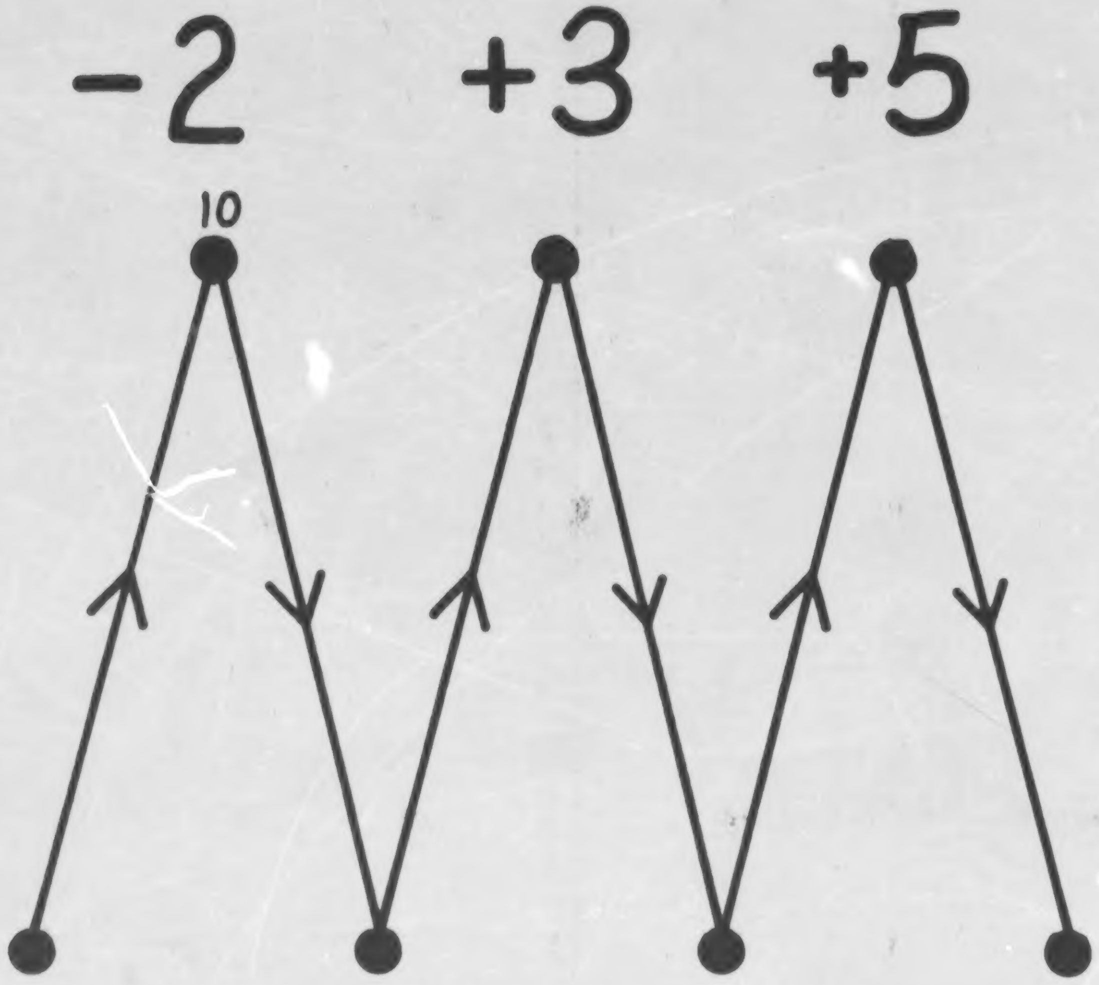
Find 17 in this arrow picture and circle it.



Name ___________________________

N10 *

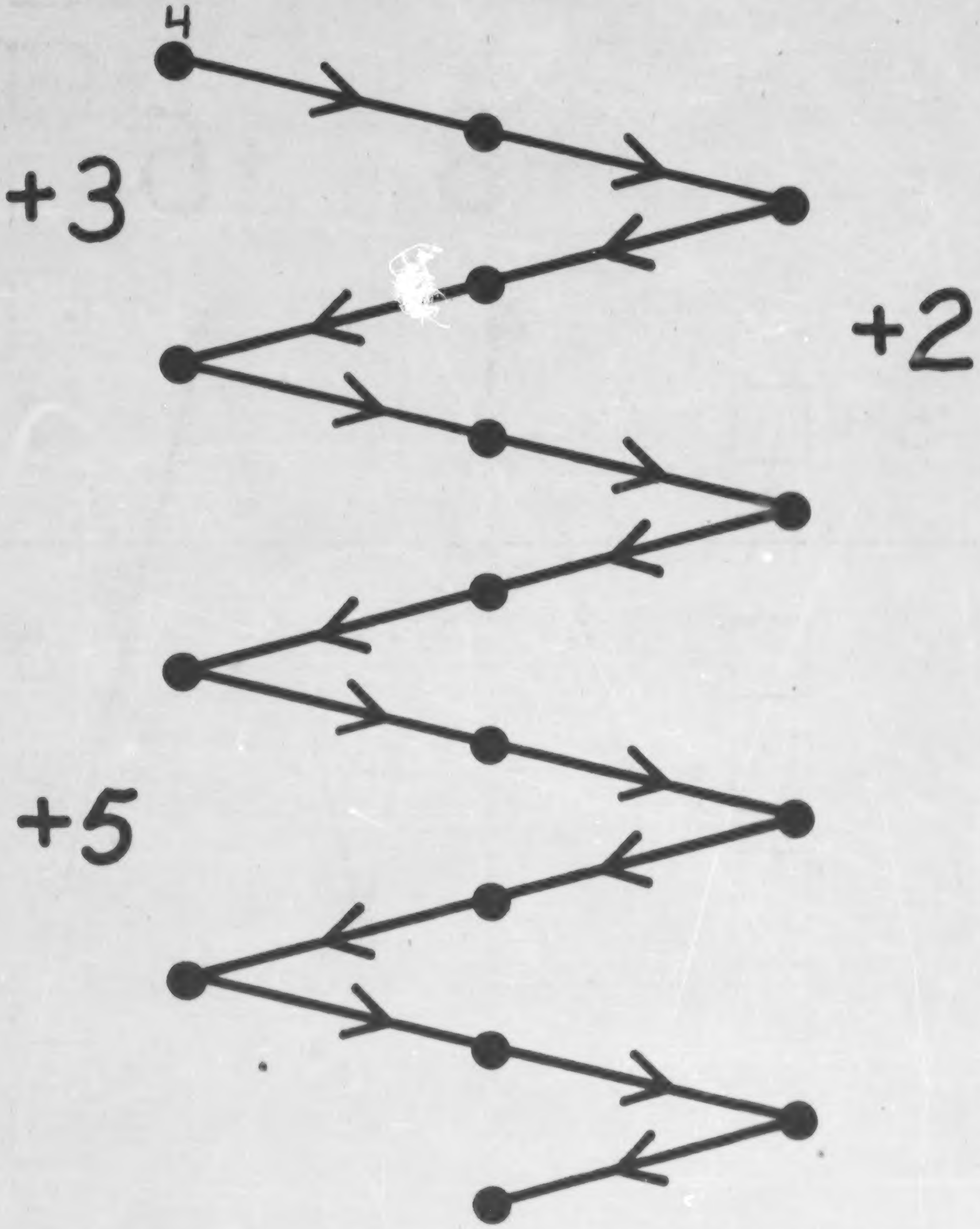
Label the dots. Draw all the +5 arrows.



Name ______________________

N10 **

Label the dots. Draw all the +5 arrows.



Name ____________________

N 12 *

Complete these number stories.

5 + 5 = □

5 + 6 = □

5 + 7 = □

5 + 8 = □

5 + □ = 14

10 + 3 = □

10 + 4 = □

10 + 5 = □

10 + 6 = □

10 + □ = 17

16 + □ = 17

16 + □ = 18

16 + □ = 19

16 + □ = 20

16 + 5 = □

8 + □ = 10

8 + □ = 12

8 + □ = 14

8 + 8 = □

8 + 10 = □

N 12 **

Name ______________________

Complete these number stories.

6 + 6 = □

7 + 7 = □

8 + 8 = □

9 + 9 = □

10 + □ = 20

14 − 4 = □

13 − 3 = □

12 − □ = 10

11 − □ = 10

□ − 0 = 10

10 + 5 = □

20 + 5 = □

30 + □ = 35

40 + □ = 45

□ + 5 = 55

□ = 5 + 5

□ = 6 + 4

□ = 7 + 3

10 = 8 + □

10 = □ + 1

Name ____________________

N 12 ***

Complete these number stories.

5 + 5 + 5 = ☐
5 + 5 + 6 = ☐
5 + 5 + ☐ = 17
5 + 5 + ☐ = 18
5 + 5 + 9 = ☐

7 − 3 = ☐
7 − 4 = ☐
7 − 5 = ☐
7 − ☐ = 1
7 − ☐ = 0

☐ = 50 + 50
☐ = 51 + 51
104 = ☐ + 52
106 = ☐ + 53
108 = 54 + ☐

23 + 10 = ☐
23 + 20 = ☐
23 + 30 = ☐
23 + ☐ = 63
23 + ☐ = 73

Name ______________________________

N 12 ****

Complete these number stories.

100 + □ = 200
101 + 101 = □
102 + 102 = □
103 + □ = 206
□ + 104 = 208

50 + □ = 100
49 + 49 = □
48 + 48 = □
□ + 47 = 94
□ + 46 = 92

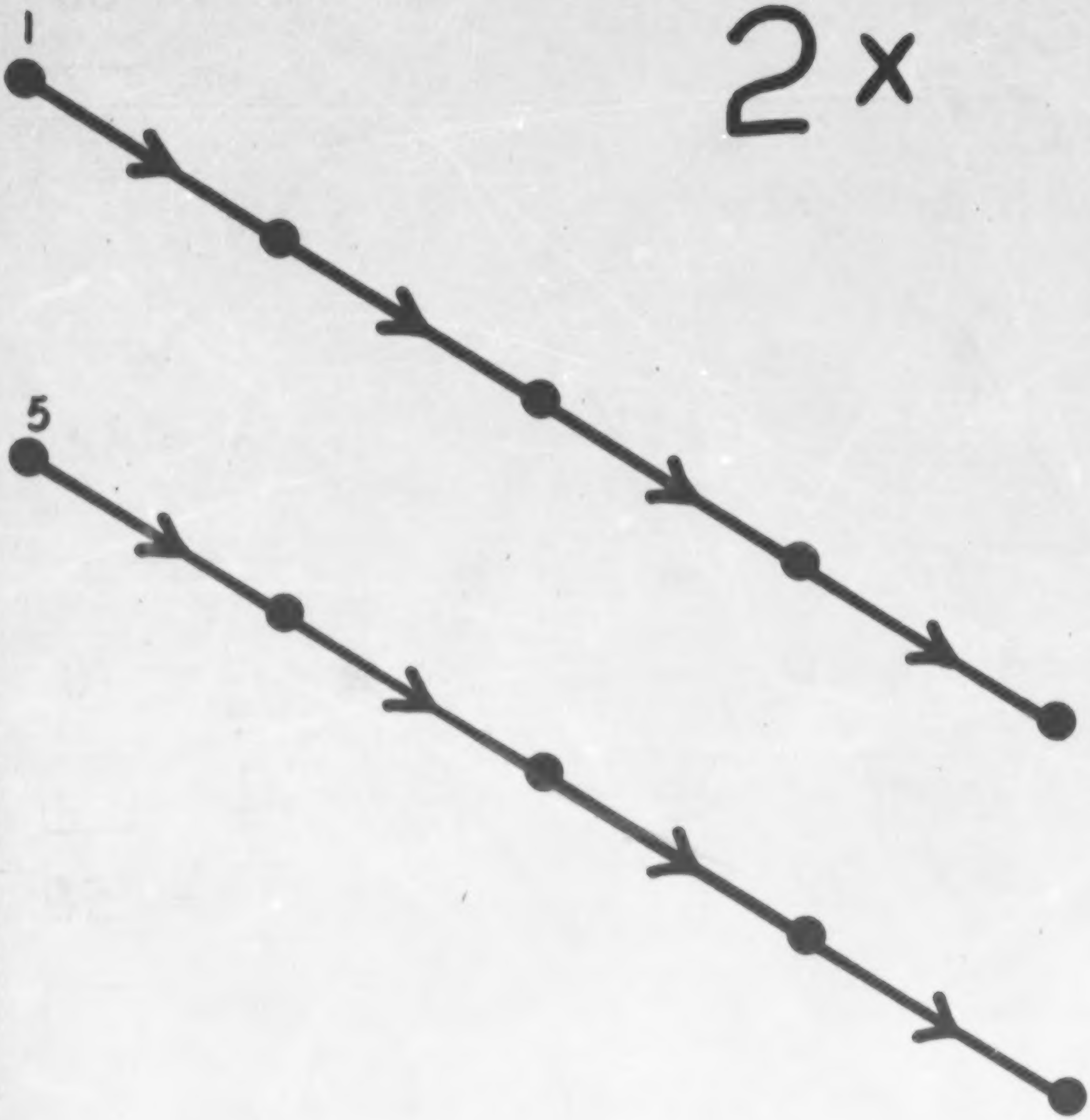
30 + 30 = □
31 + 29 = □
32 + 28 = □
33 + 27 = □
34 + 26 = □

5 + □ = 10
15 + 15 = □
25 + □ = 50
35 + 35 = □
45 + 45 = □

Name ______________________________ **N16** *

Label the dots.

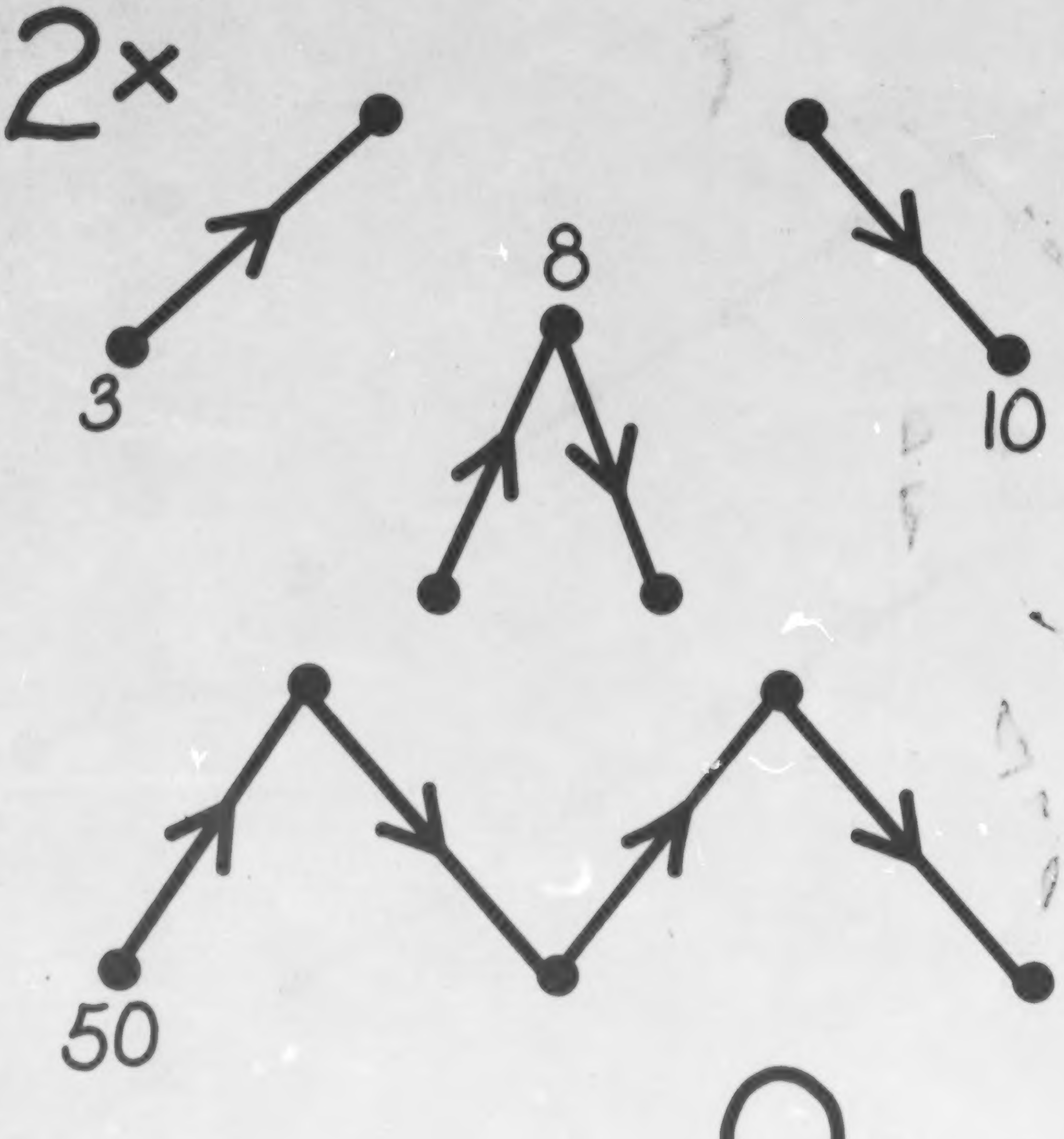
2 x



Name ______________________

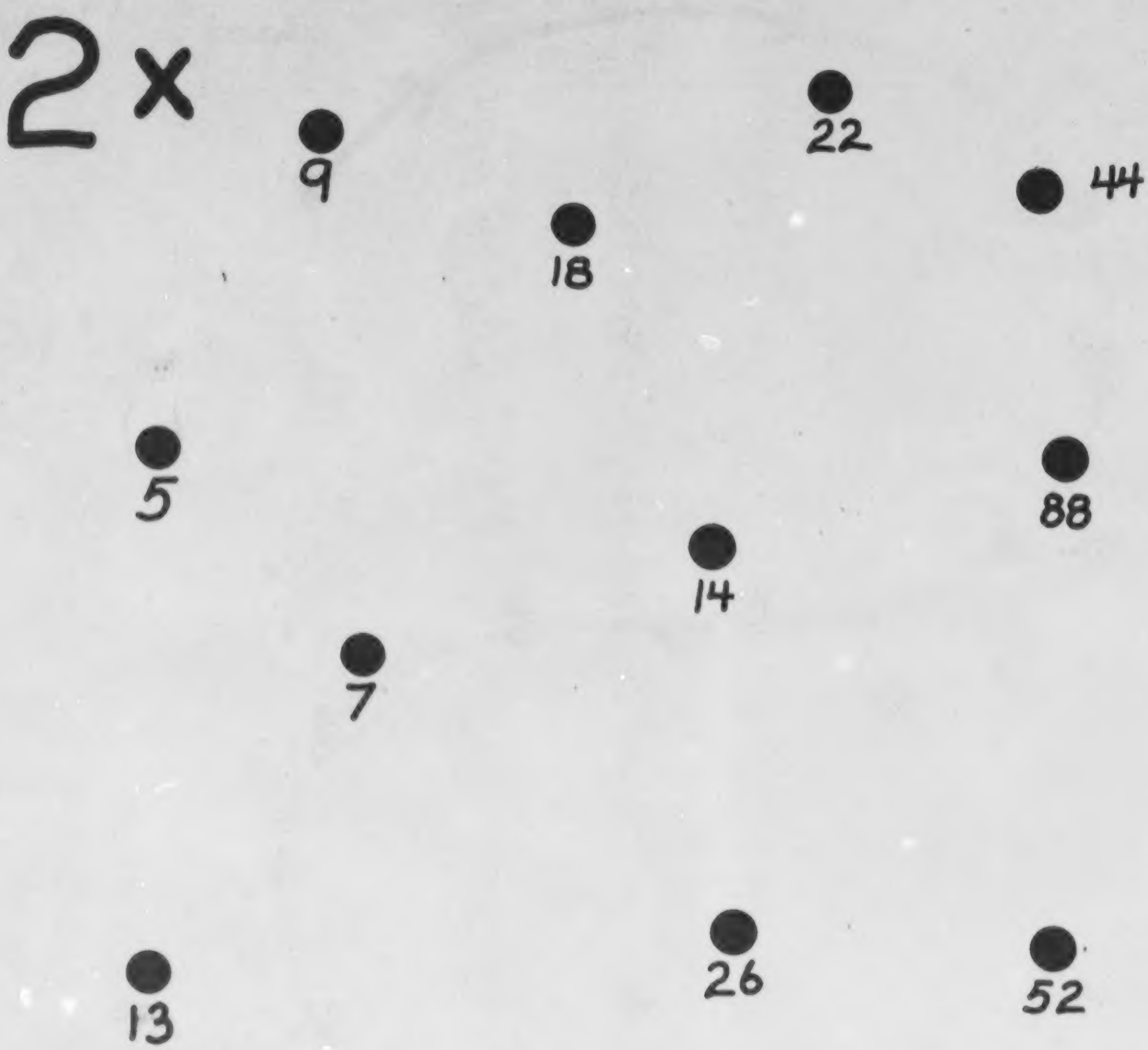
N16 **

Label the dots.



Name ______________________ **N16 *****

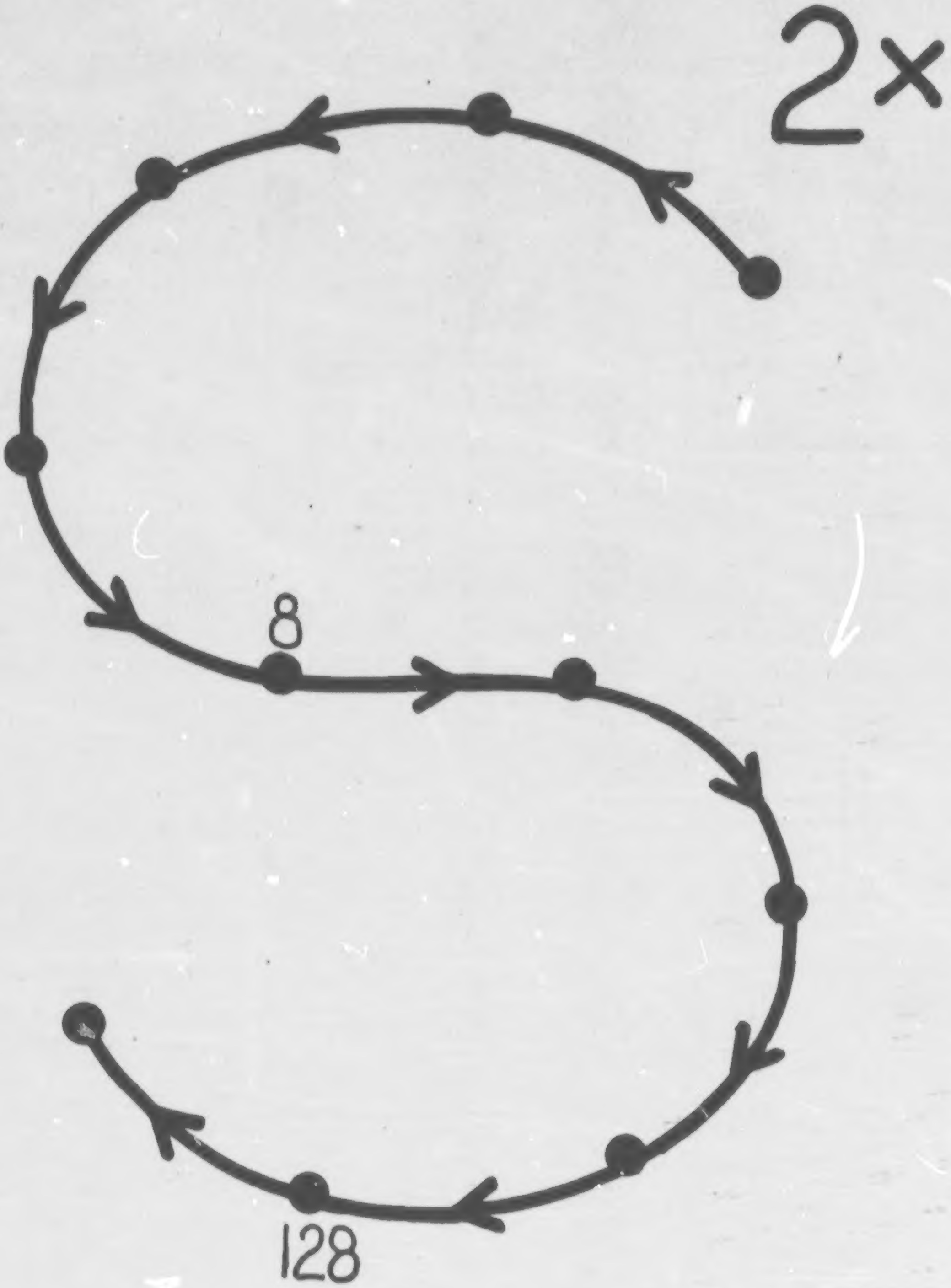
Draw all the 2x arrows.


2x
9
22
44
18
5
88
14
7
13
26
52

You should have six arrows.

Name ______________________________ **N 16 ******

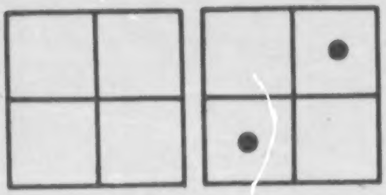
Label the dots.



Name ____________________

N 17 *

What number is on the Minicomputer?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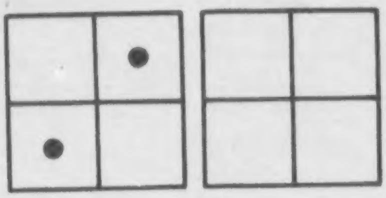
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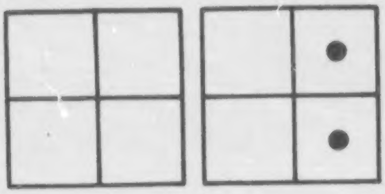
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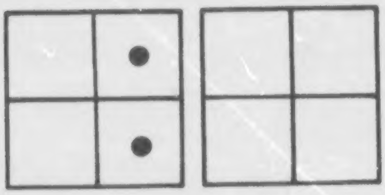
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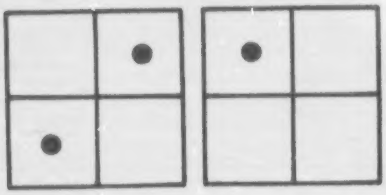
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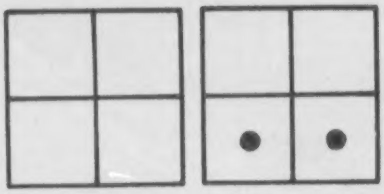
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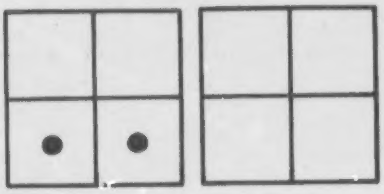
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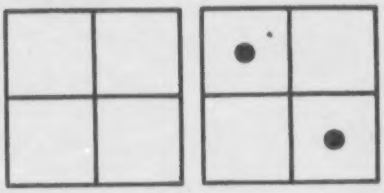
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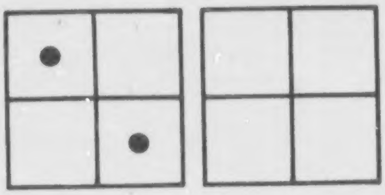
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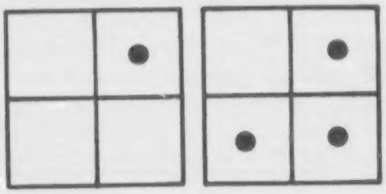
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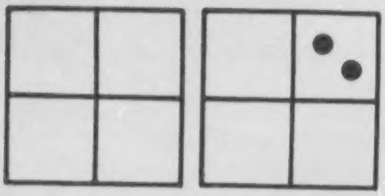
Name ______________________________

N17 **

What number is on the Minicomputer?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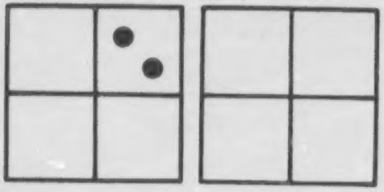
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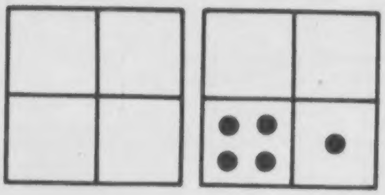
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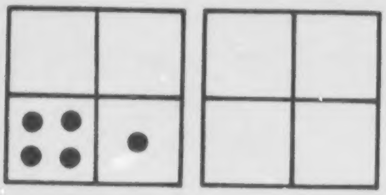
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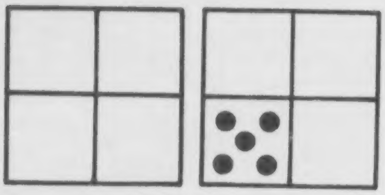
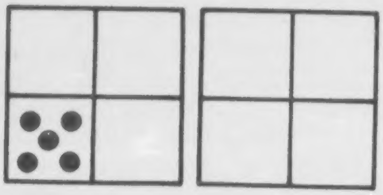
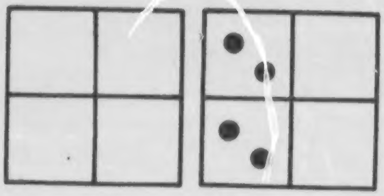
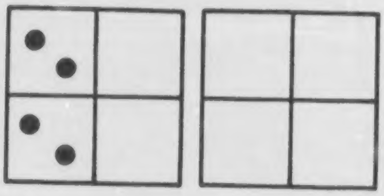
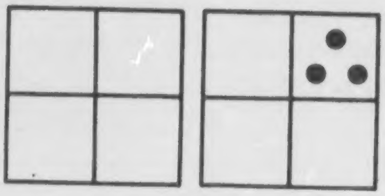
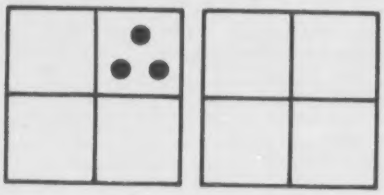

= ______

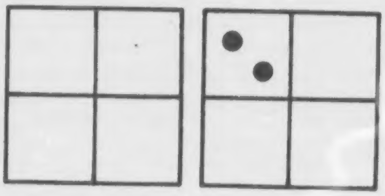

= ______

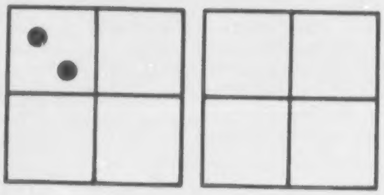

= ______


= ______


= ______


= ______

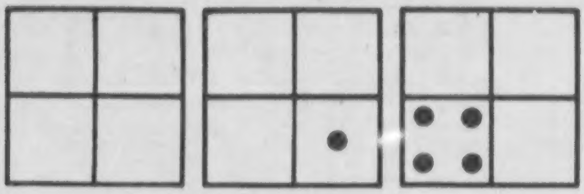

= ______


= ______

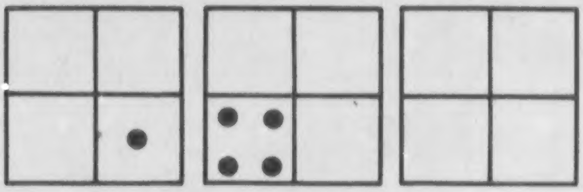
Name ______________________

N17 ***

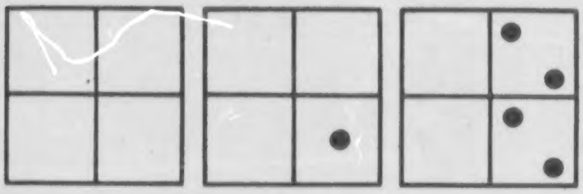
What number is on the Minicomputer?



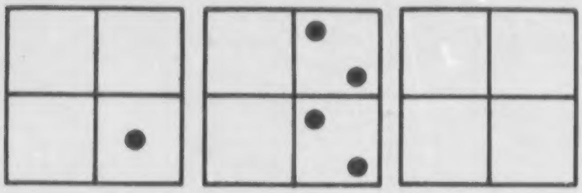
= ______



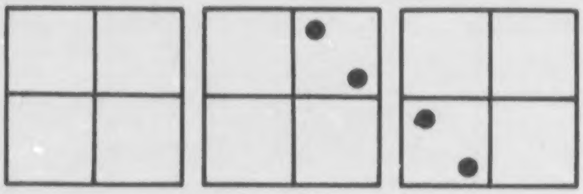
= ______



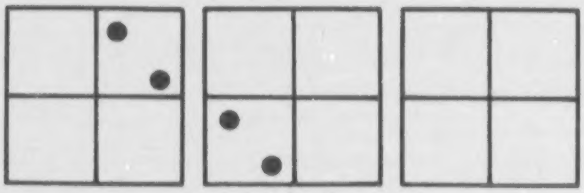
= ______



= ______



= ______

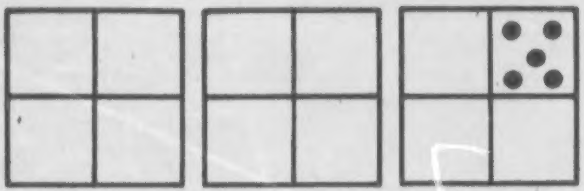


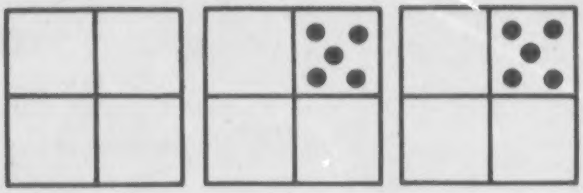
= ______

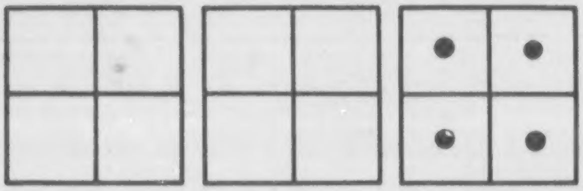
Name ______________________

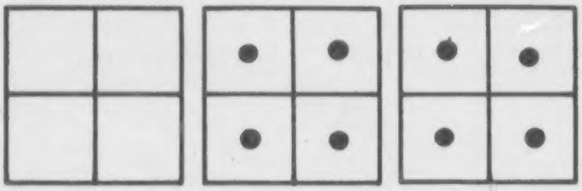
N17 ****

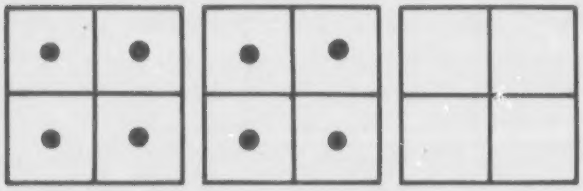
What number is on the Minicomputer?

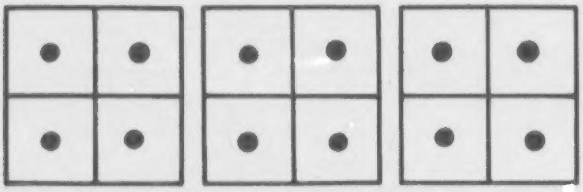
 = ______

 = ______

 = ______

 = ______

 = ______

 = ______

Name ______________________ **N 20** *

Complete these number stories.

$\square + \square = 10$

$\square + \square = 12$

$\square + \square = 14$

$\square + \square = 16$

$\square + \square = 18$

$\square + \square = 20$

$\square + \square = 40$

$\square + \square = 60$

$\square + \square = 80$

$\square + \square = 100$

$0 = \triangle + \triangle + \triangle$

$15 = \triangle + \triangle + \triangle$

$30 = \triangle + \triangle + \triangle$

$60 = \triangle + \triangle + \triangle$

$90 = \triangle + \triangle + \triangle$

$150 = \triangle + \triangle + \triangle$

Name ______________________________

N 20 **

Complete these number stories.

☐ + ☐ + 5 = 25

☐ + ☐ + 7 = 47

☐ + ☐ + 3 = 83

☐ + ☐ + 8 = 108

---

68 = 8 + ☐ + ☐ + ☐

68 = △ + △ + 20 + 20 + 20

72 = △ + △ + 20 + 20 + 20

☐ + ☐ = 200

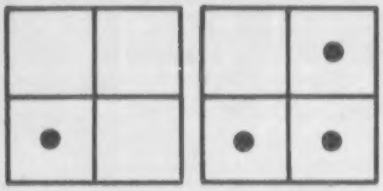
☐ + ☐ = 300

☐ + ☐ = 400

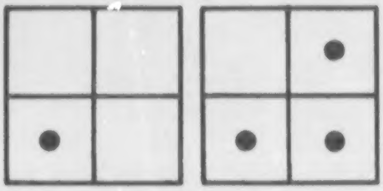
☐ + ☐ = 500

Name ____________________ **N 21 ***

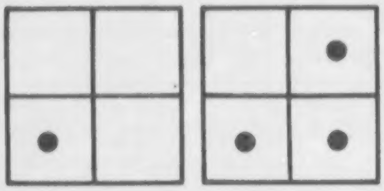
Complete these number stories.



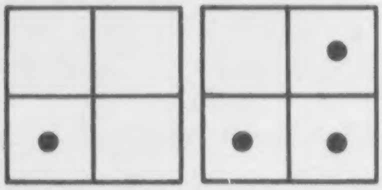
27 - 4 = ____



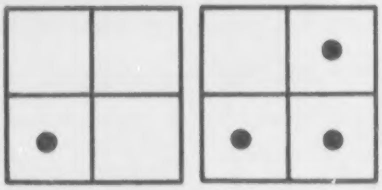
27 - 20 = ____



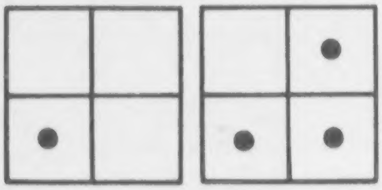
27 - 3 = ____



27 - 5 = ____



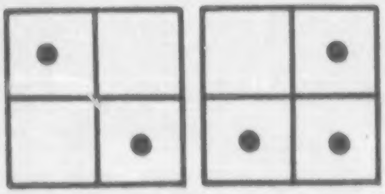
27 - 7 = ____



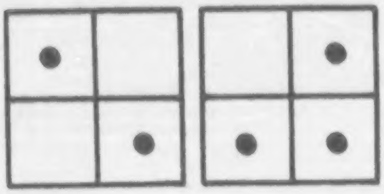
27 - 27 = ____

Name ______________________________ **N 21 ****

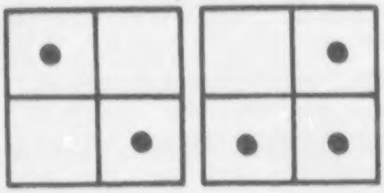
Complete these number stories.



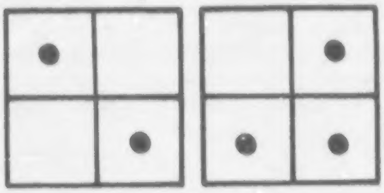
97 - 80 = ____



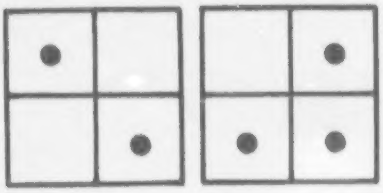
97 - 5 = ____



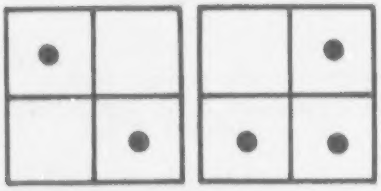
97 - 14 = ____



97 - 82 = ____



97 - 85 = ____

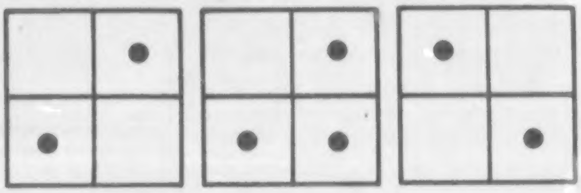


97 - 17 = ____

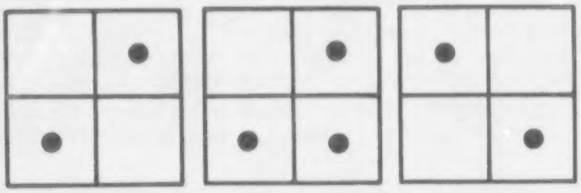
Name ______________________

N 21 ***

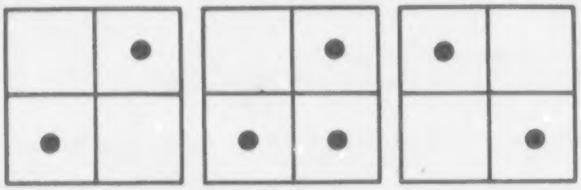
Complete these number stories.



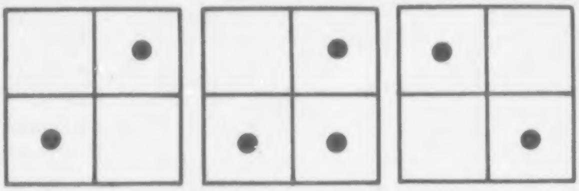
679 - 58 = ___



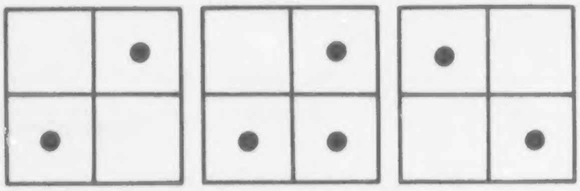
679 - 248 = ___



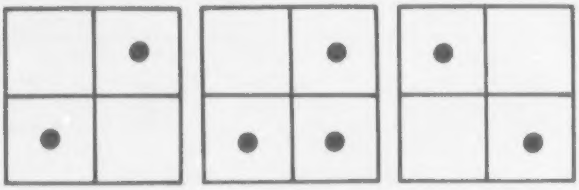
679 - 431 = ___



679 - 609 = ___



679 - 478 = ___

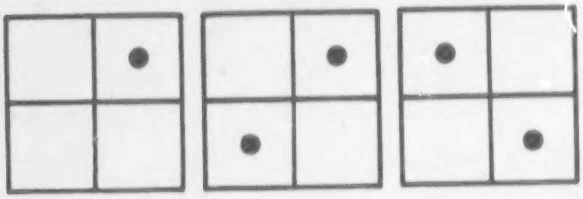


679 - 668 = ___

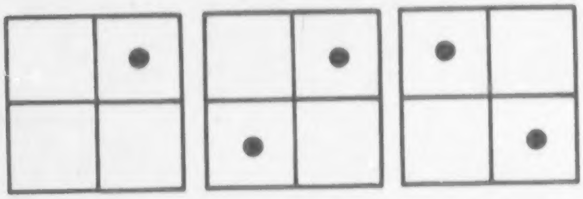
Name ______________________

N 21 ****

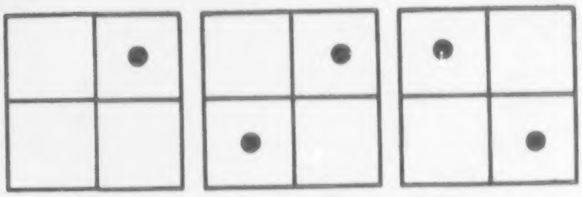
Complete these number stories.



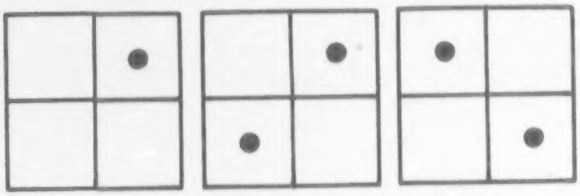
469 - 10 = ___



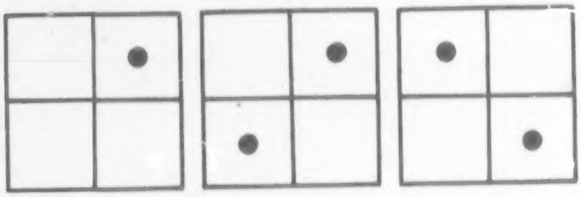
469 - 200 = ___



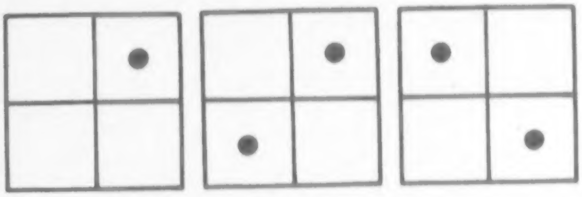
469 - 24 = ___



469 - 248 = ___



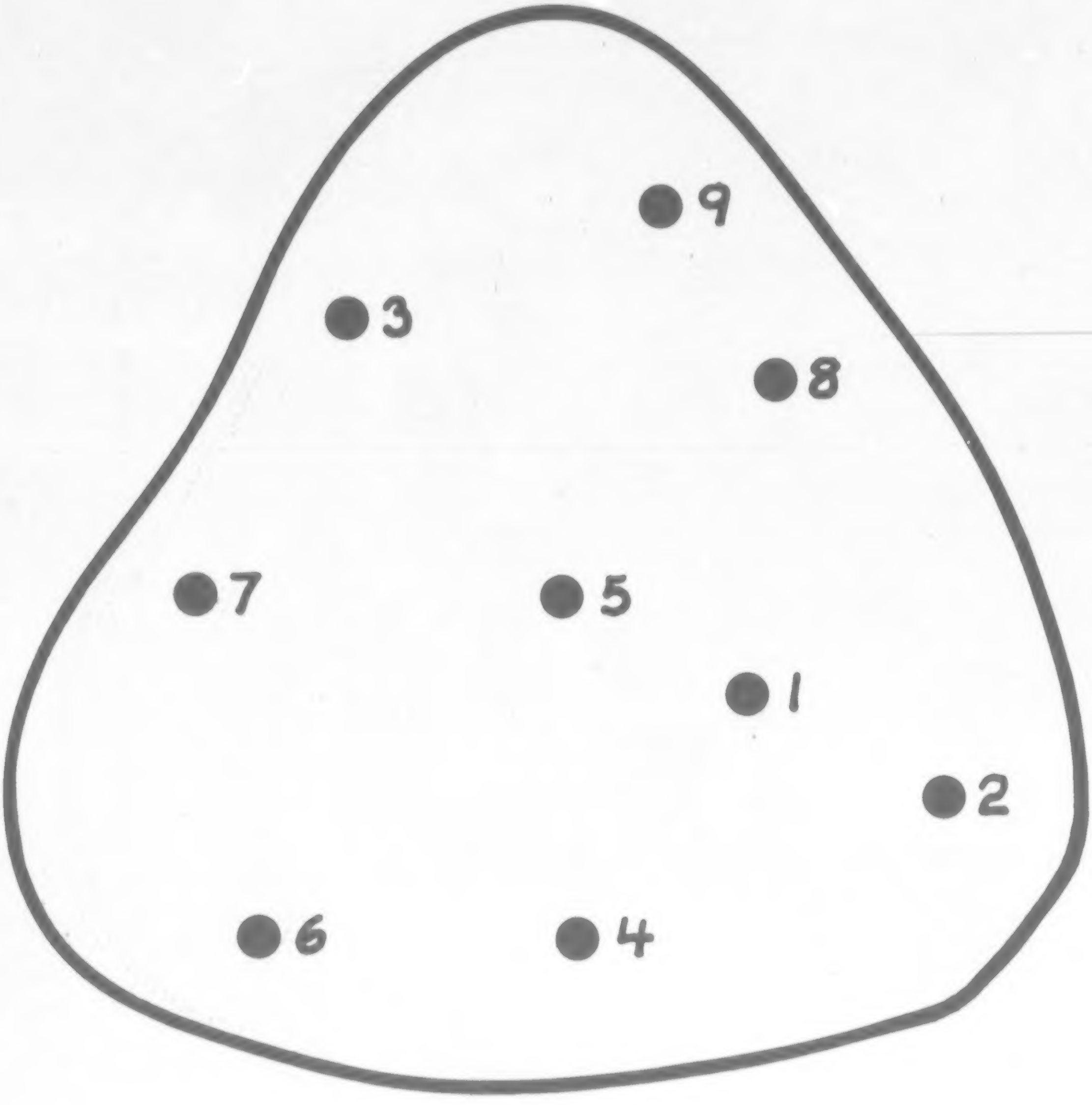
469 - 411 = ___



469 - 255 = ___

Name ____________________ **N 25** *

Draw a string around each pair of numbers whose sum is 10.

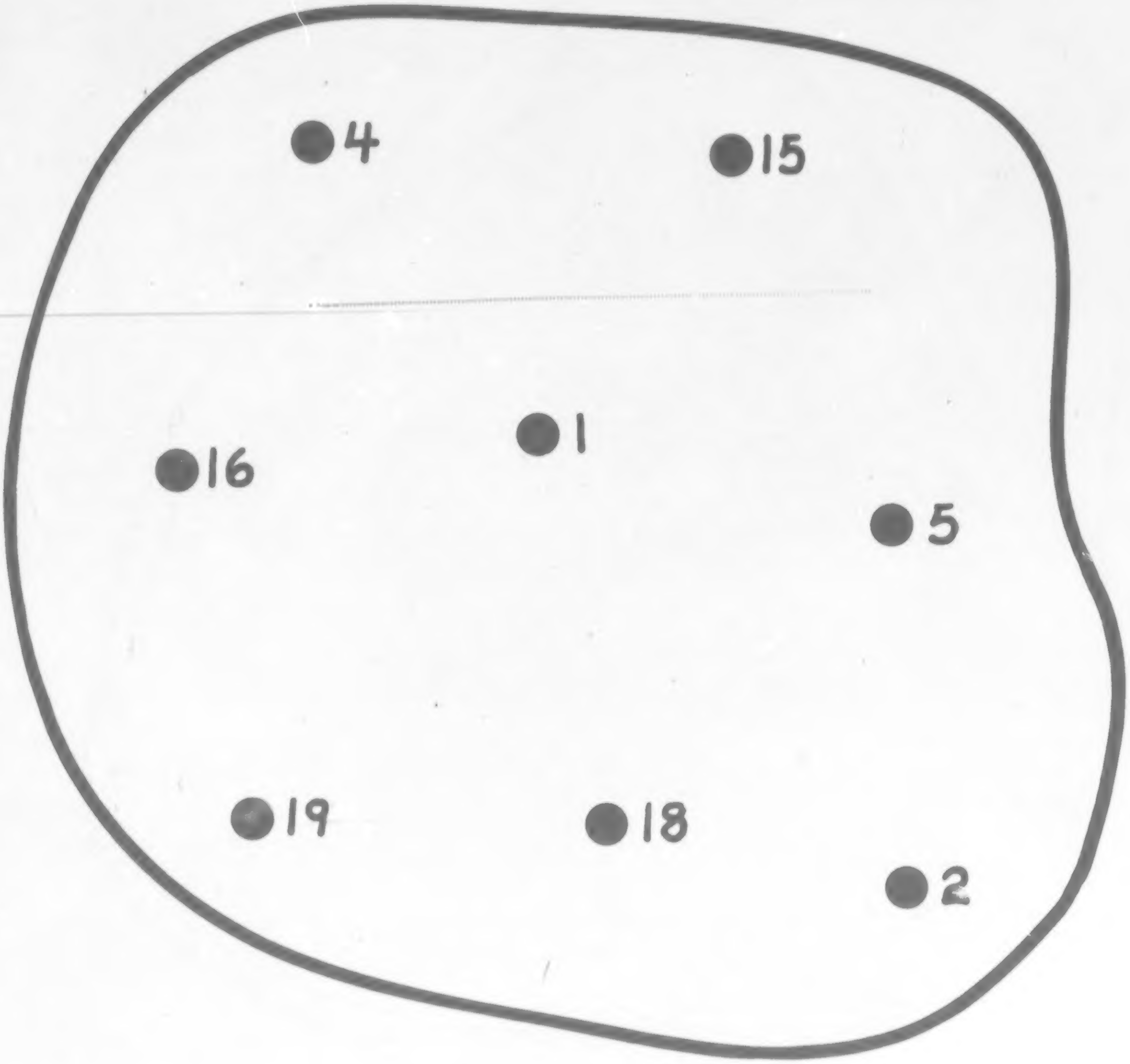


10 + 10 + 10 + 10 + 5 = __________.

Name ____________________

N 25 **

Draw a string around each pair of numbers whose sum is 20.

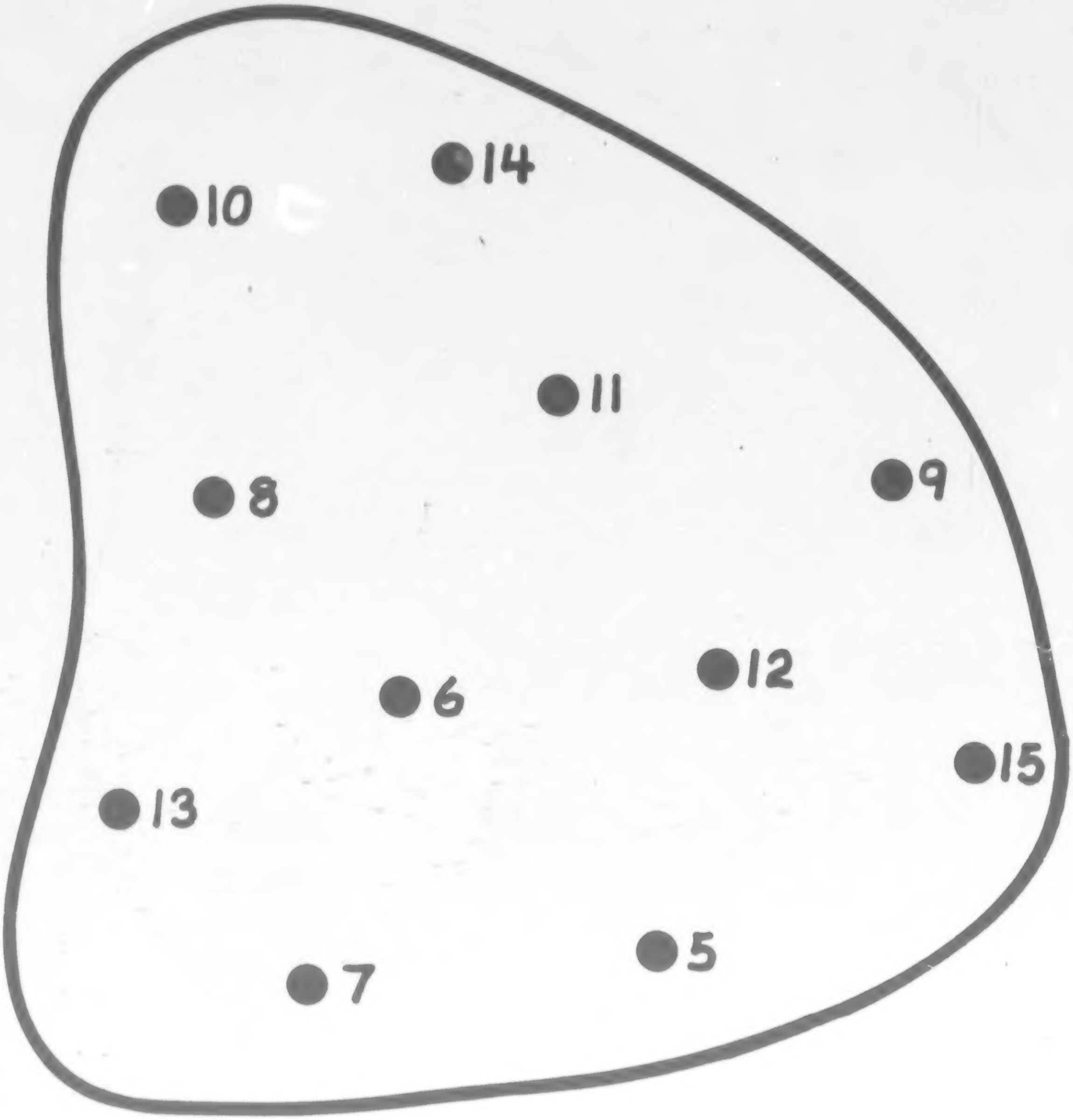

4
15
1
16
5
19
18
2

20 + 20 + 20 + 20 = __________.

Name ______________________________

N 25 ***

Draw a string around each pair of numbers whose sum is 20.

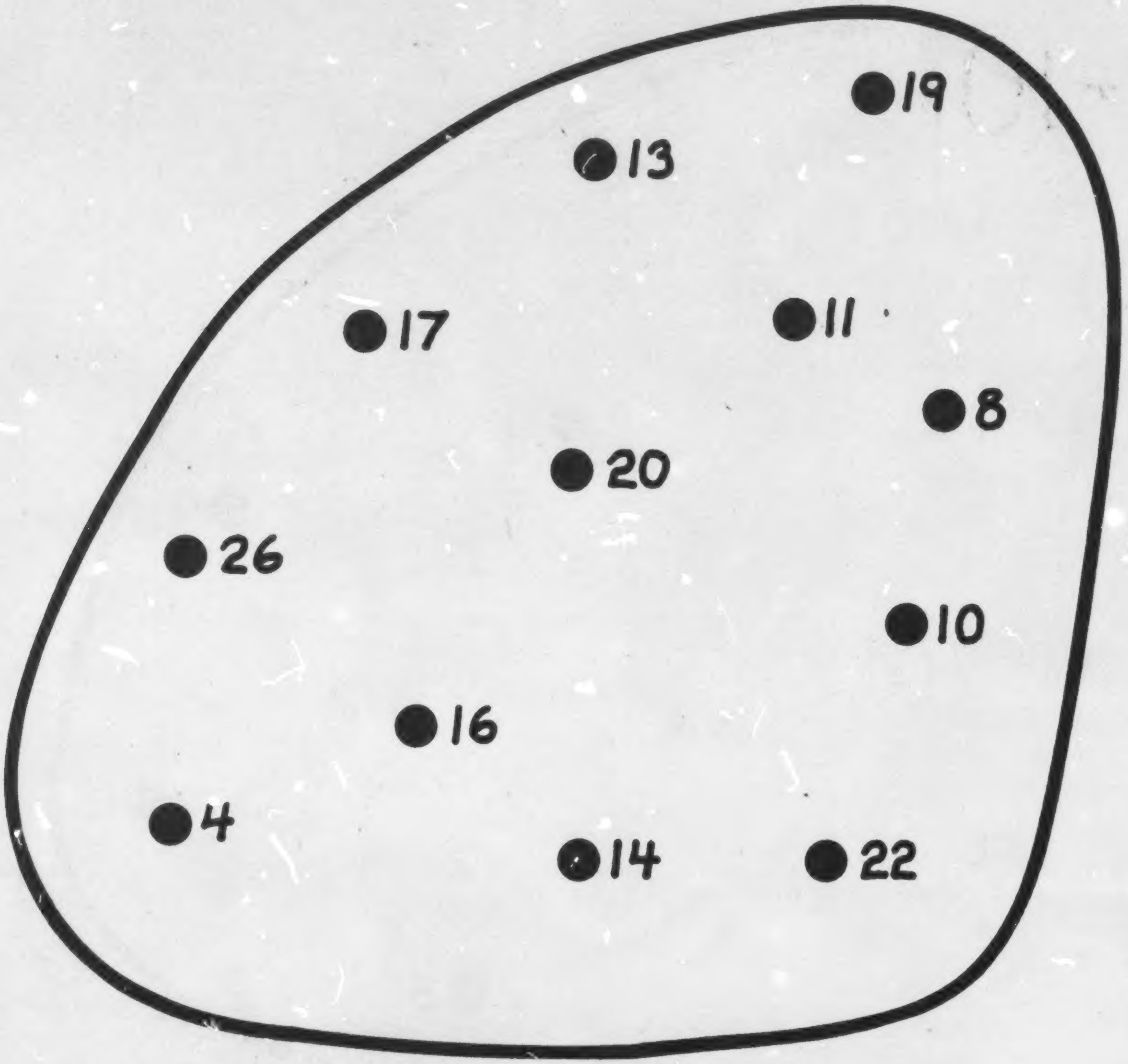

14
10
11
8
9
12
6
15
13
5
7

20 + 20 + 20 + 20 + 20 + ________ = ________.

Name ______________________

N 25 ****

Draw a string around each pair of numbers whose sum is 30.

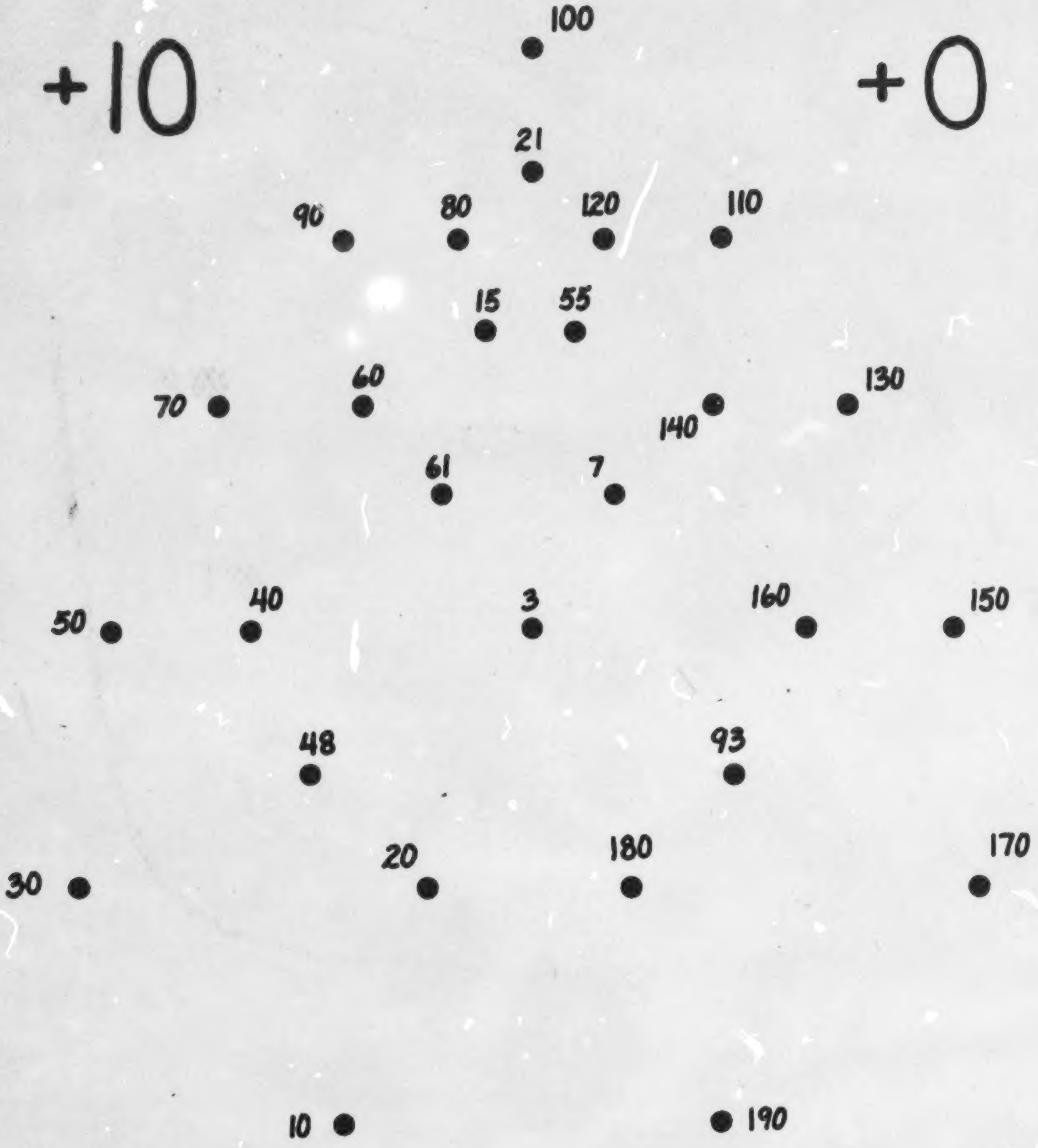

19
13
17
11
8
20
26
10
16
4
14
22

Write a number story for this string picture.

Name ______________________________

N 28 *

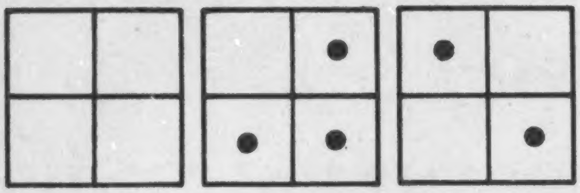
Draw all the green and red arrows.


+10
100
+0
21
90
80
120
110
15
55
60
70
140
130
61
7
50
40
3
160
150
48
93
20
180
170
30
10
190

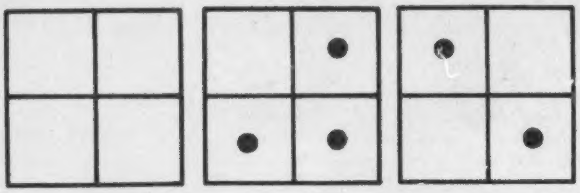
Name ______________________

N 29 *

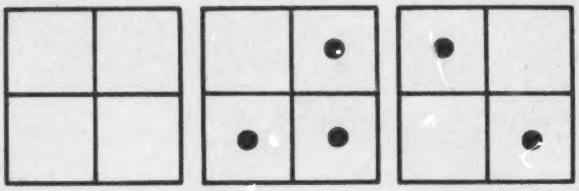
Complete these number stories.



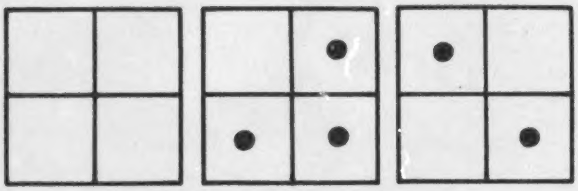
79 - 8 = ____



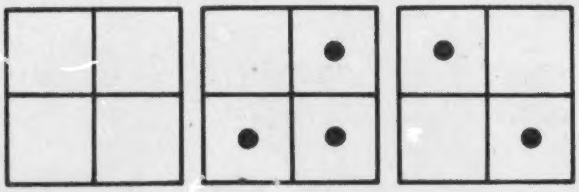
79 - 40 = ____



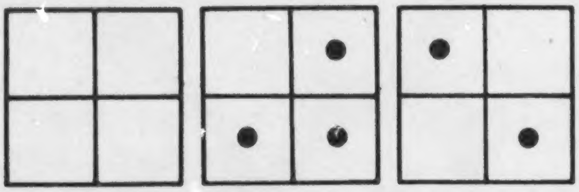
79 - 30 = ____



79 - 9 = ____



79 - 28 = ____

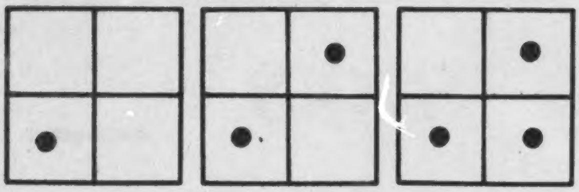


79 - 19 = ____

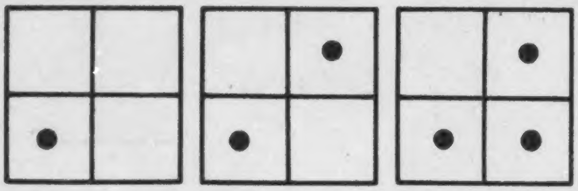
Name ______________________________

N 29 **

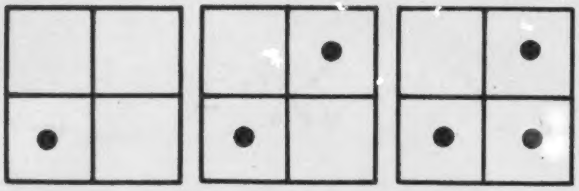
Complete these number stories.



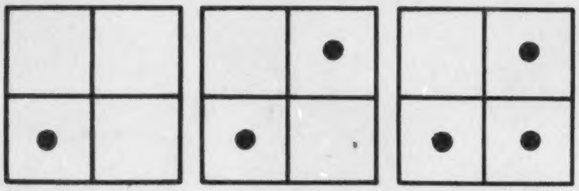
267 - 41 = ____



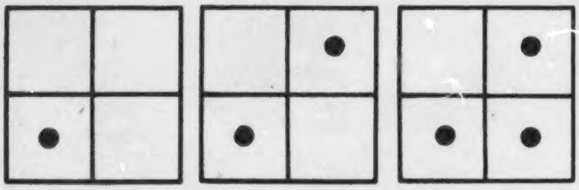
267 - 25 = ____



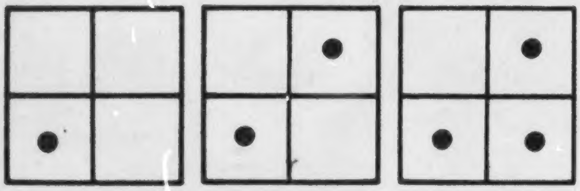
267 - 260 = ____



267 - 63 = ____



267 - 222 = ____

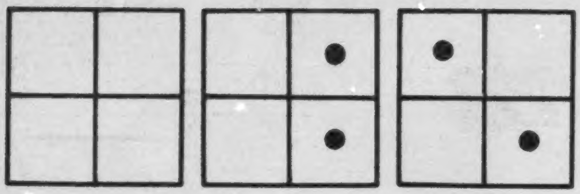


267 - 47 = ____

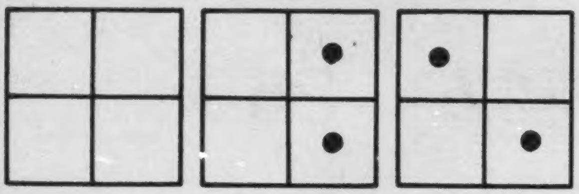
Name ______________________

N 29 ***

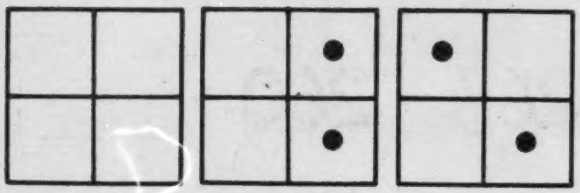
Complete these number stories.



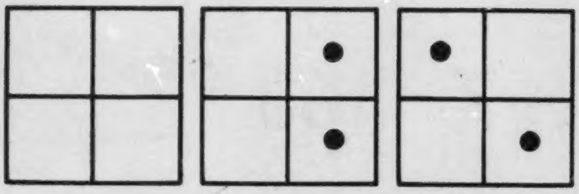
59 - 4 = ____



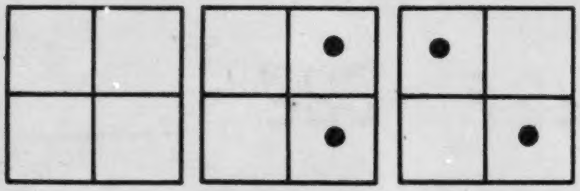
59 - 20 = ____



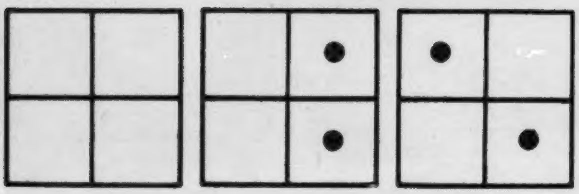
59 - 30 = ____



59 - 5 = ____



59 - 21 = ____

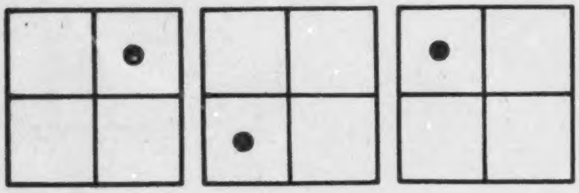


59 - 14 = ____

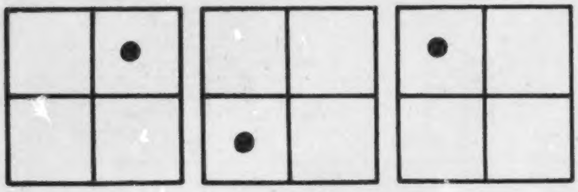
Name ______________________

N 29 *****

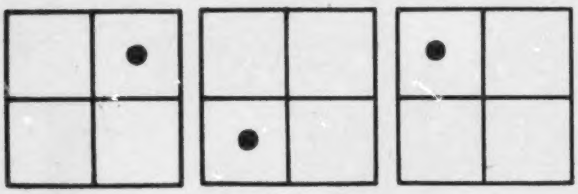
Complete these number stories.



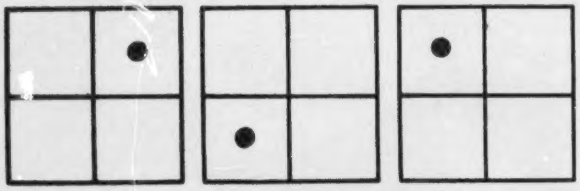
428 - 10 = ____



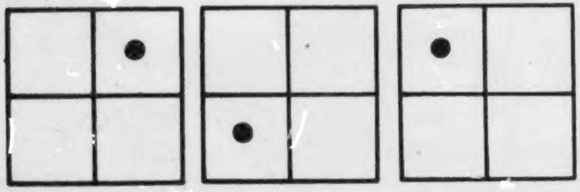
428 - 4 = ____



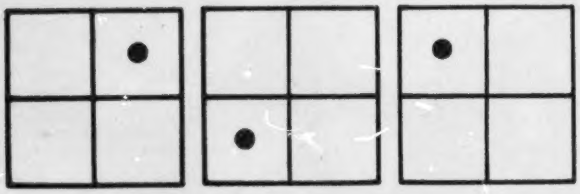
428 - 200 = ____



428 - 14 = ____



428 - 210 = ____



428 - 214 = ____

Name ______________________

N 30 *

Build a red-blue road from 6 to 13.

+2

−1

● 13

6 ●

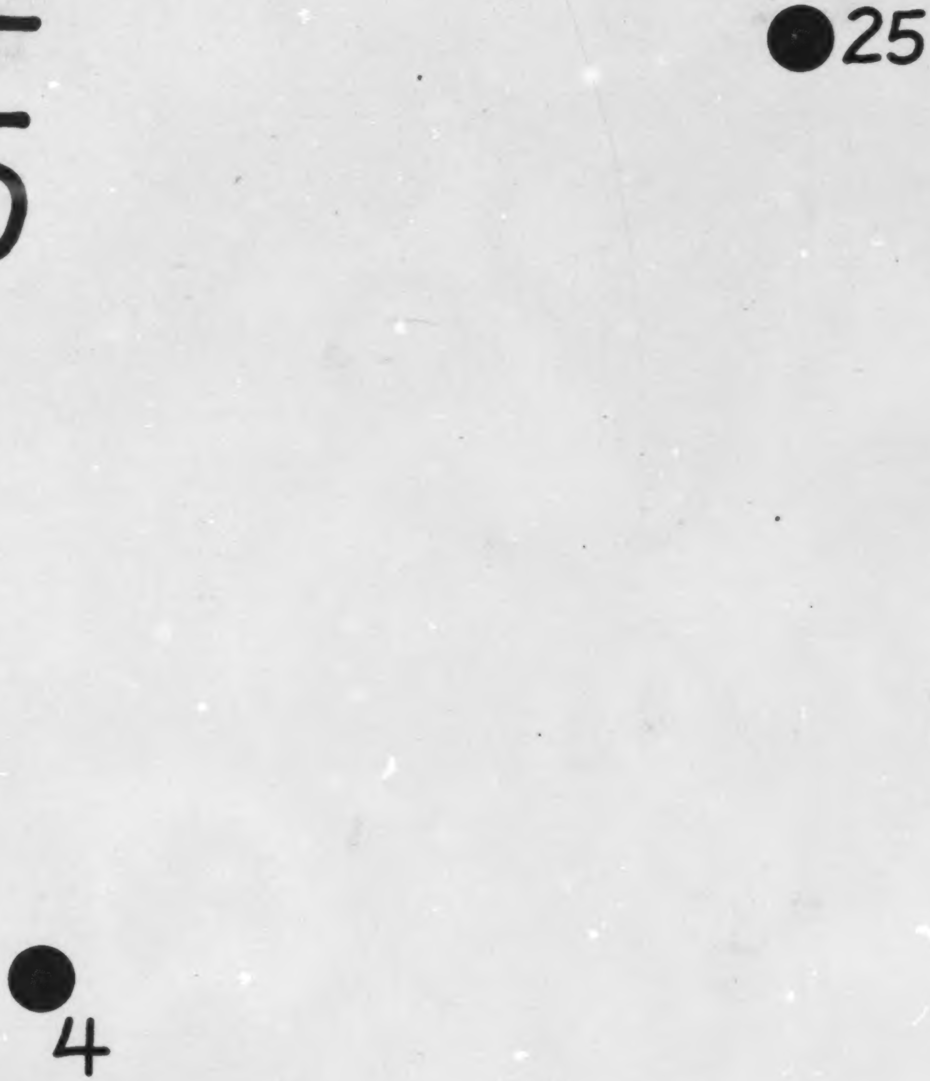
Name ______________________

N 30 **

Build a red-blue road from 4 to 25.

-2

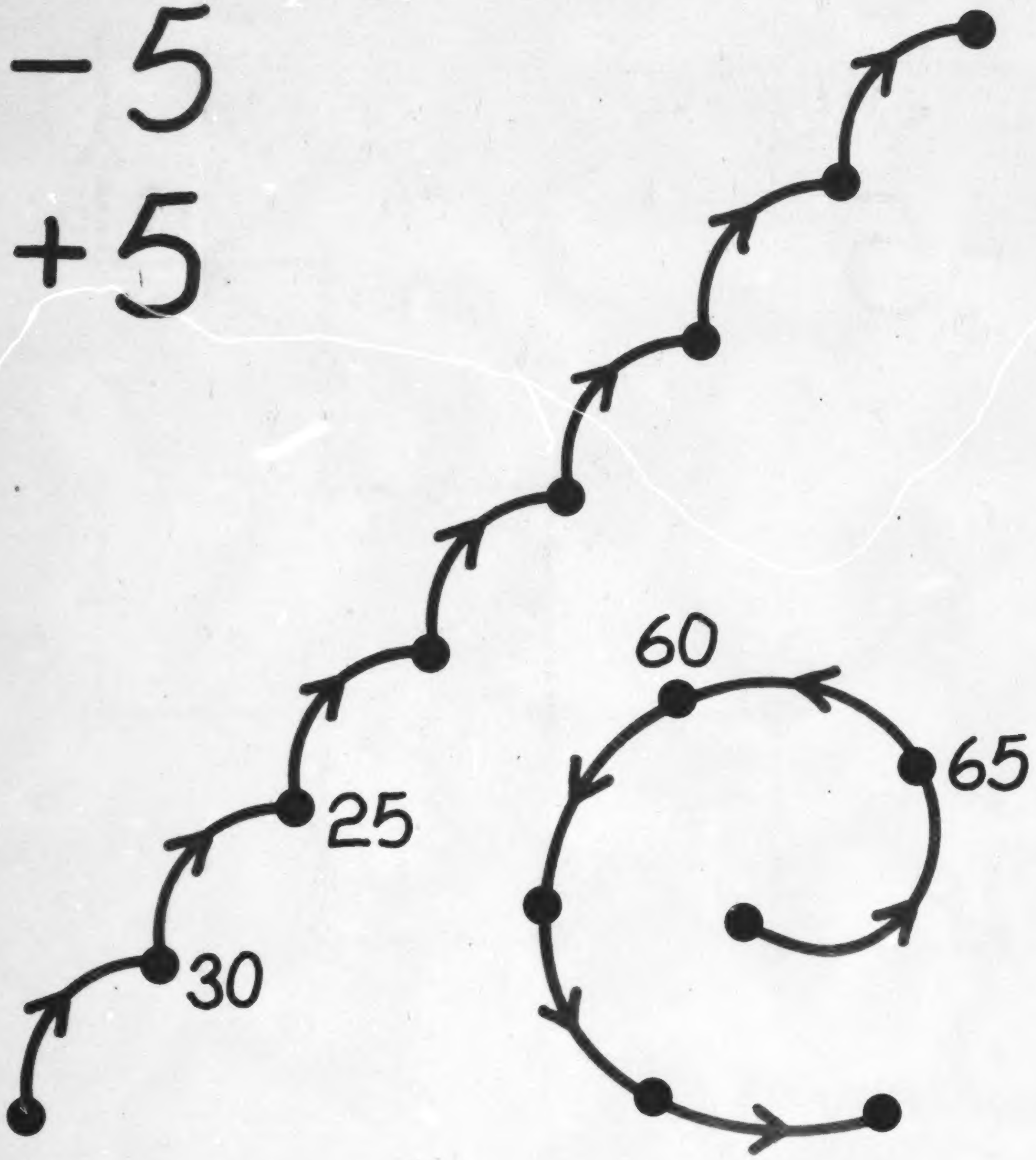
+5



Name ______________________

N 32 *

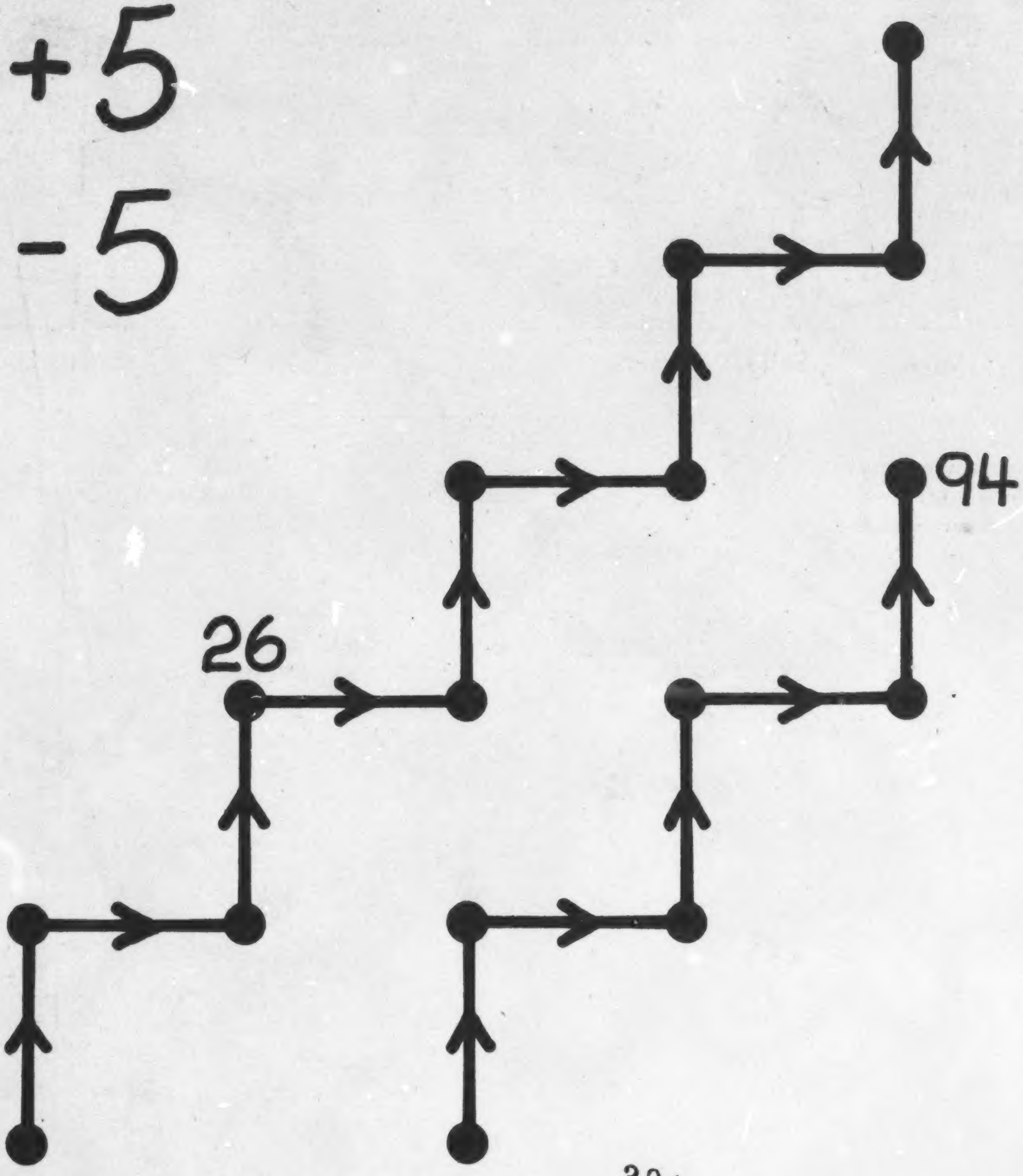
Label the dots. Draw all the missing +5 arrows.


−5
+5
60
65
25
30

Name

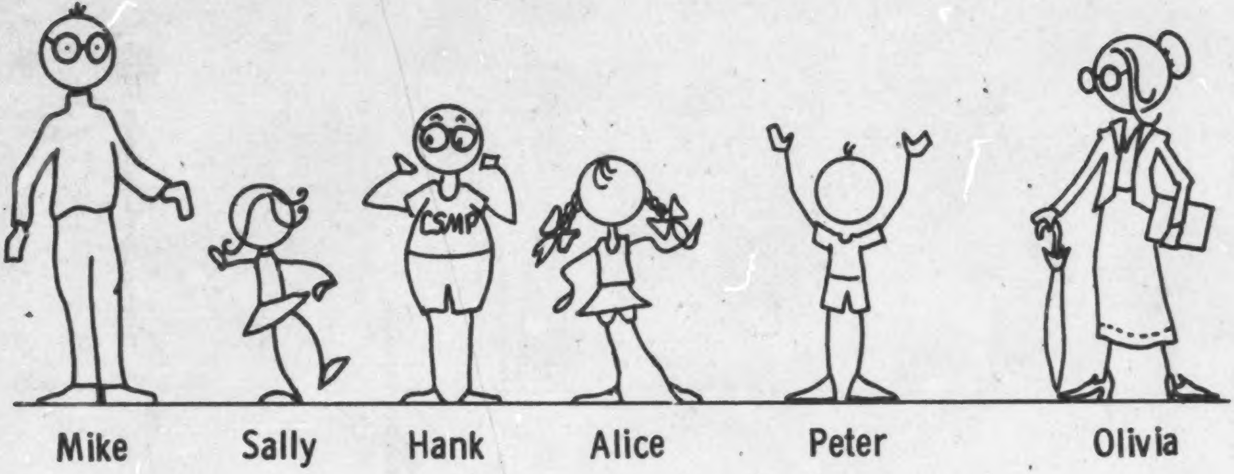
N 32 **

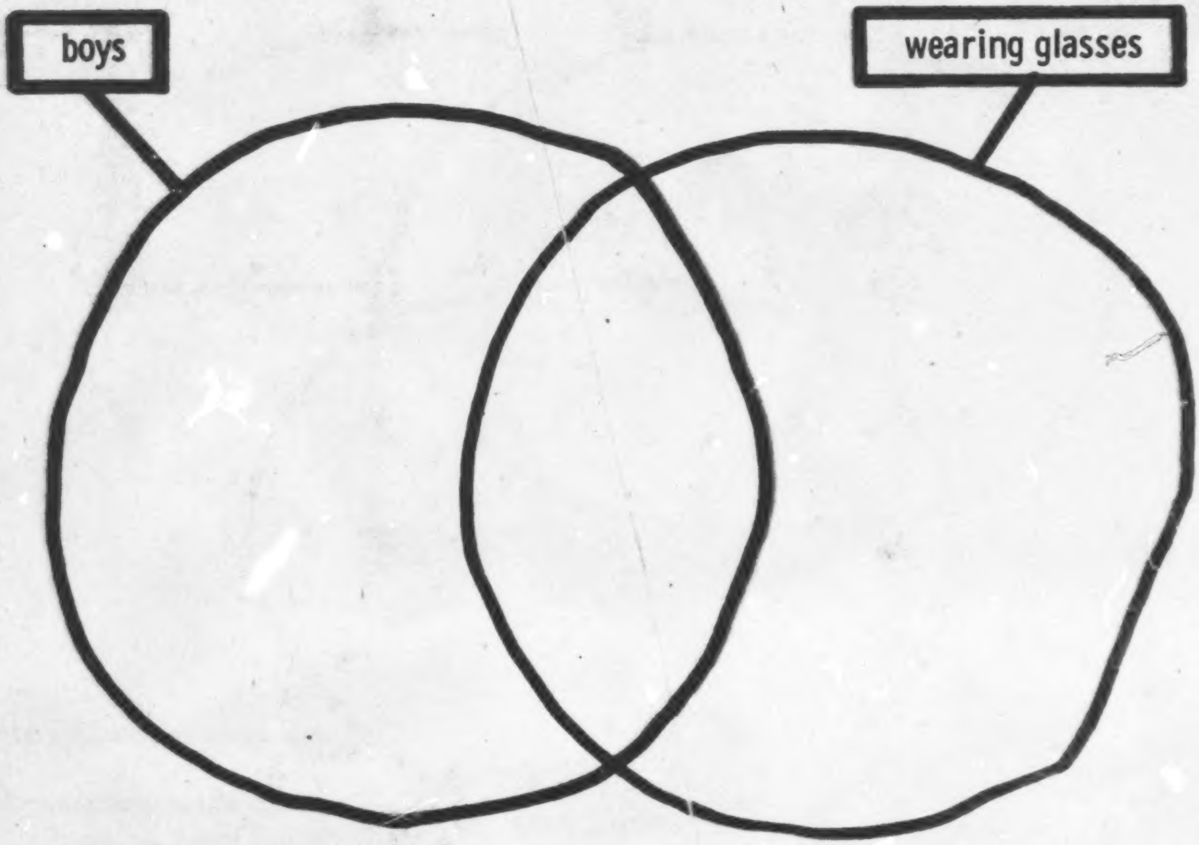
Label the dots. Draw all the missing -5 arrows.



Name ______________________ L 3 *

Draw and label a dot for each person.

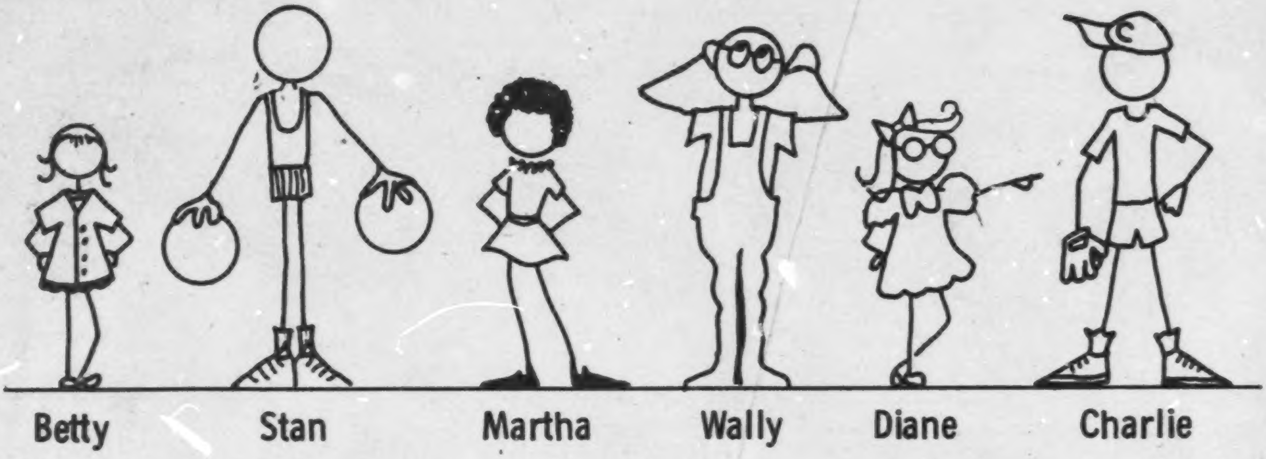

CSMP
Mike
Sally
Hank
Alice
Peter
Olivia

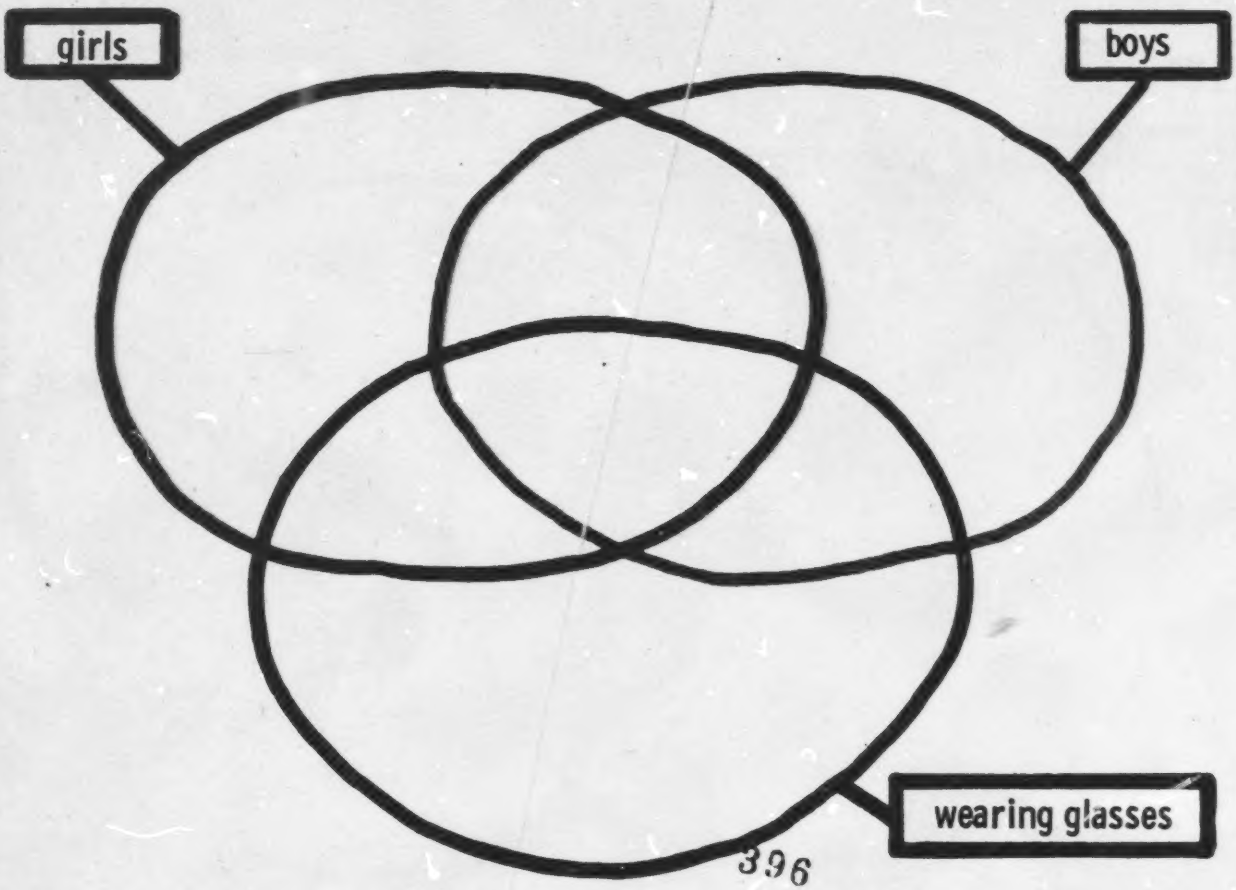

boys
wearing glasses

Name ______________________________

L 3 **

Draw and label a dot for each person.

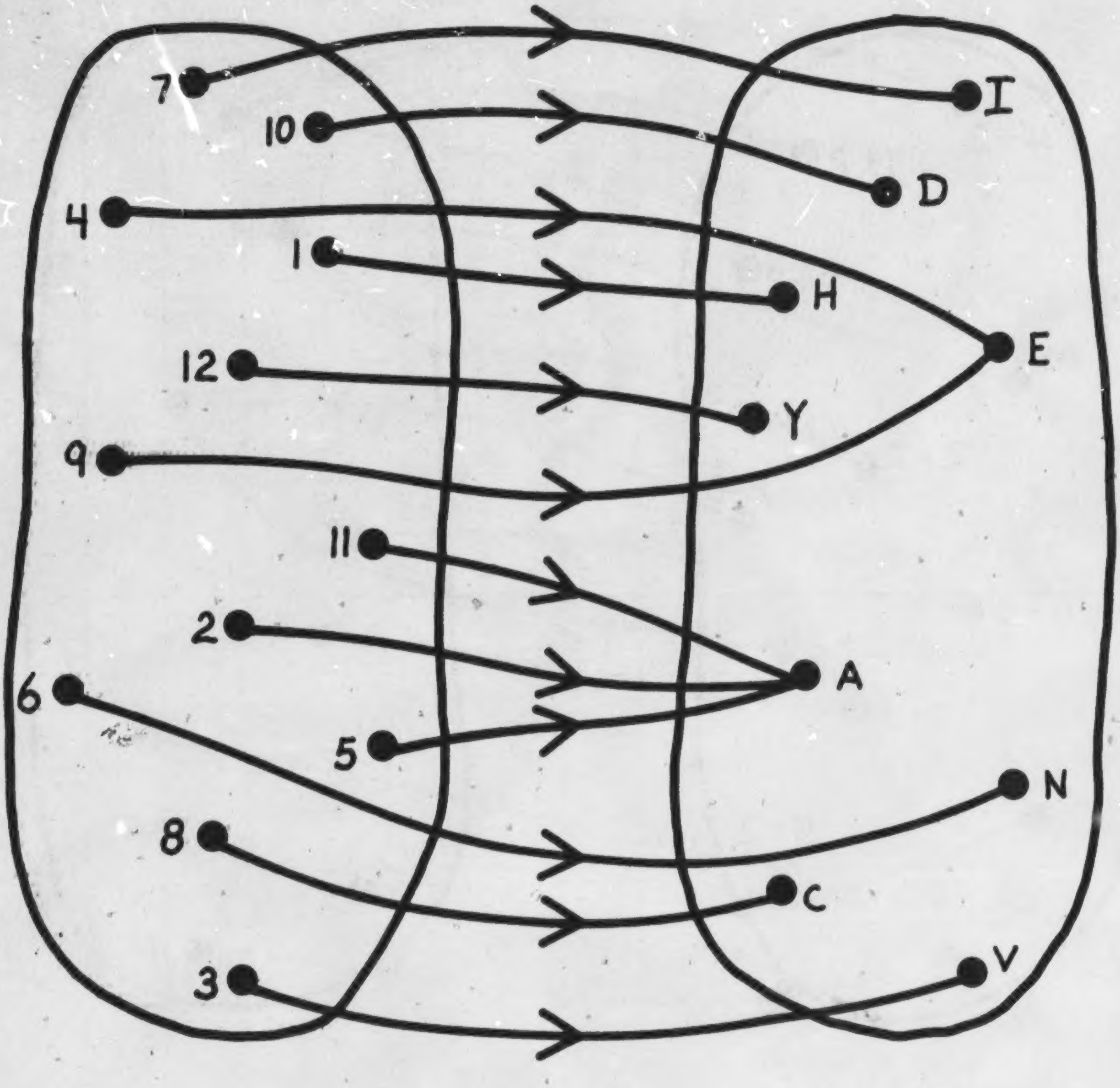

Betty
Stan
Martha
Wally
Diane
Charlie


girls
boys
wearing glasses

Name ______________________________

L8 *

Decode this message.



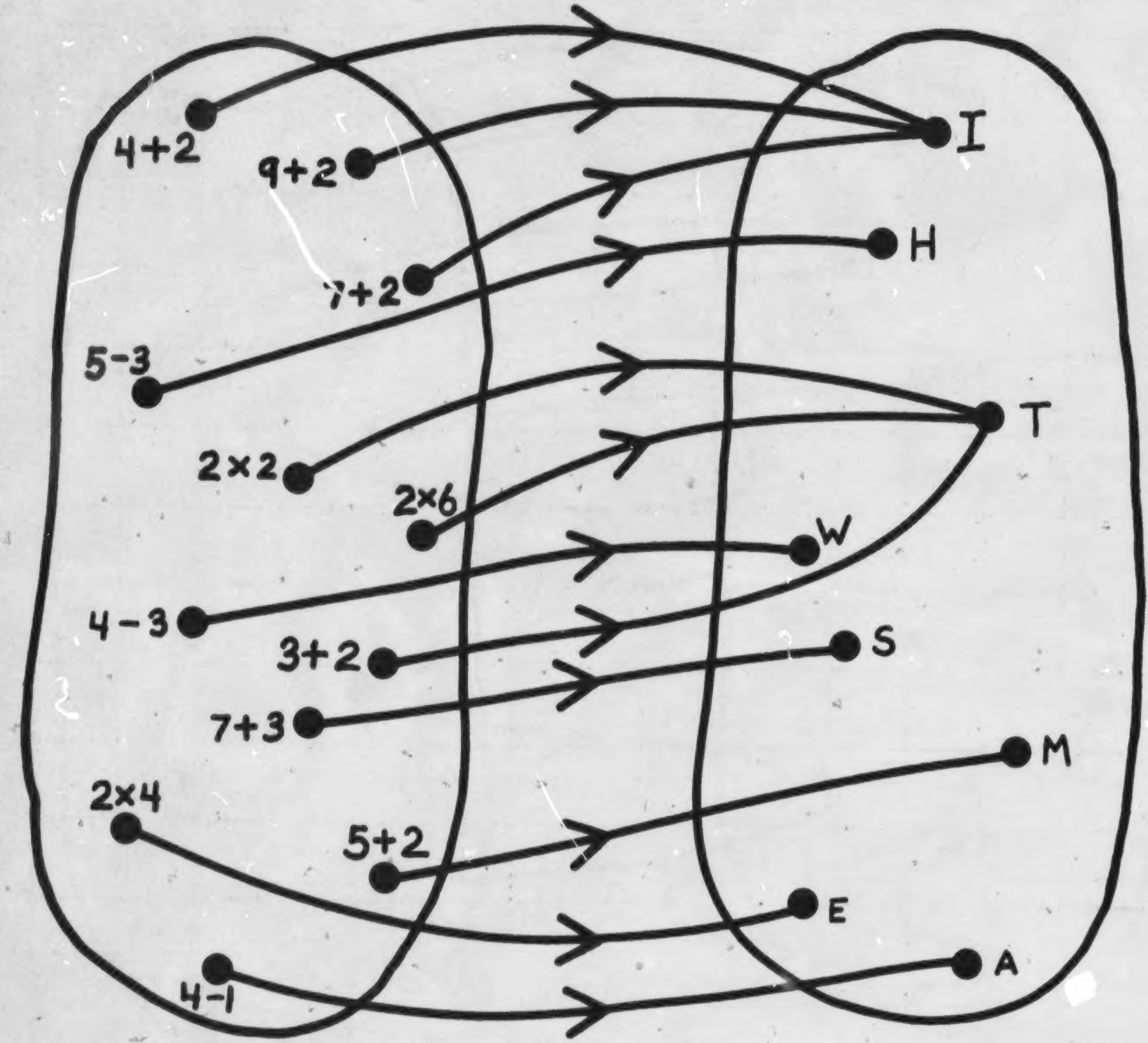
7
10
4
1
12
9
11
2
6
5
8
3
I
D
H
E
Y
A
N
C
V

_ _ _ _   _   _ _ _ _ _   _ _ _.

Name ____________________

L8 **

Decode this message.


4+2
9+2
7+2
5-3
2×2
2×6
4-3
3+2
7+3
2×4
5+2
4-1
I
H
T
W
S
M
E
A

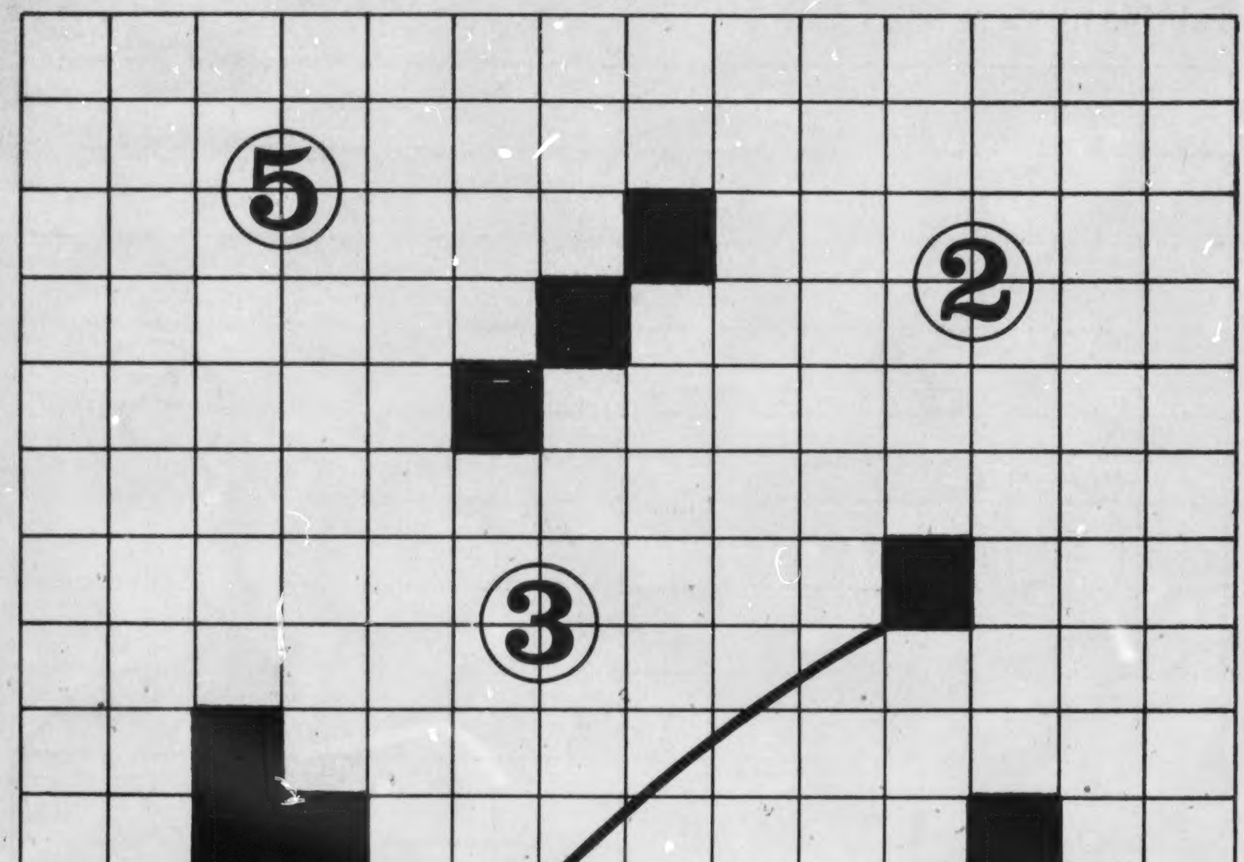
_ _ _ _ _ _ _ _ _ _ _ _ _ _?

Name ______________________

G1 *

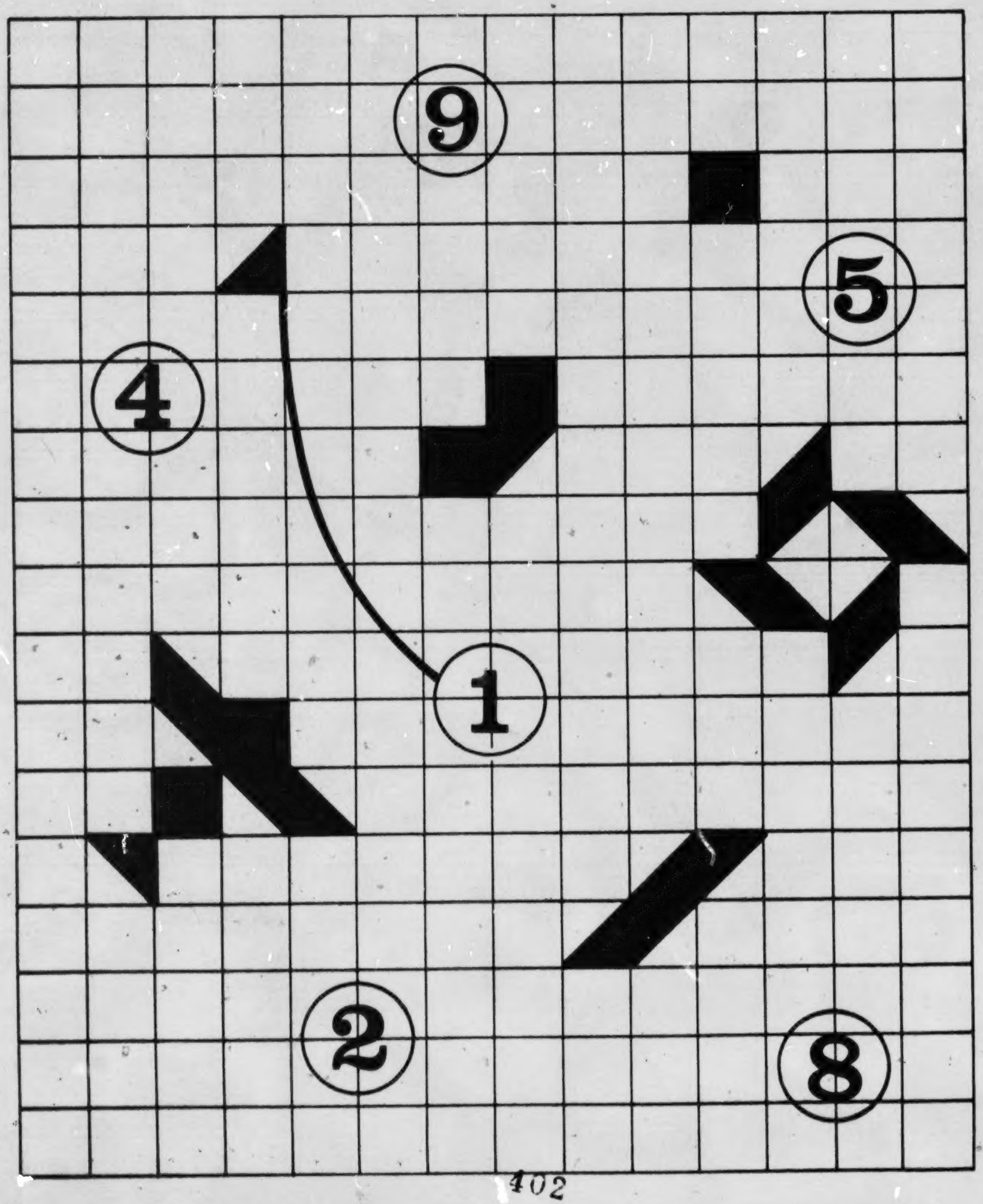
Name ______________________ G 1 **

Pair the numbers with the shapes.



(Turn the page.)

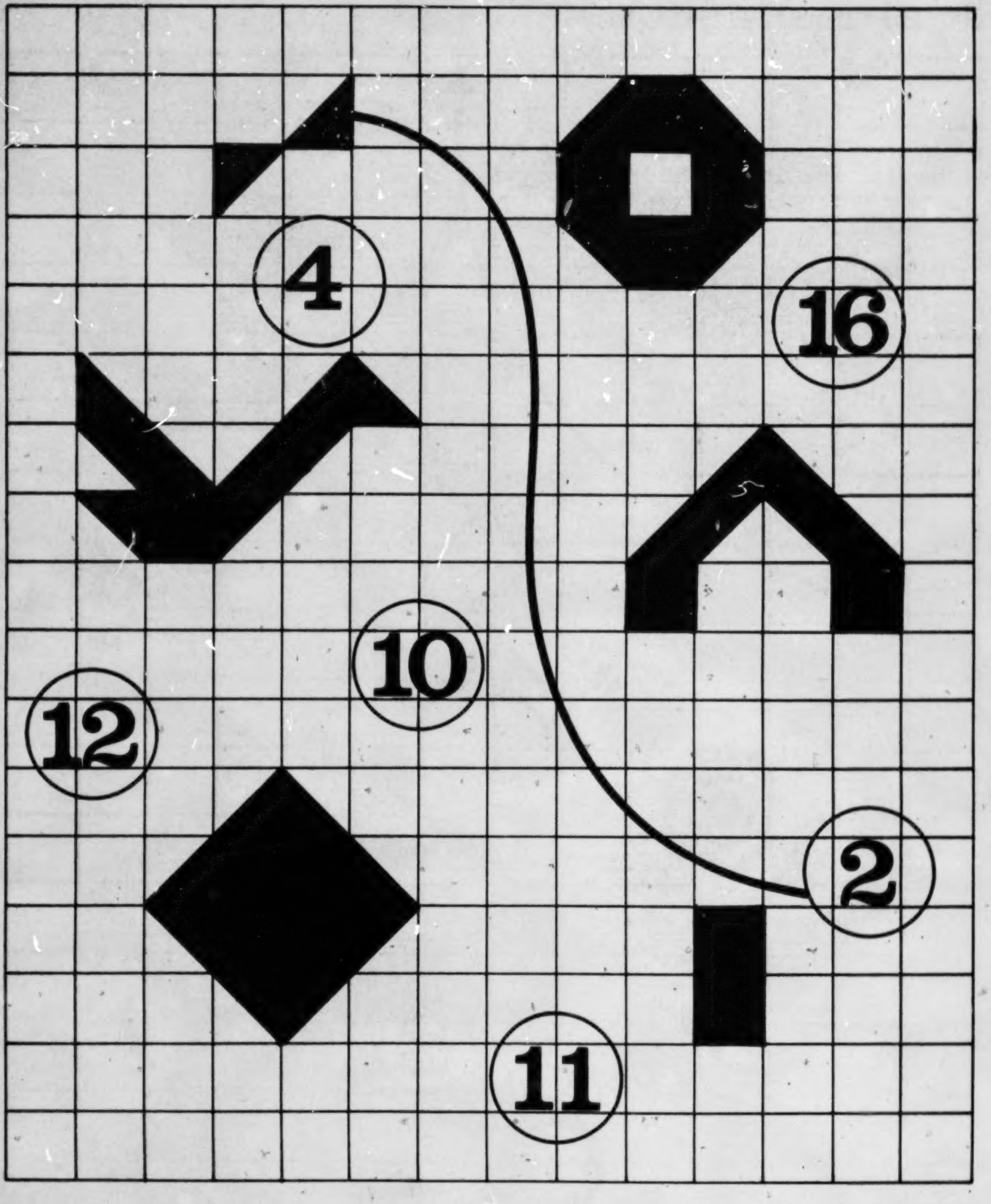
Pair the numbers with the shapes.



Name ________________________________

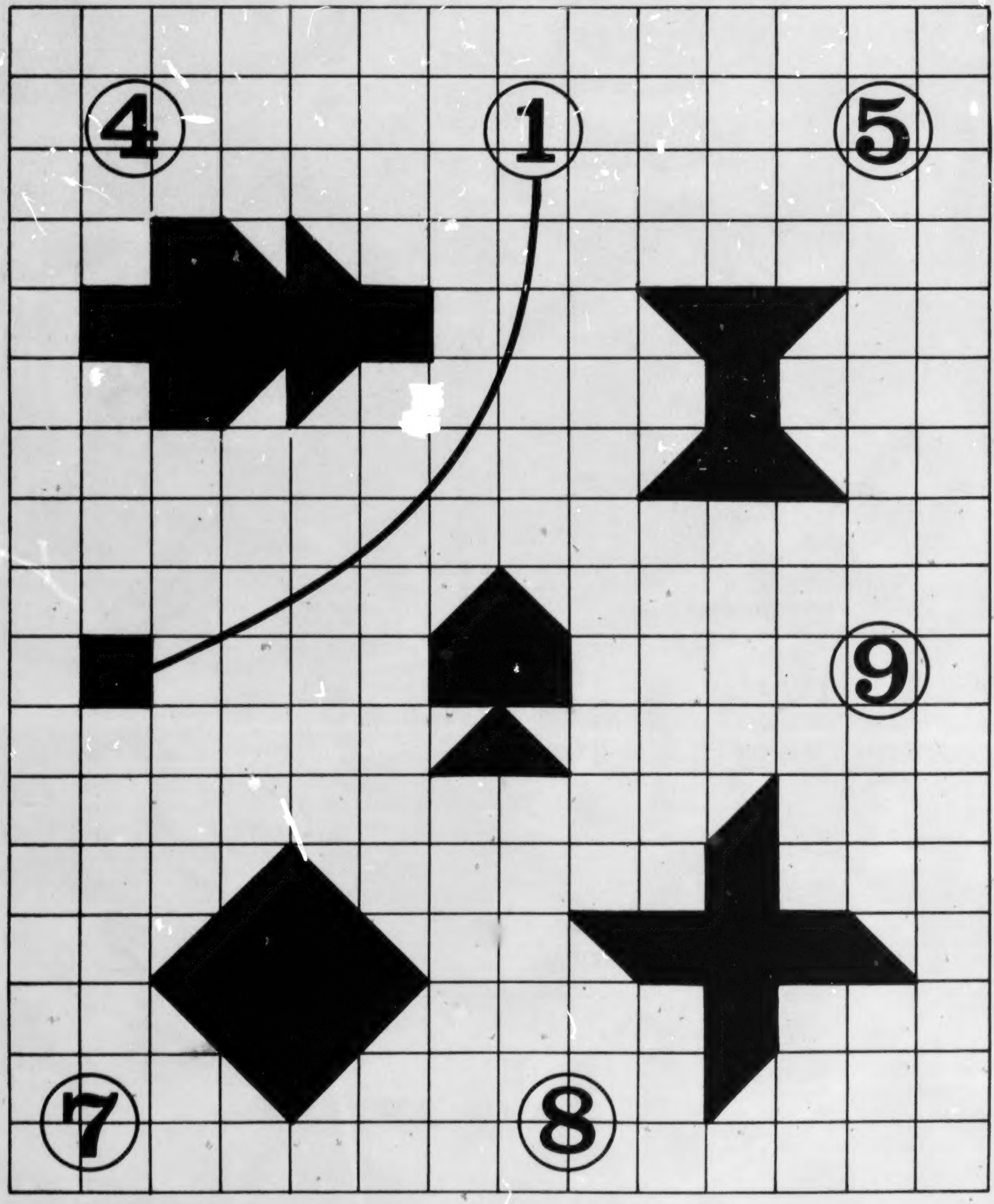
G1 ***

Pair the numbers with the shapes.


4
16
10
12
2
11

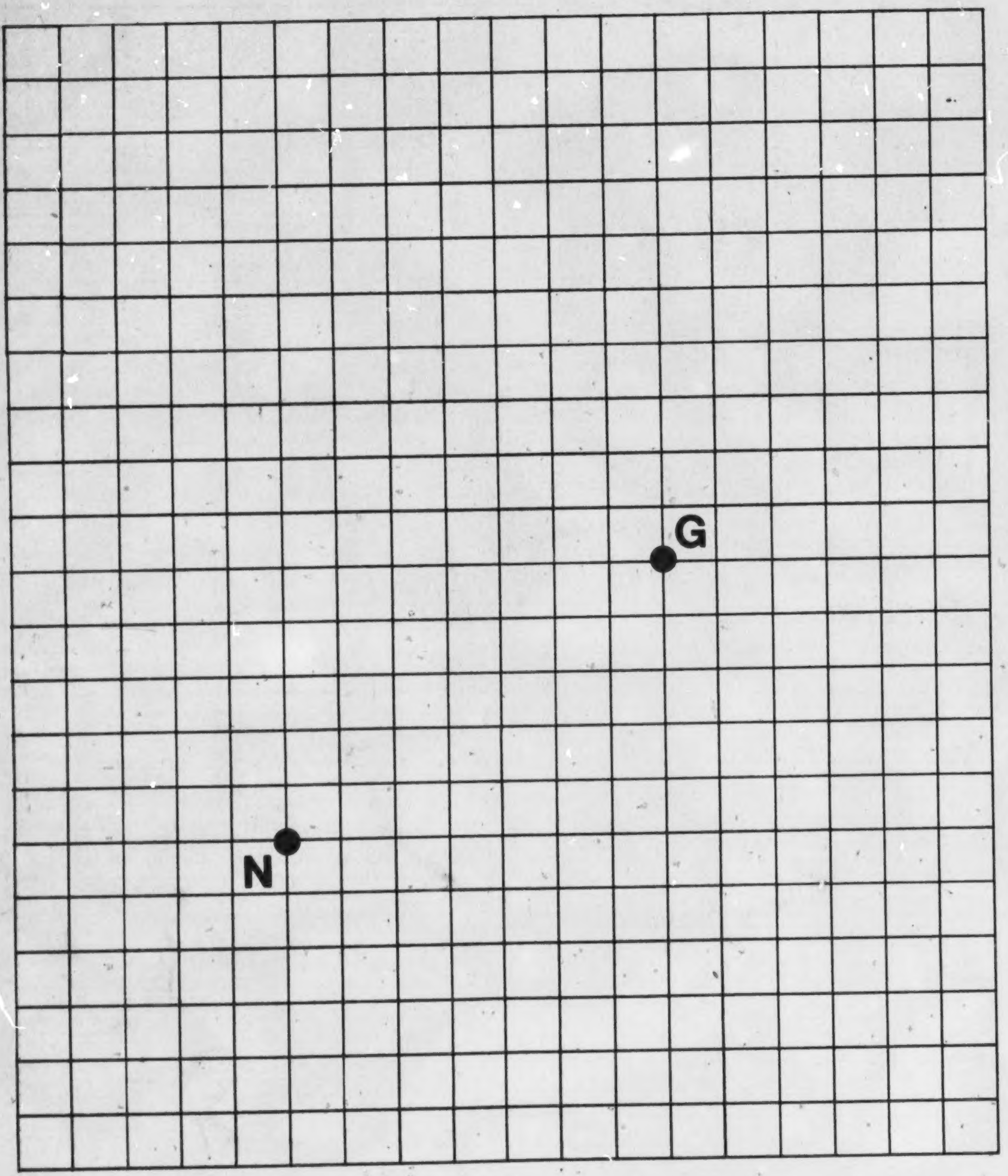
(Turn the page.)

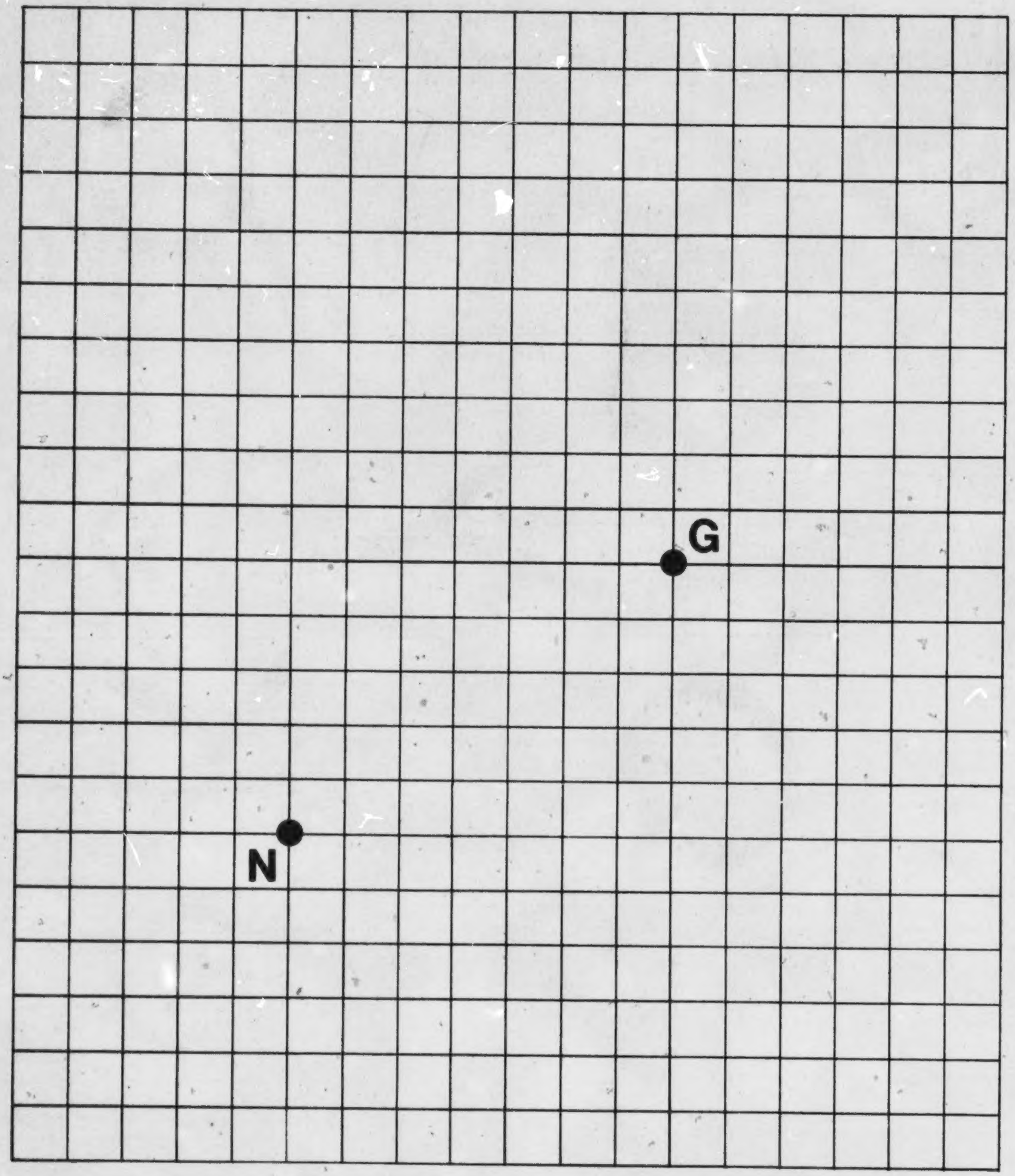
Pair the numbers with the shapes.



Name ______________________

G 2 *

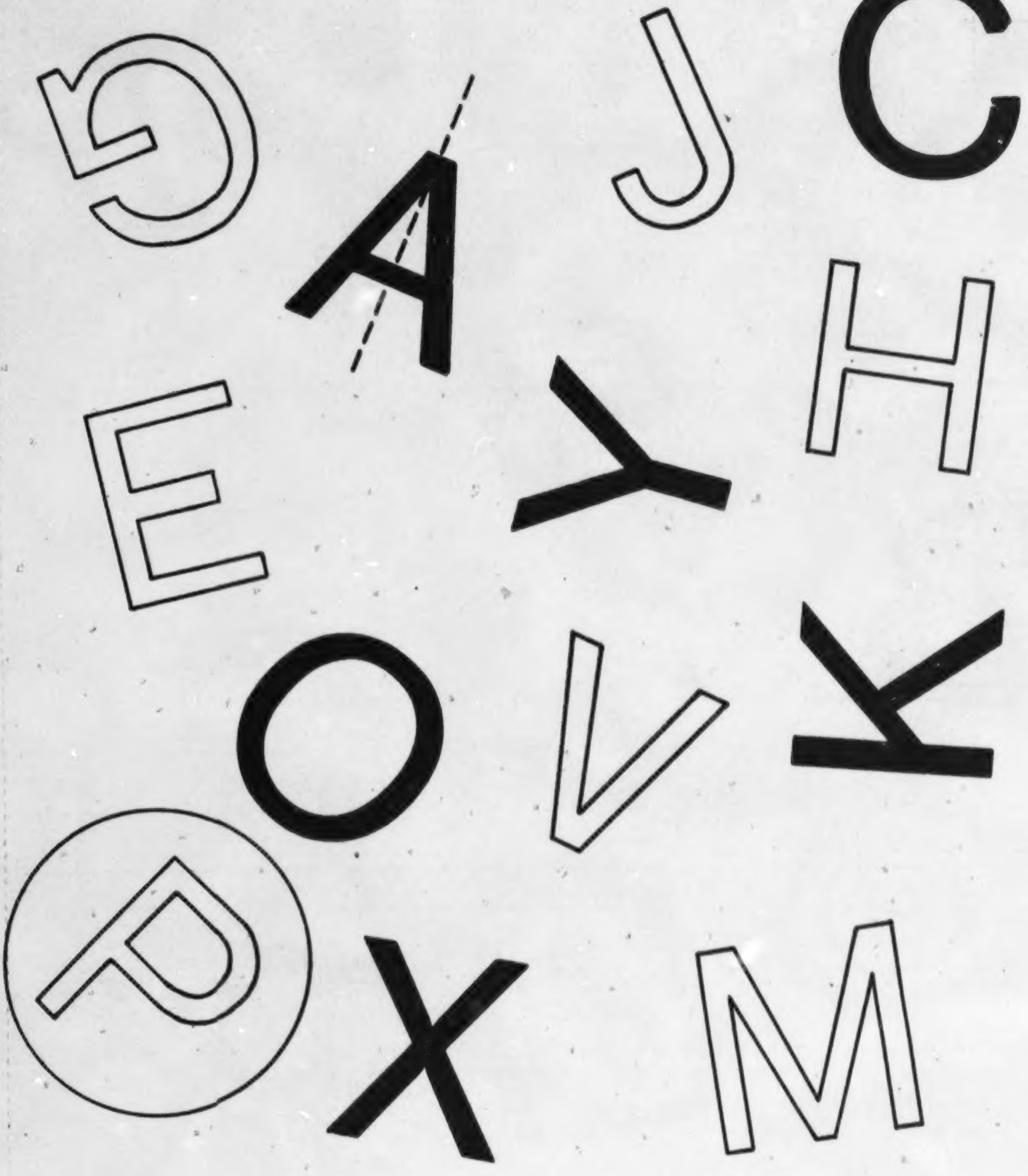



G
N

Name ______________________

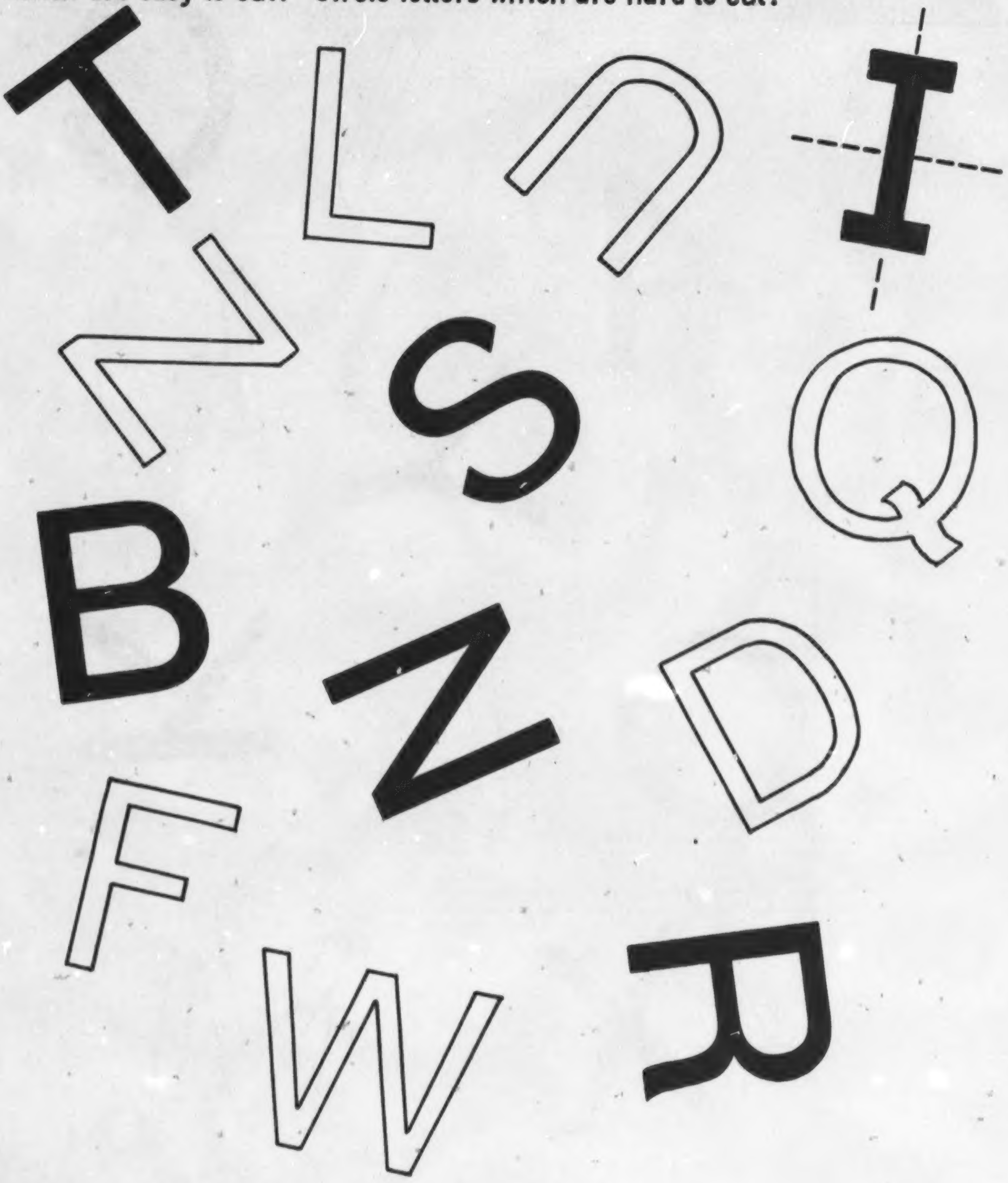
G 3 *

Try to cut all these letters. Show one cutting line for letters which are easy to cut. Circle letters which are hard to cut.


G
A
J
C
H
E
Y
O
V
K
P
X
M

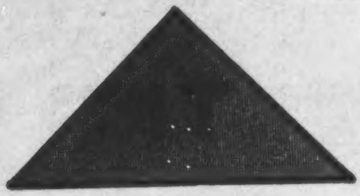
Name ________________________________ **G3 ****

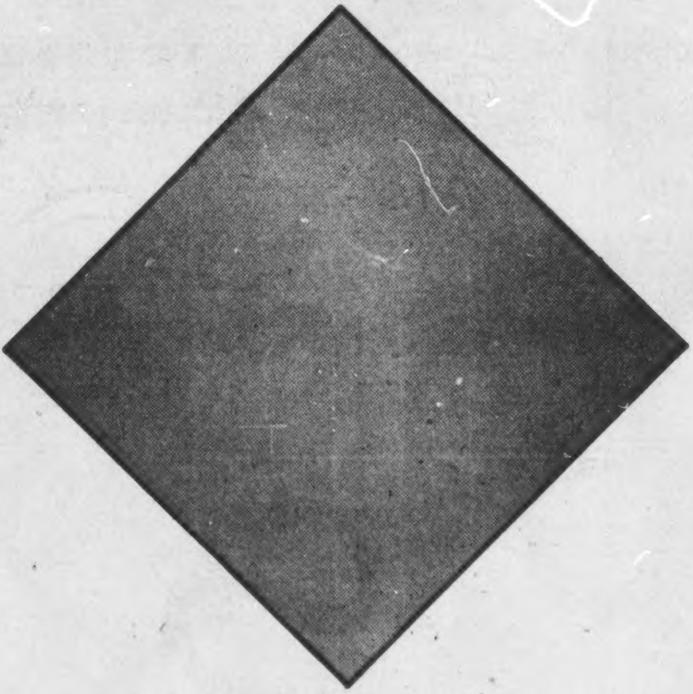
Try to cut these letters. Show one or more than one cutting line for letters which are easy to cut. Circle letters which are hard to cut.

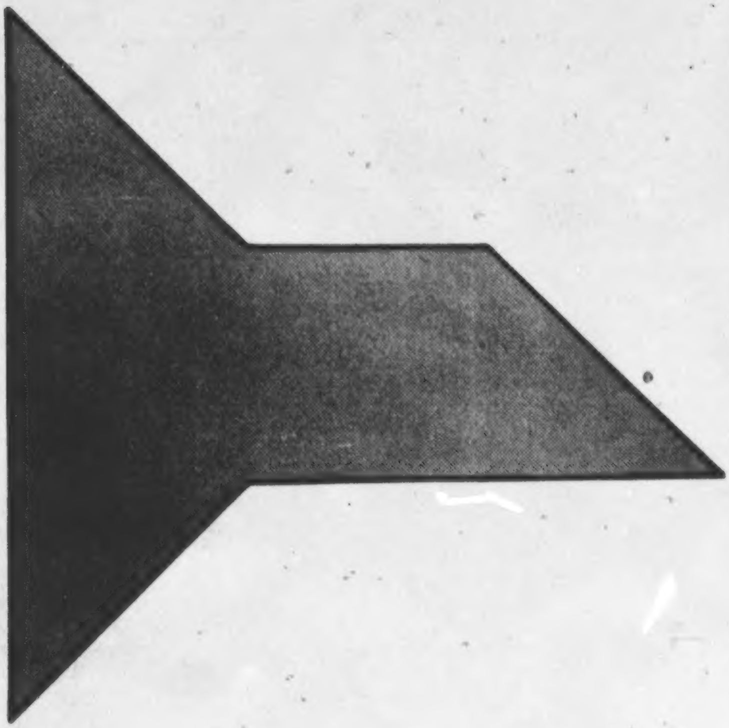

T
L
U
I
Z
S
Q
B
N
D
F
W
R

Name ________________________

G5 *







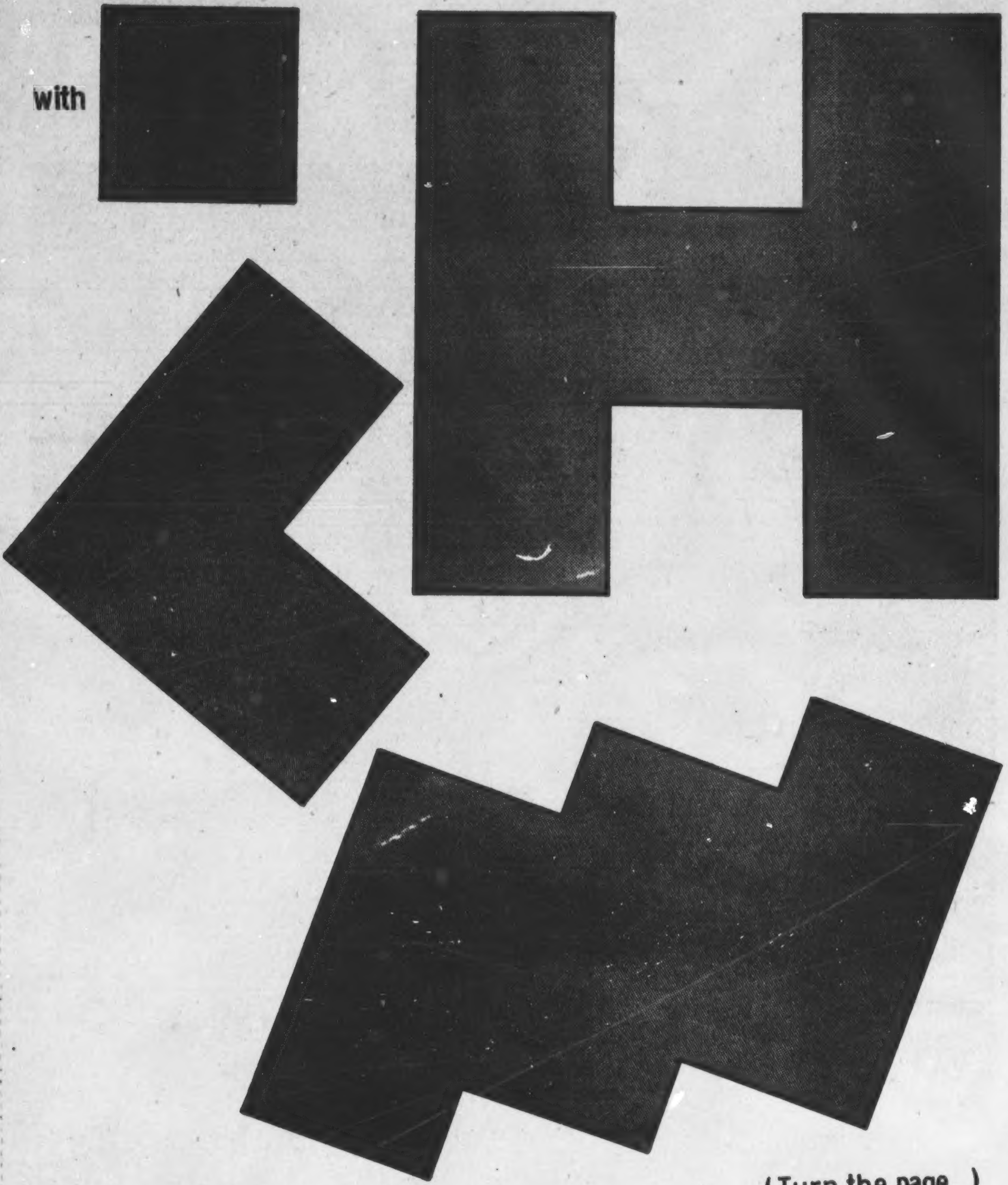
15

Name ______________________

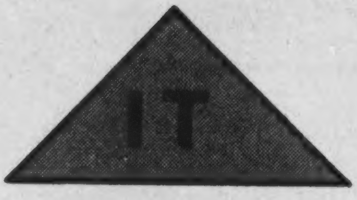
G 5 **

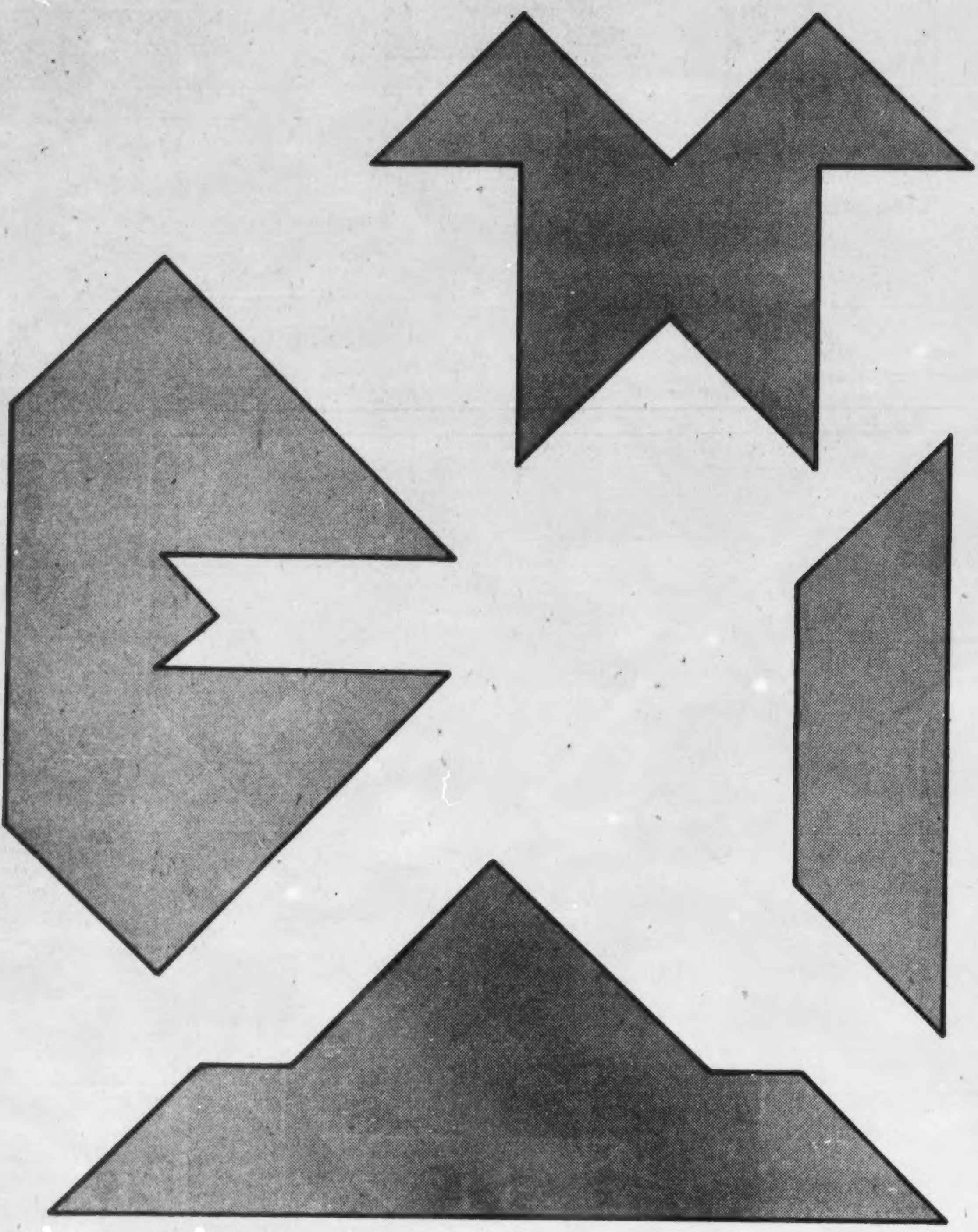
Find the area of each shape

with



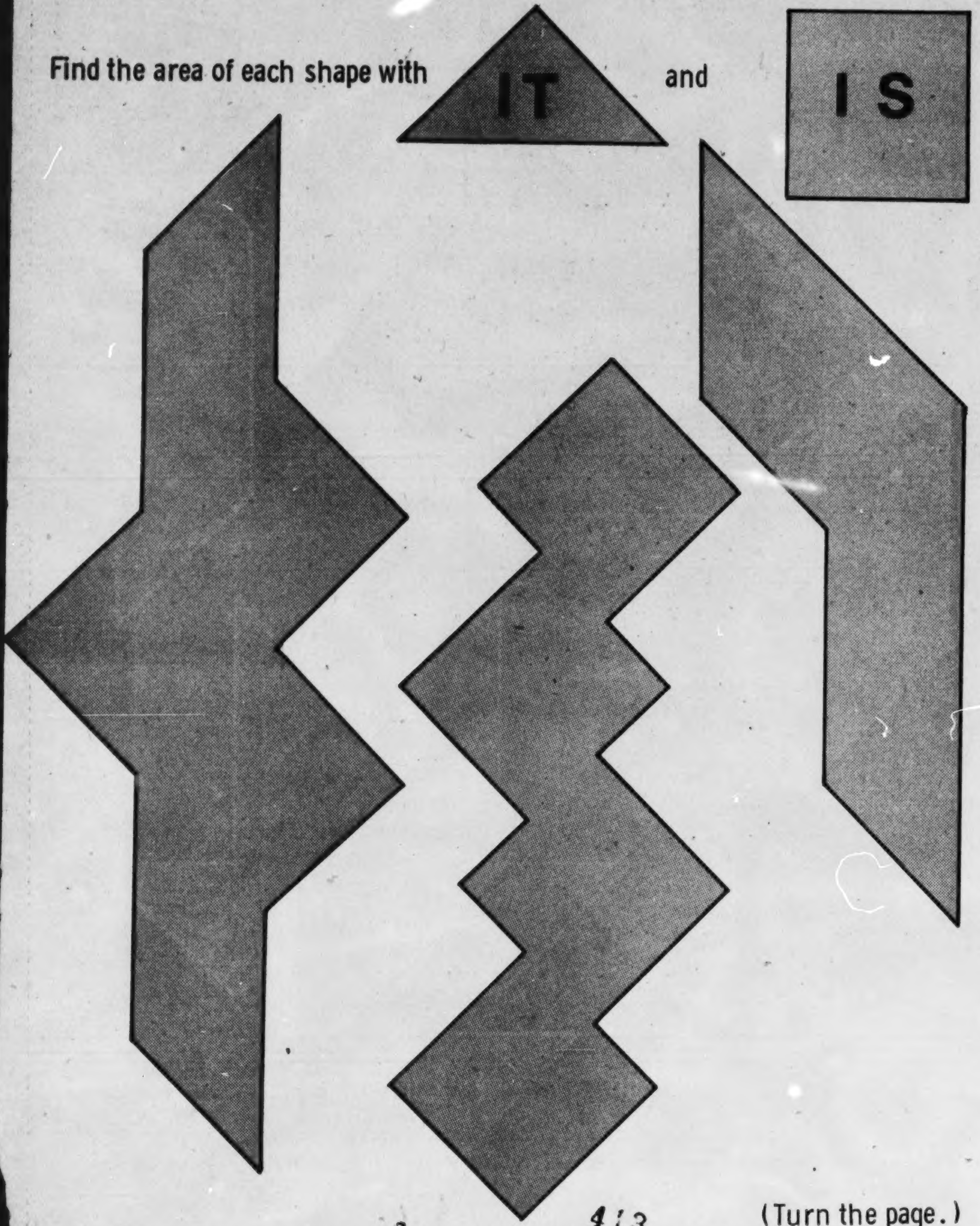
(Turn the page.)

Find the area of each shape with 

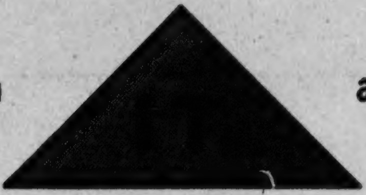
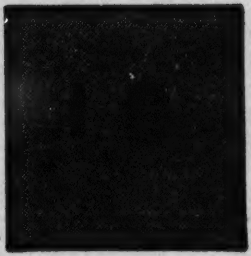


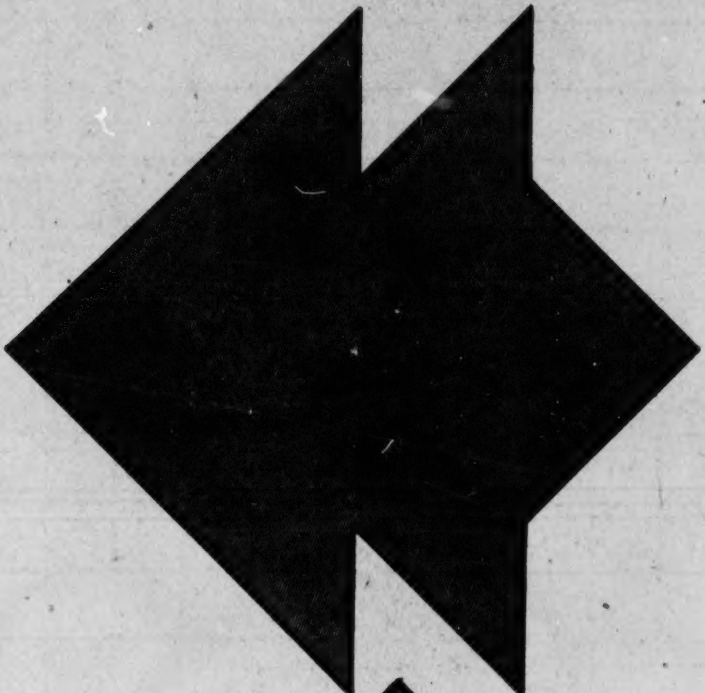
Name ________________________ **G5 *****

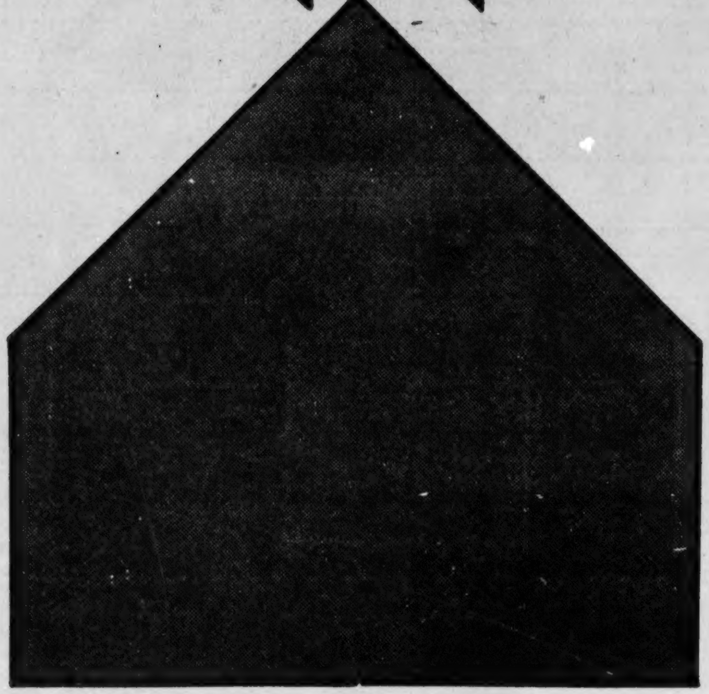
Find the area of each shape with IT and IS



(Turn the page.)

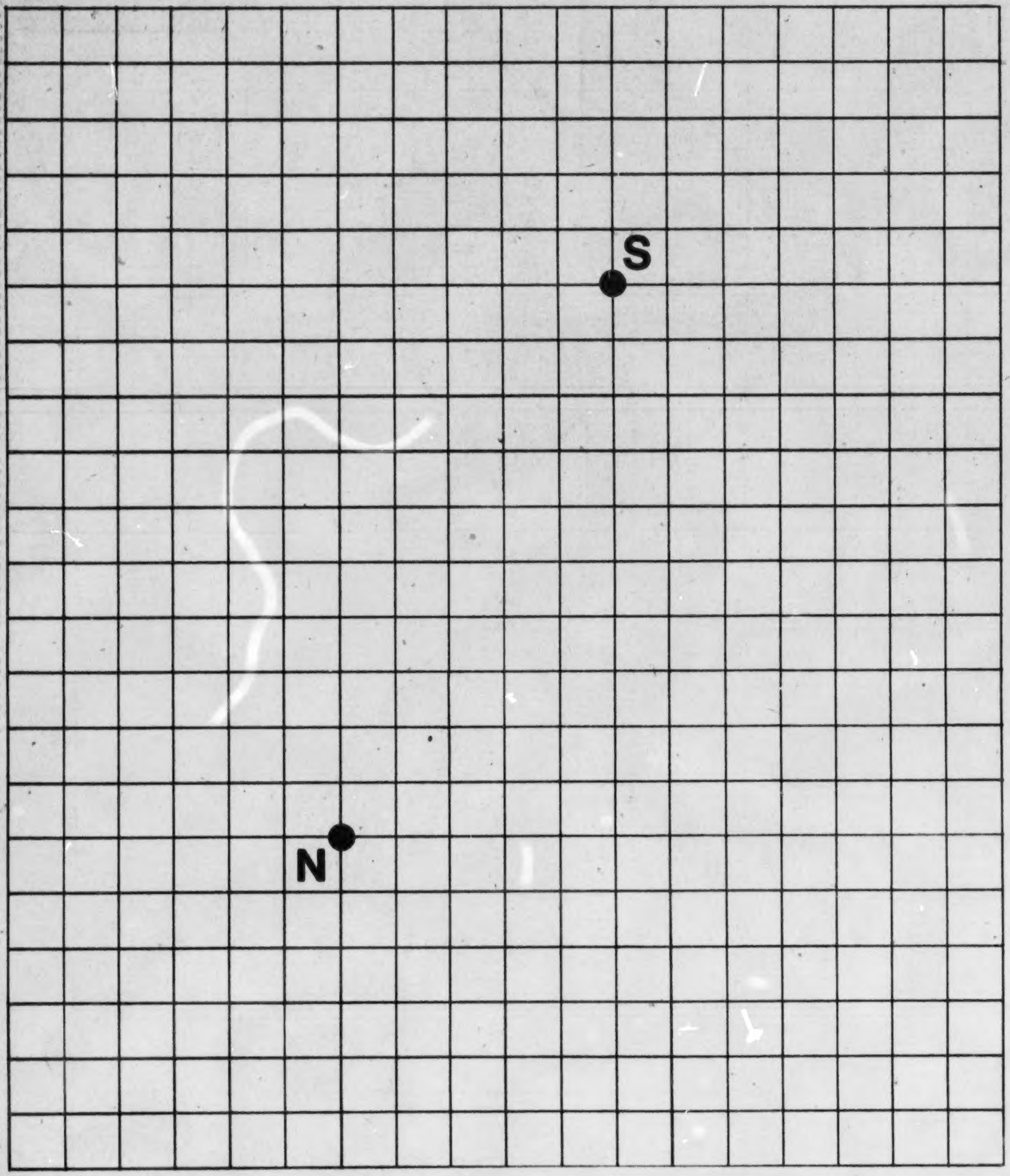
Find the area of each shape with  and 





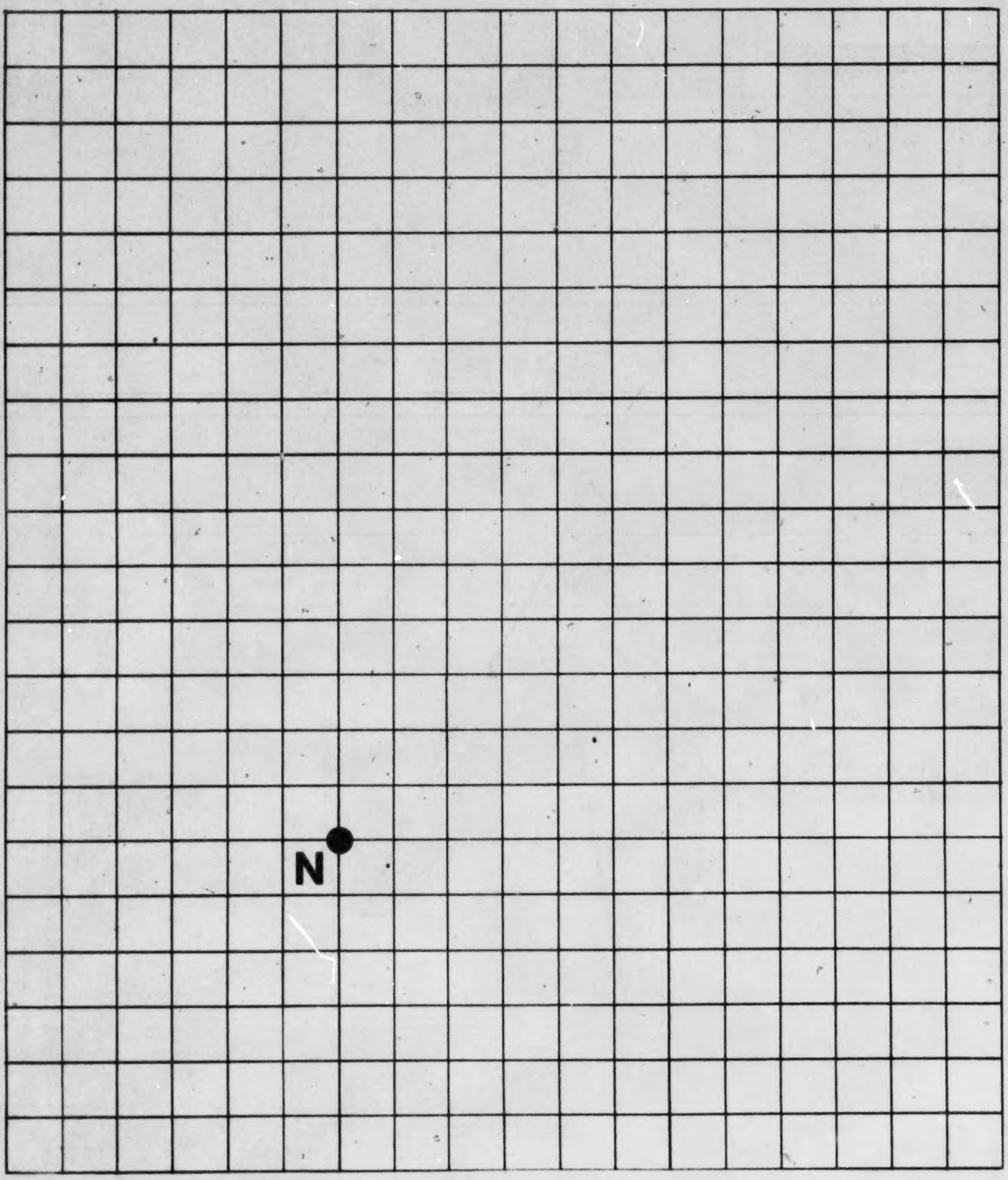
Name ______________________

G6 *



Name ____________________

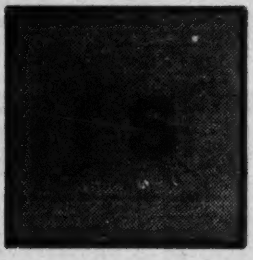
G6 **

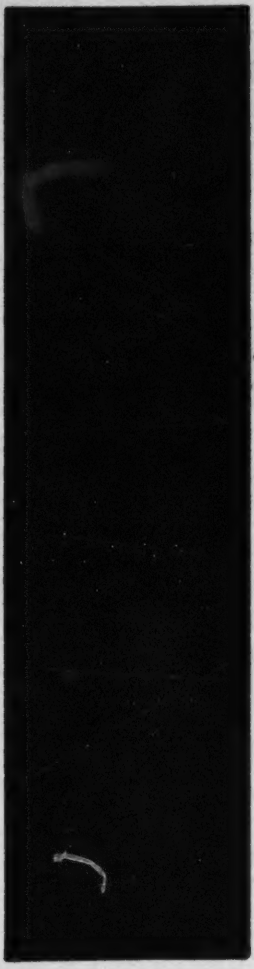


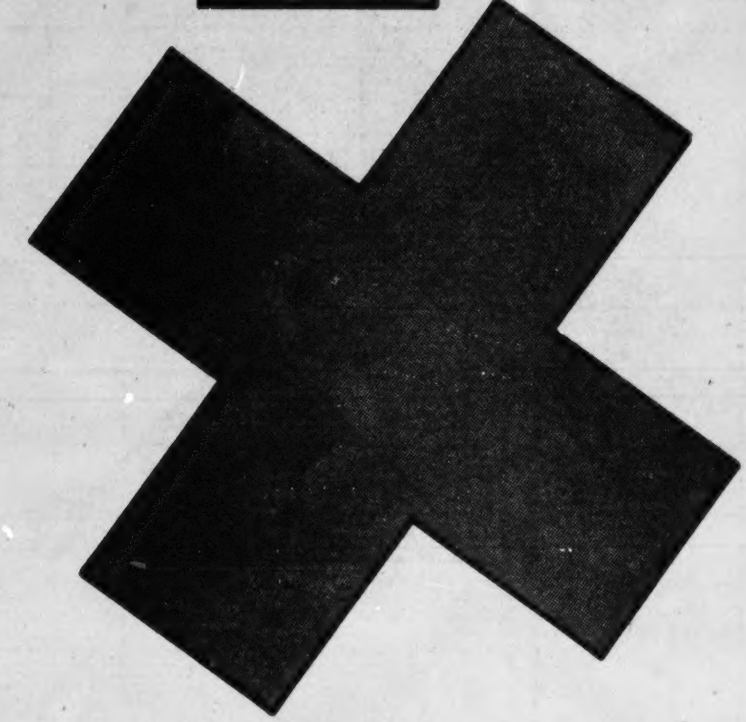
Name ______________________

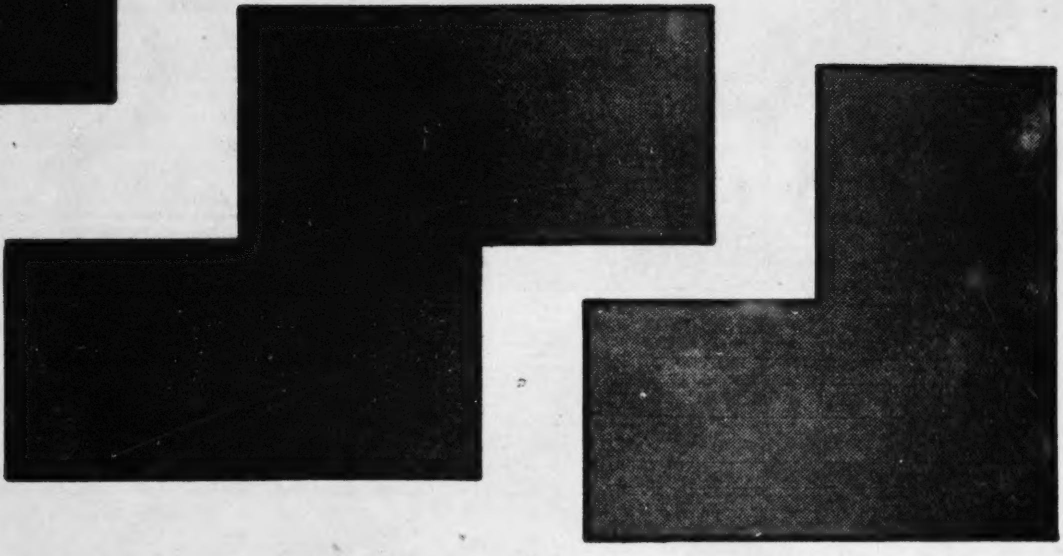
G 9 *

Find the area of each shape with



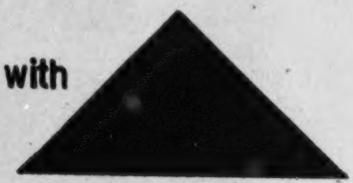


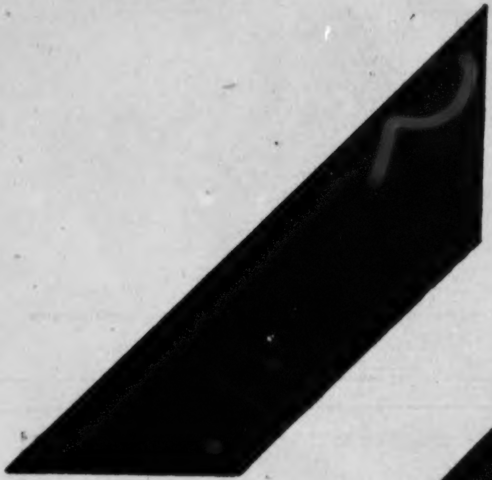


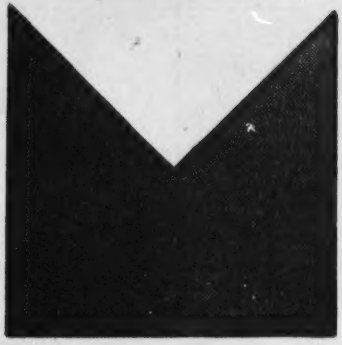


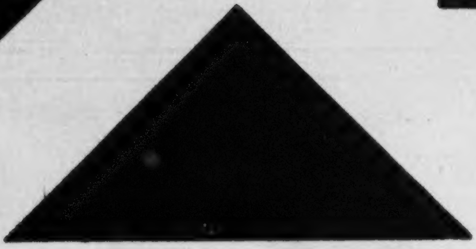
Name ______________________________ **G 9 ****

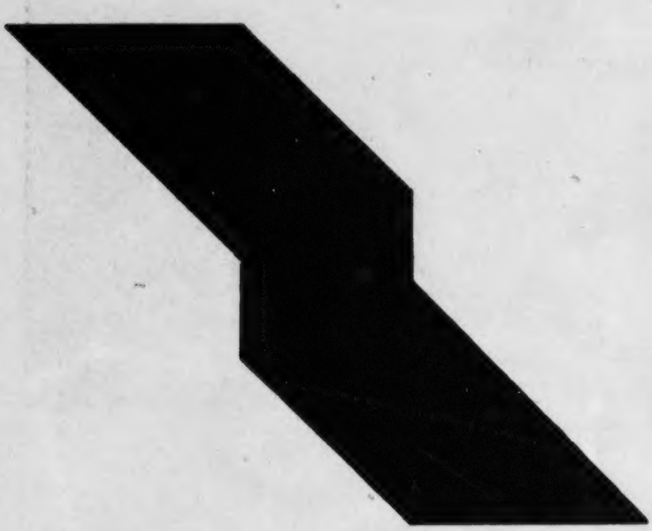
Find the area of each shape with

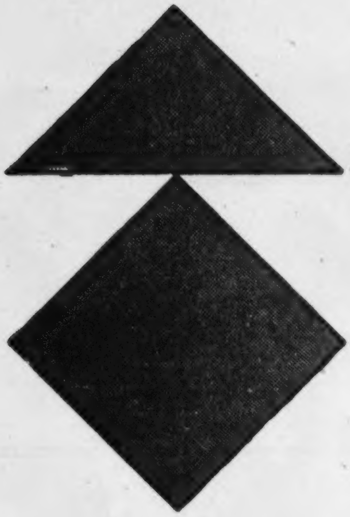






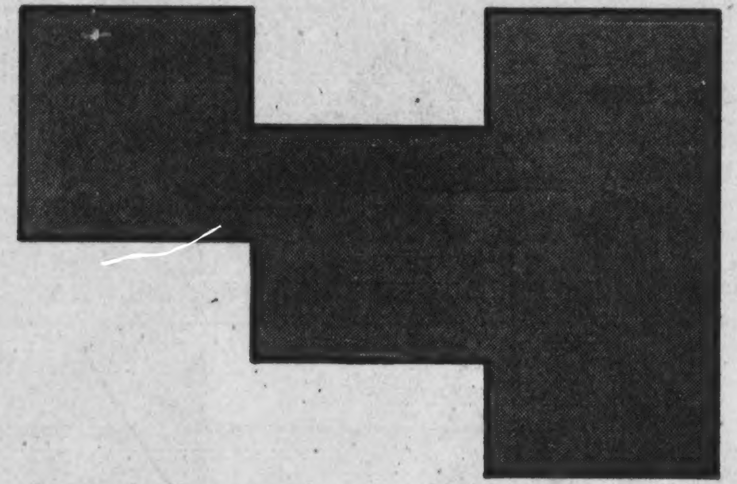




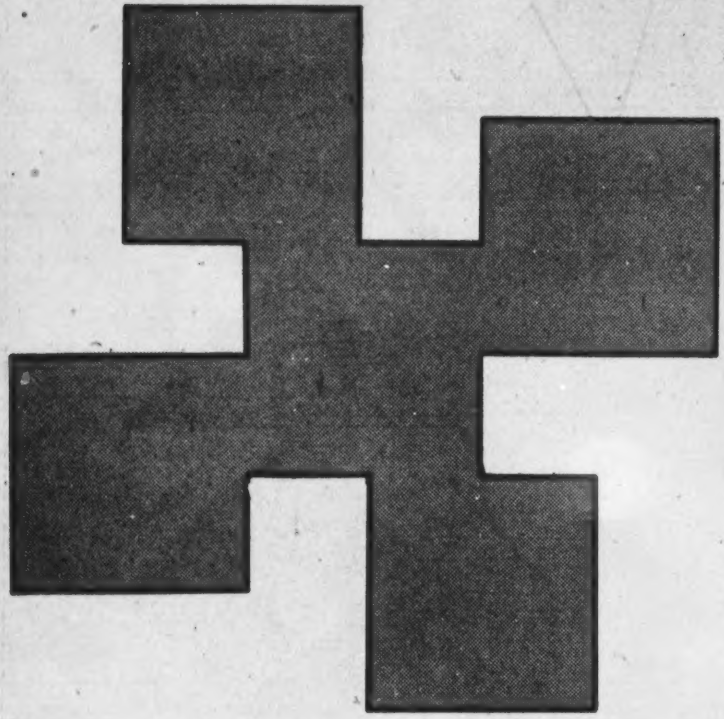


Name

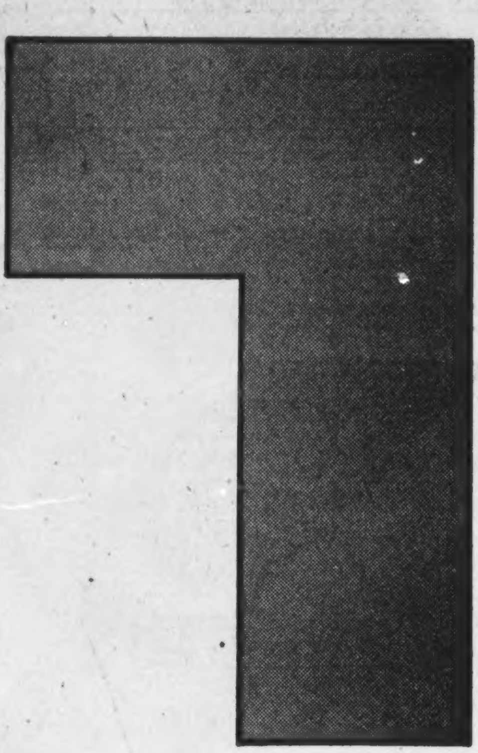
G 9 ***



Compare these shapes to the one above. Circle your answers.



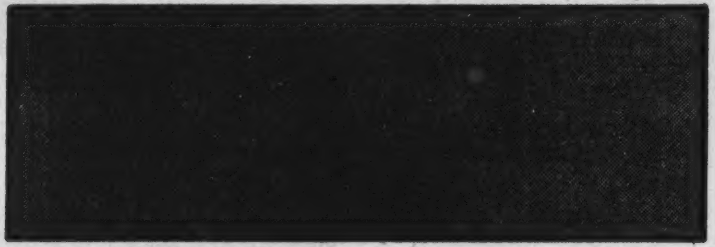
Smaller Bigger Same



Smaller Bigger Same

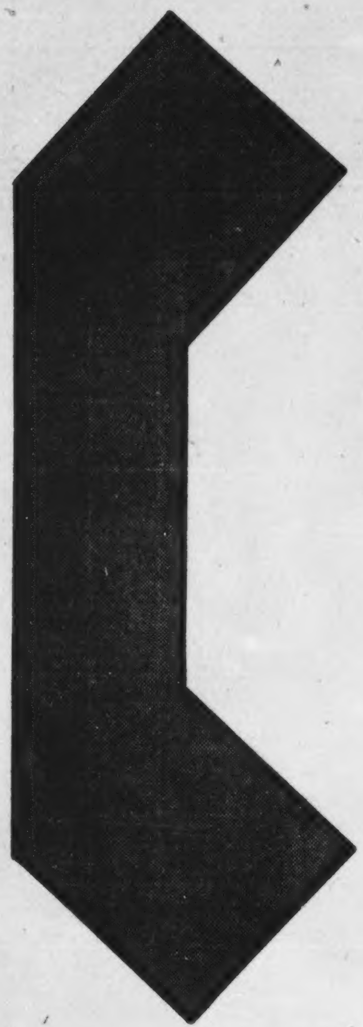
Name ____________________

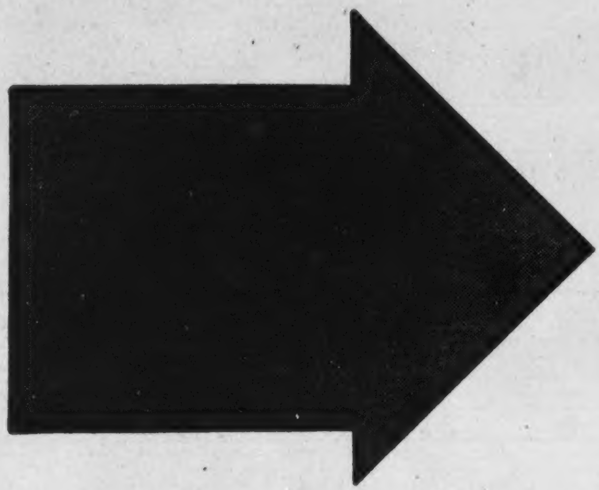
G 9 ****



Compare these shapes to the one above. Circle your answers.

Smaller Bigger Same



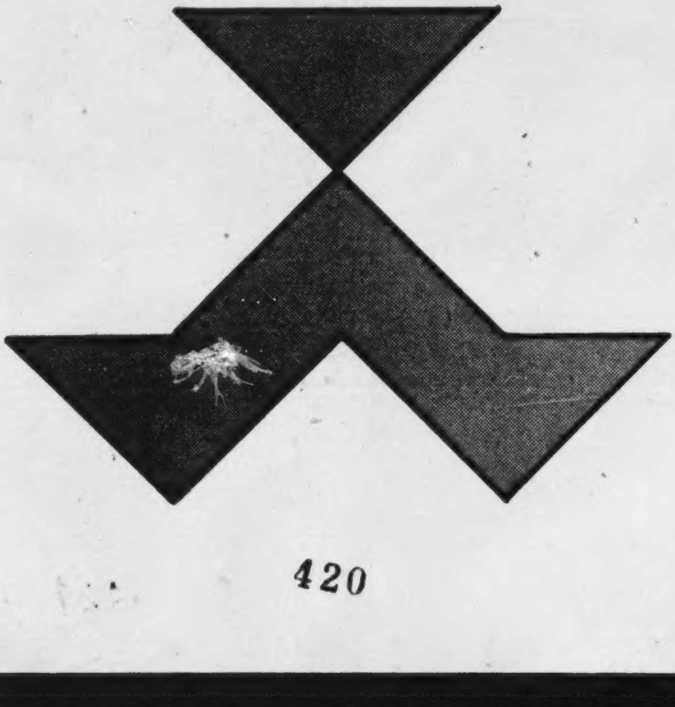


Smaller

Bigger

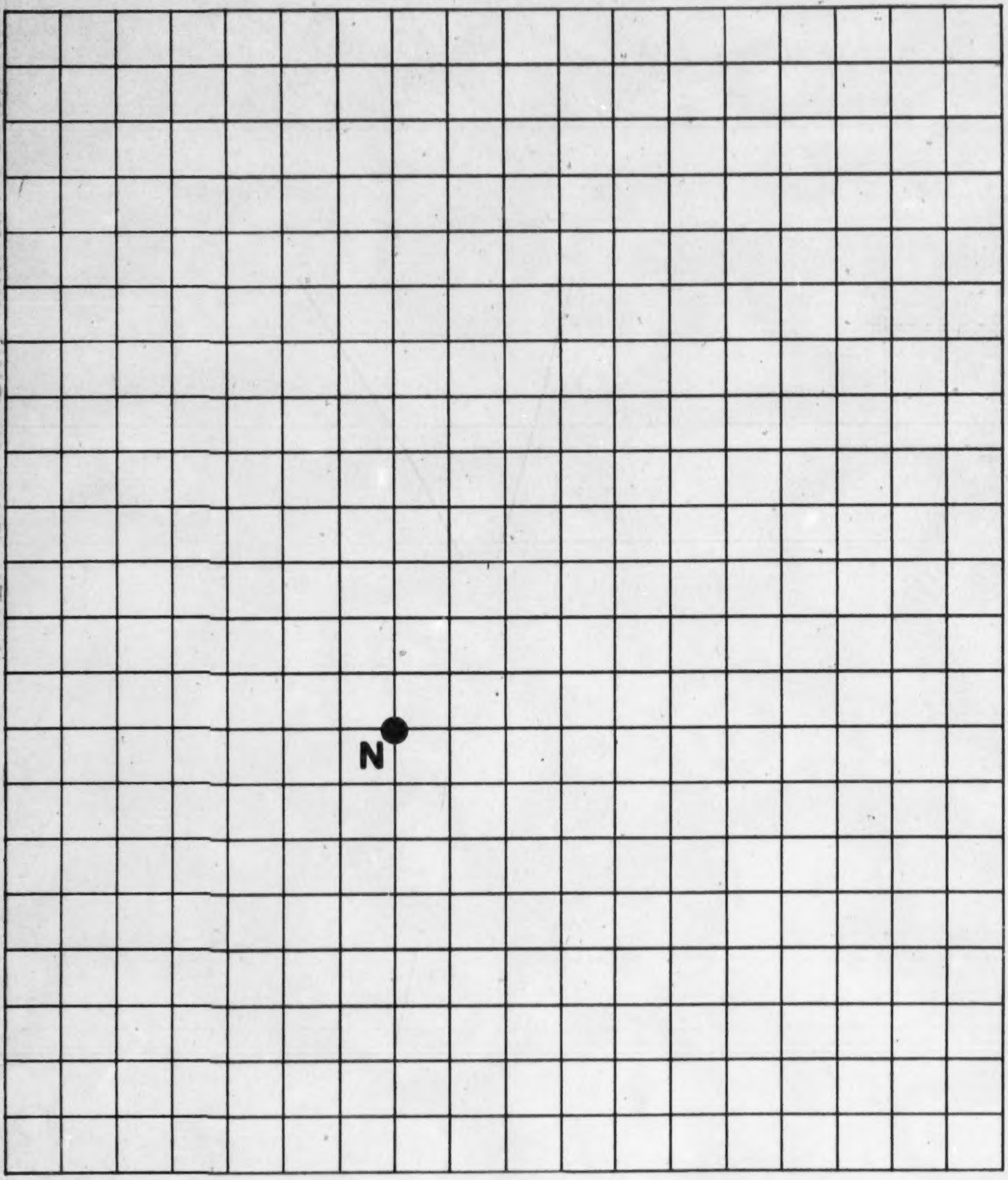
Same

Smaller Bigger Same



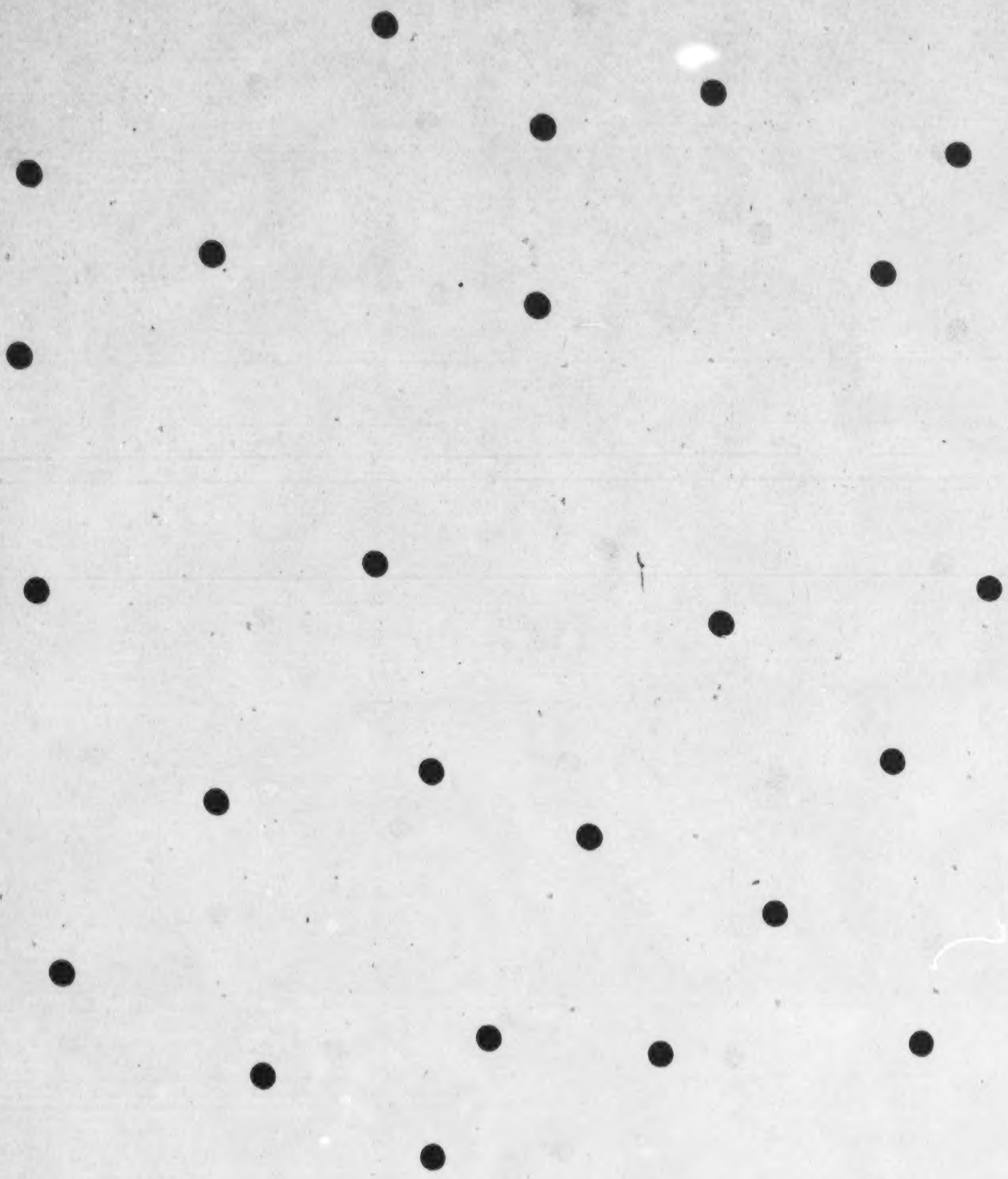
Name ______________________________

G 11

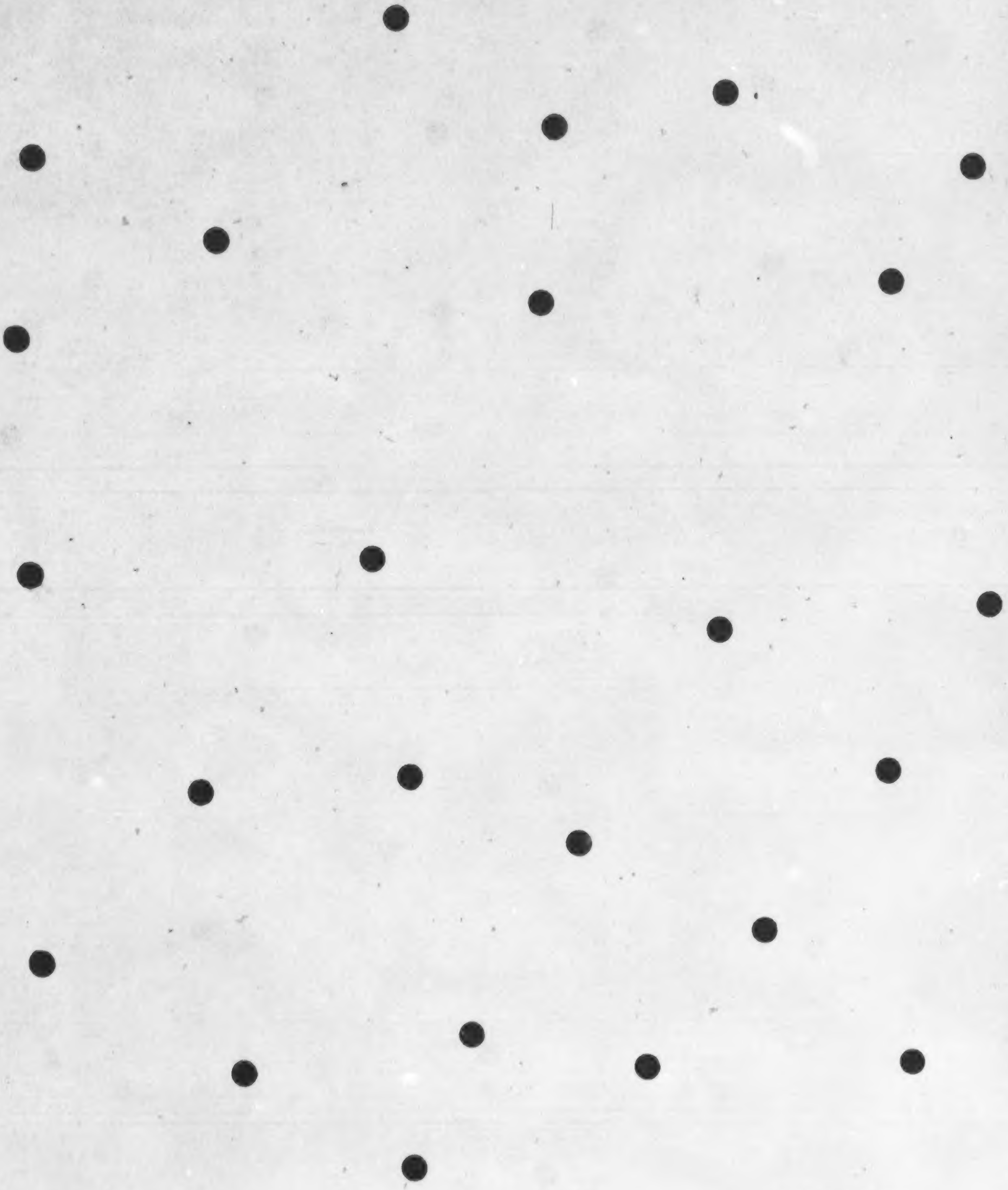


Name ______________________

G 12 *



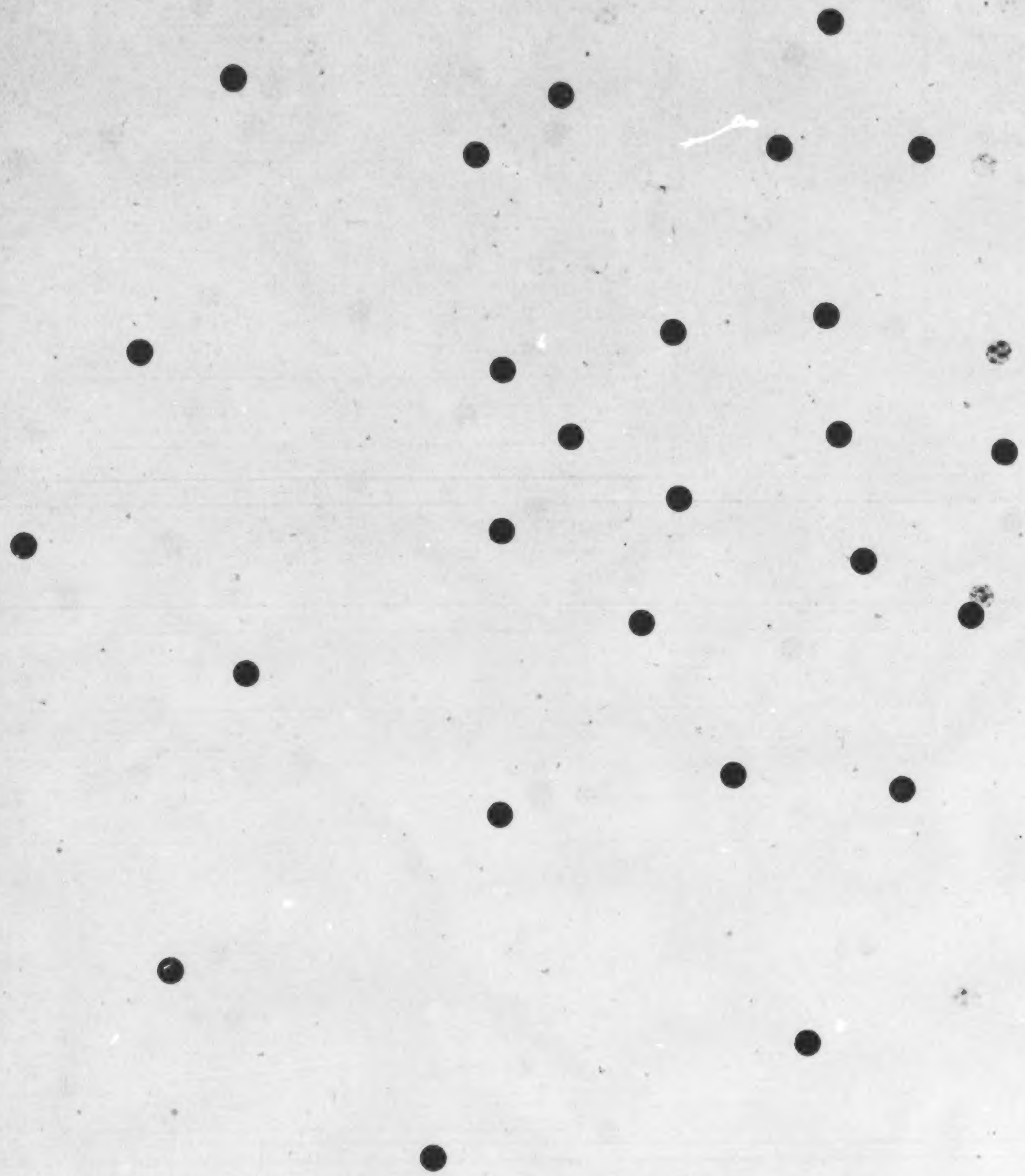
24 DOTS



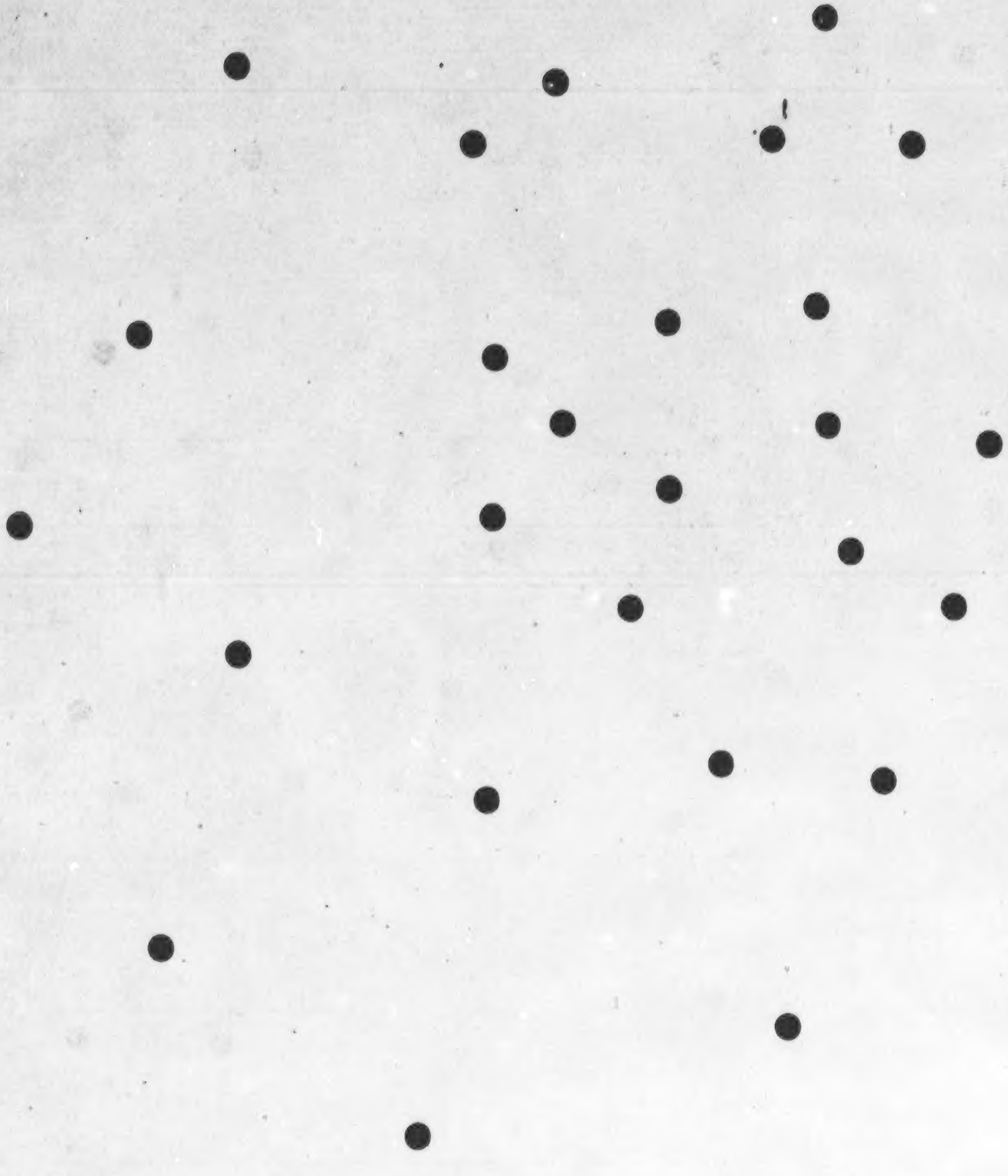
24 DOTS

Name ______________________________

G 12 **



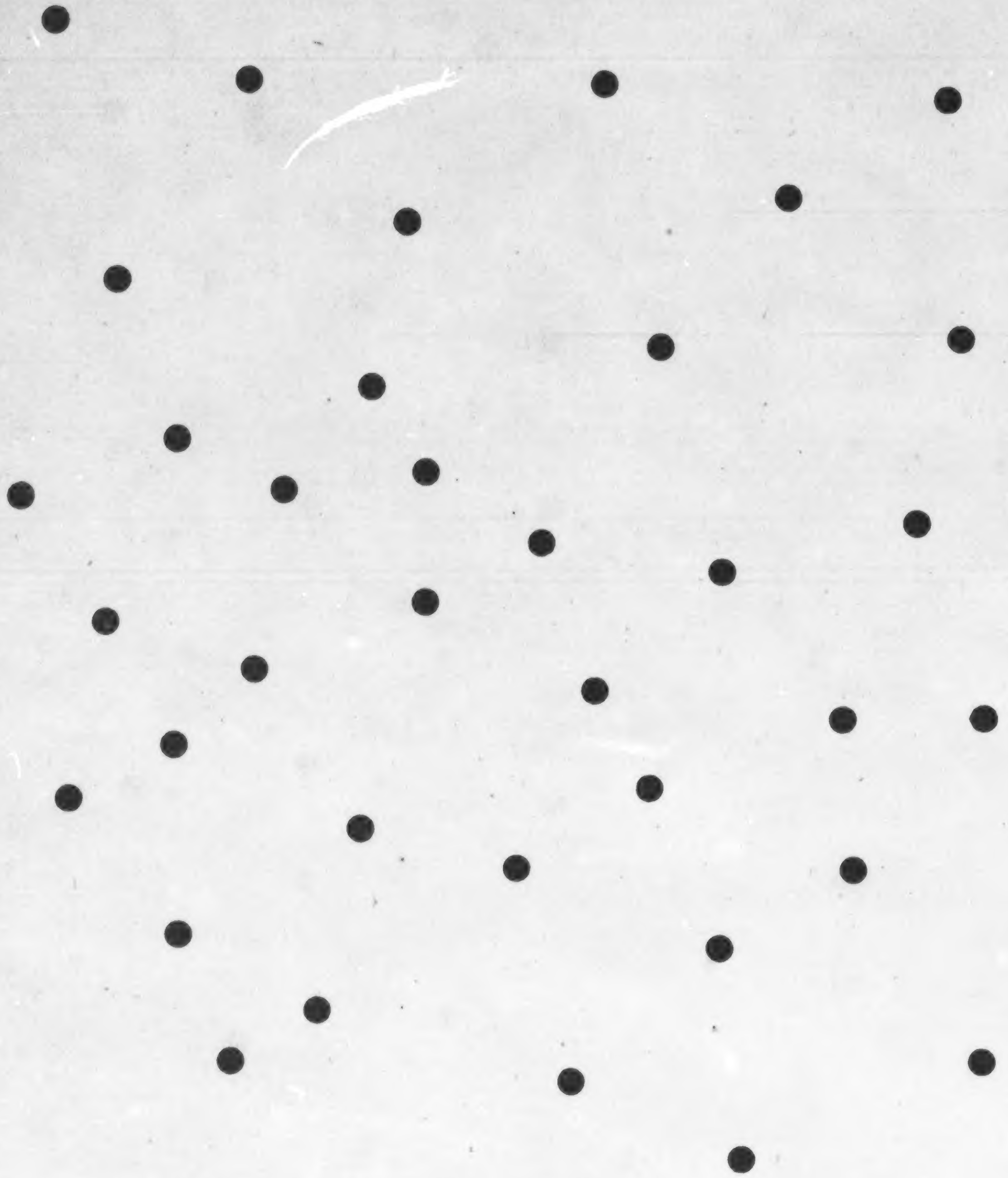
26 DOTS



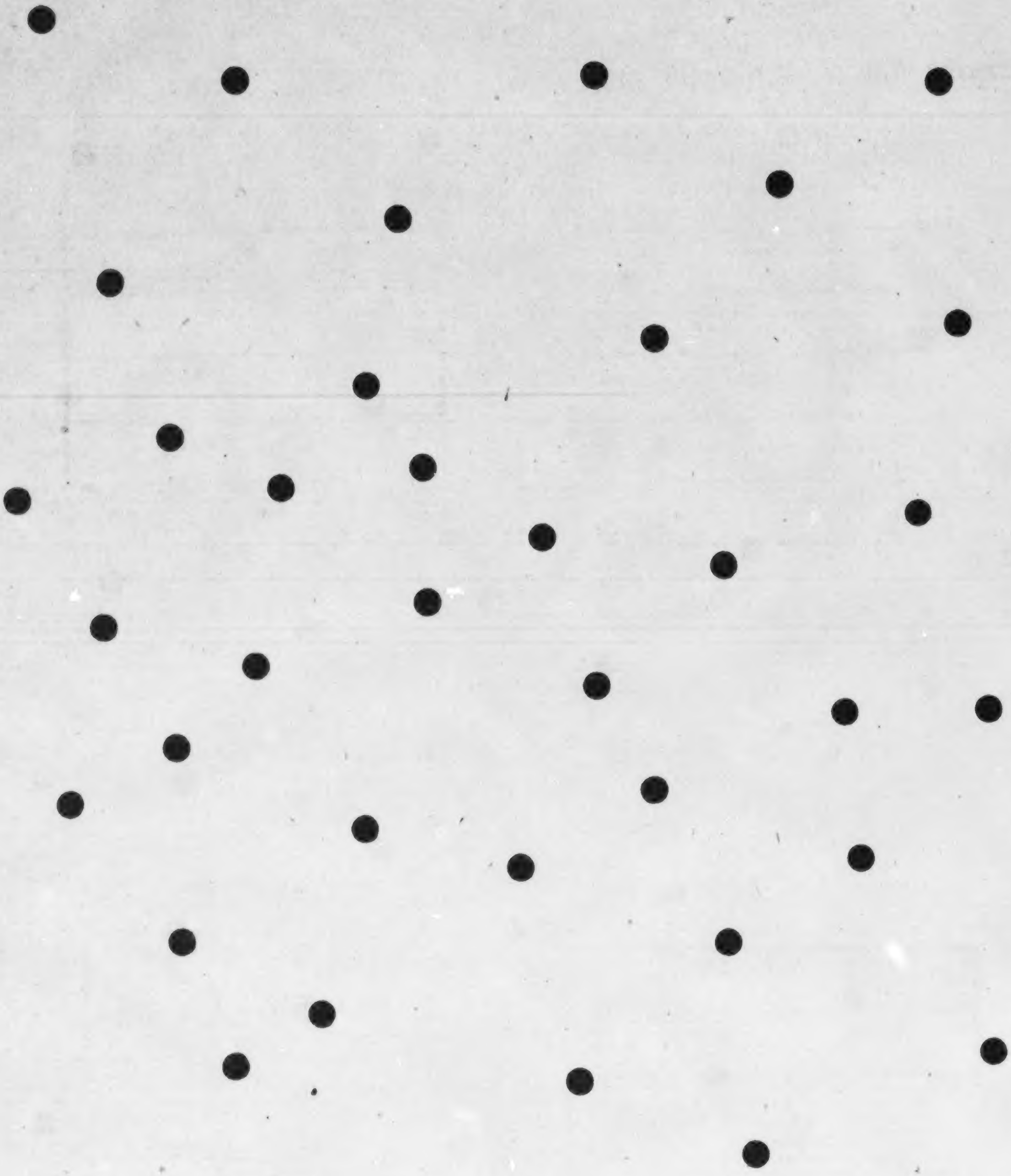
26 DOTS

Name ______________________

G 12 ***



36 DOTS

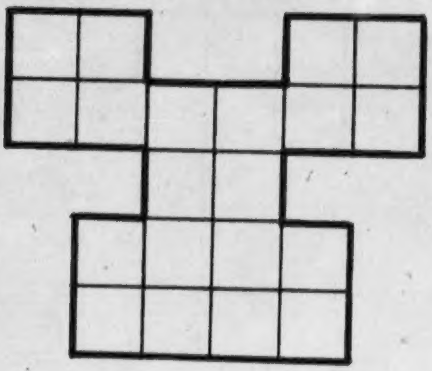


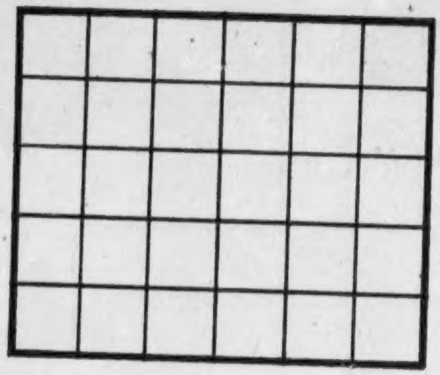
36 DOTS

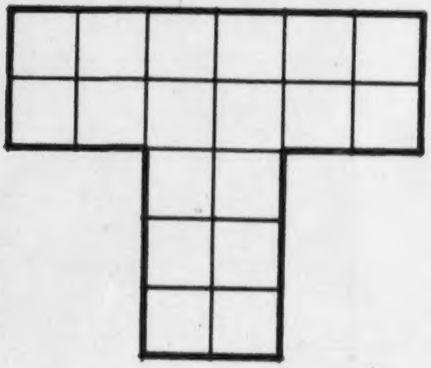
Name ____________________

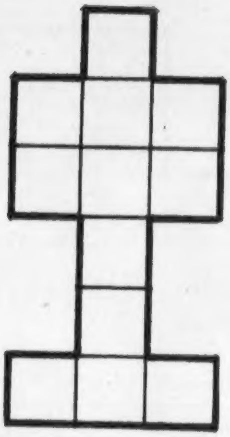
G 13 *

Color one-half of each shape red.





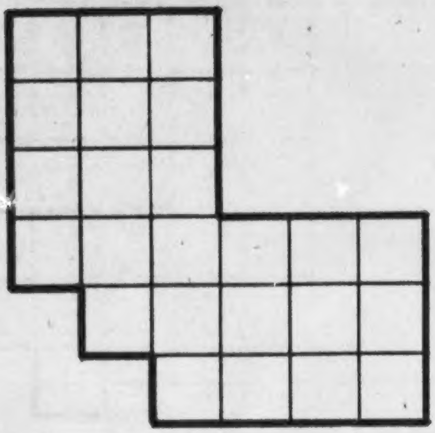


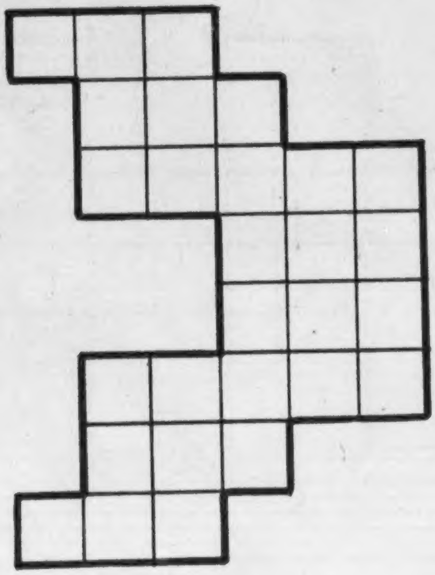


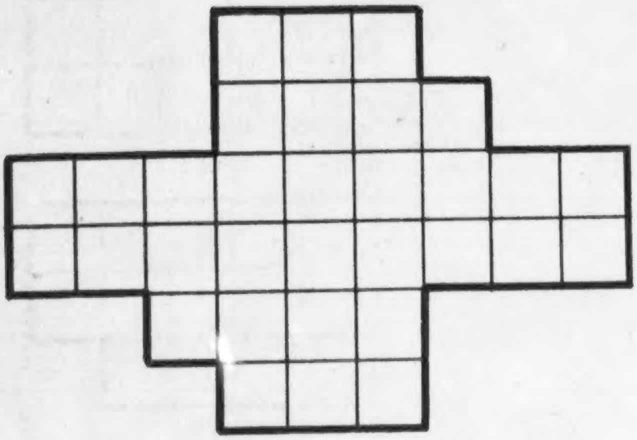
Name ______________________________

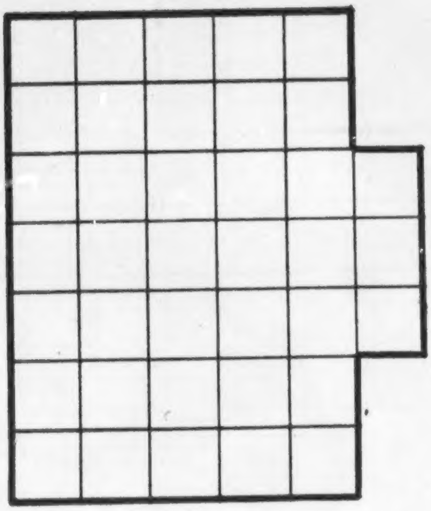
G 13 **

Color one-half of each shape red.





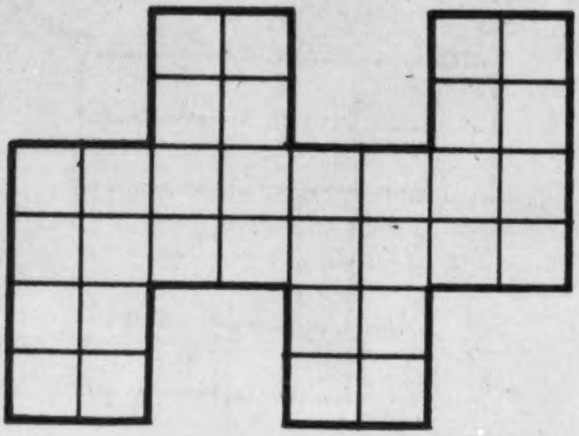


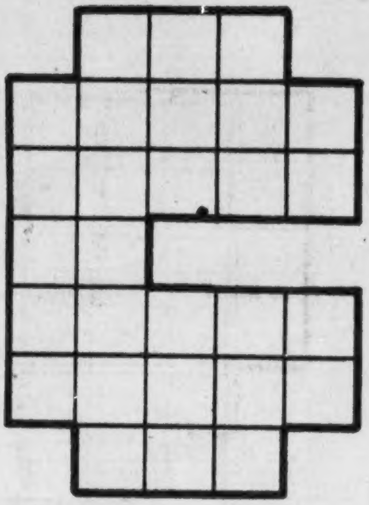


Name ______________________

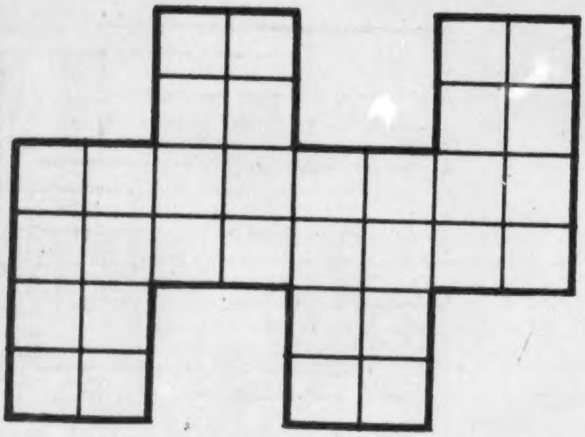
G 13 ***

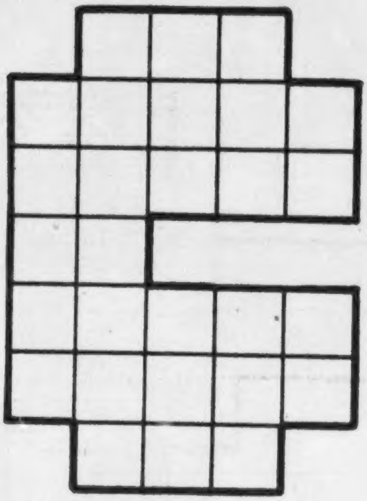
Color one-half of each shape red.





Color one-fourth of each shape blue.

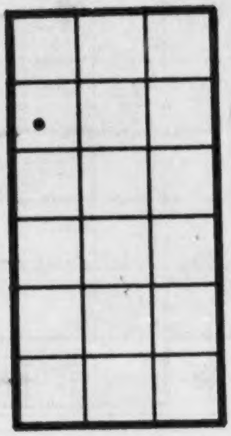


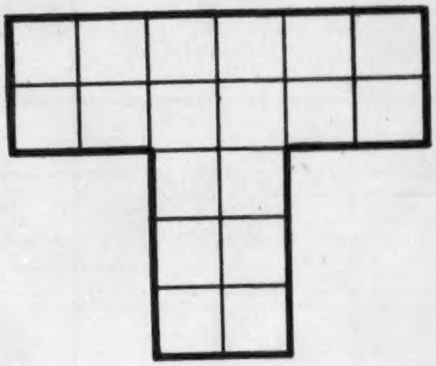


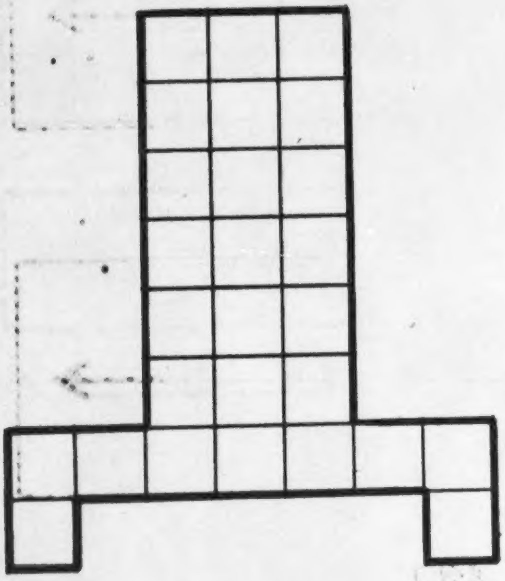
Name ______________________

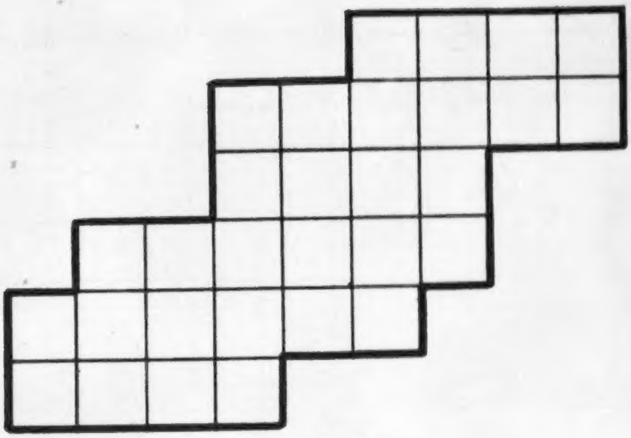
G 13 ****

Color one-third of each shape green.





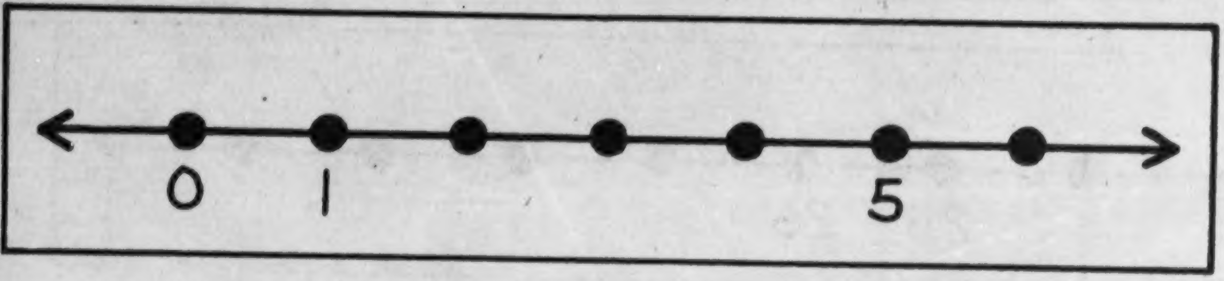


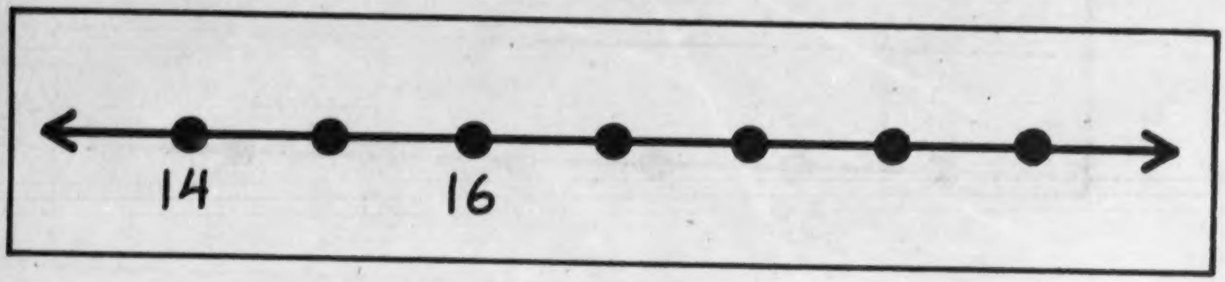


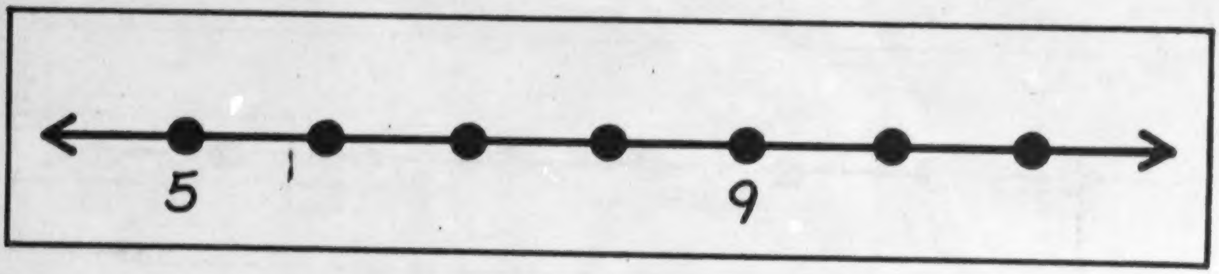
Name ______________________________

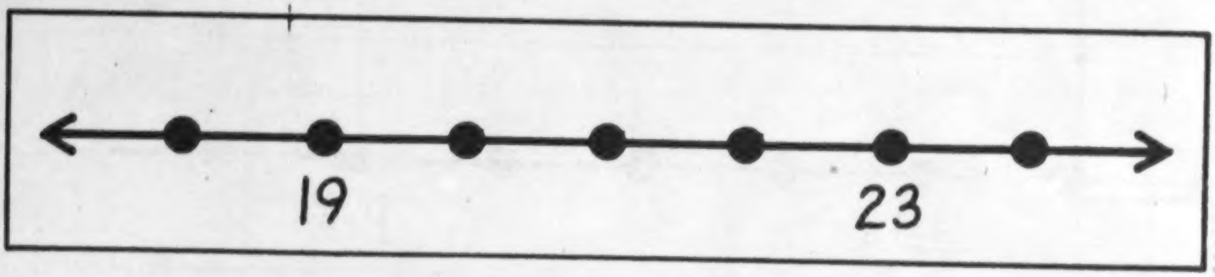
W1 *

Label the dots on these number lines.





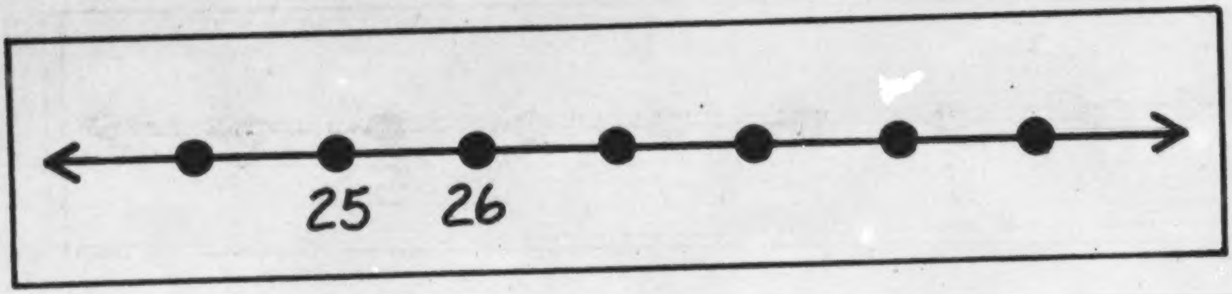


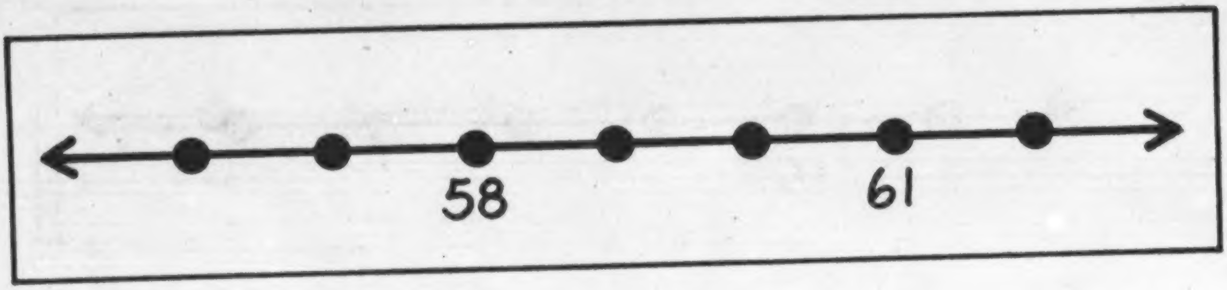


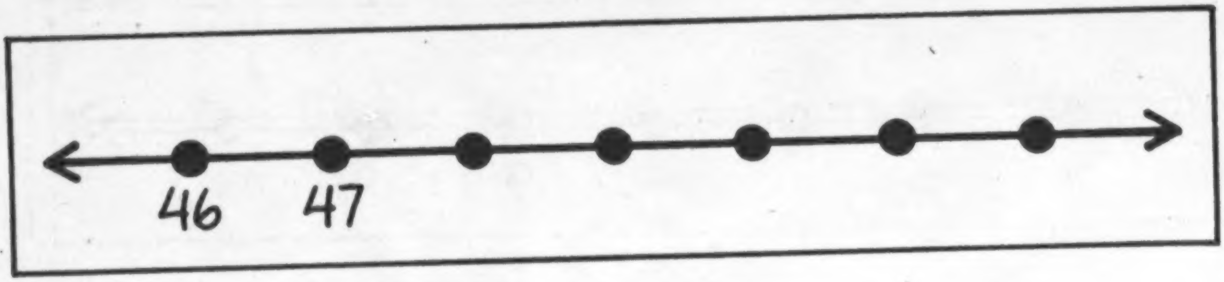
W1 **

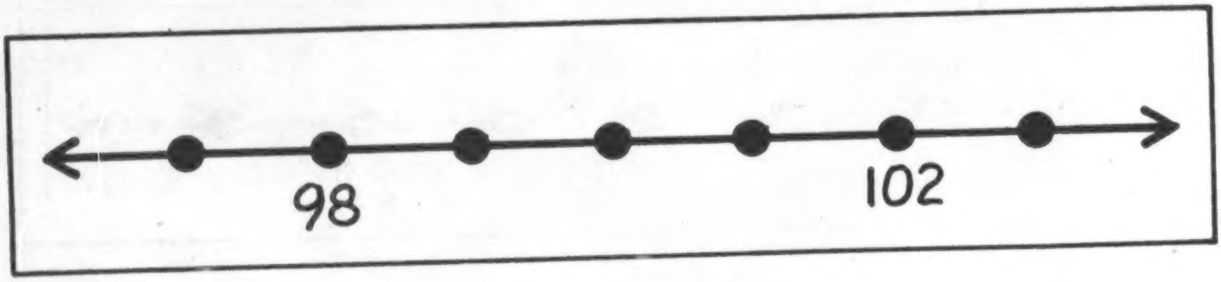
Name ______________________

Label the dots on these number lines.





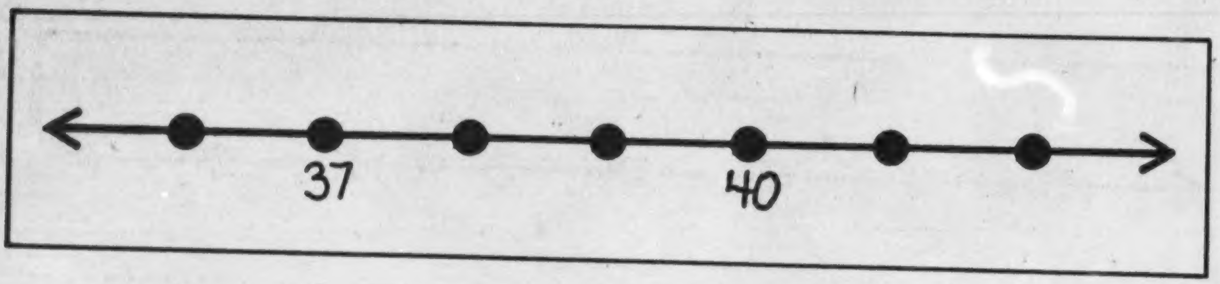


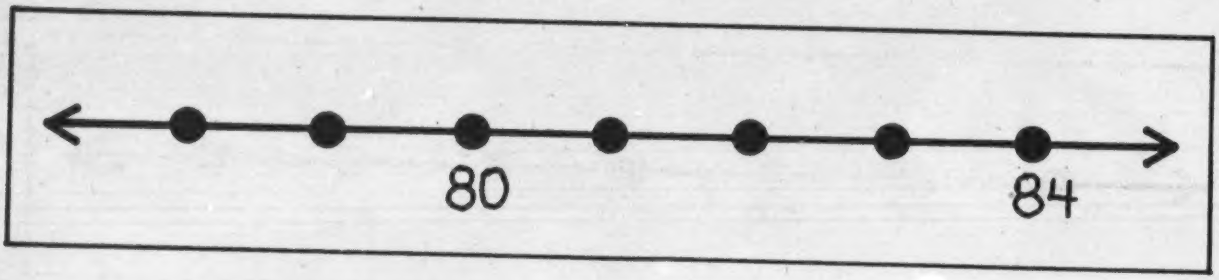


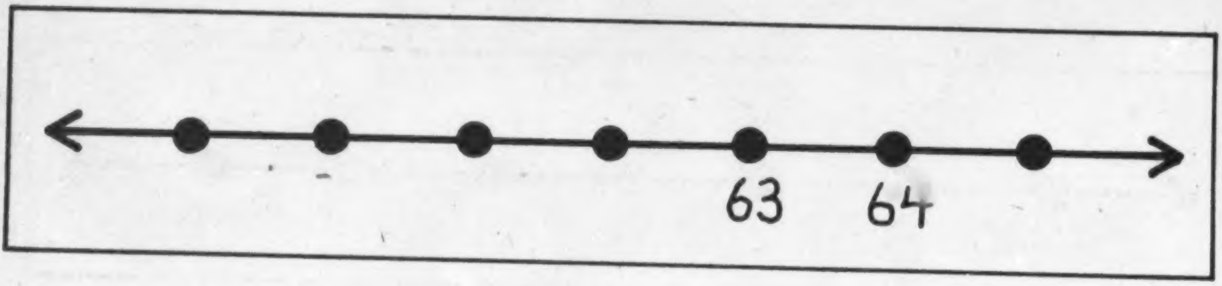
Name ______________________________

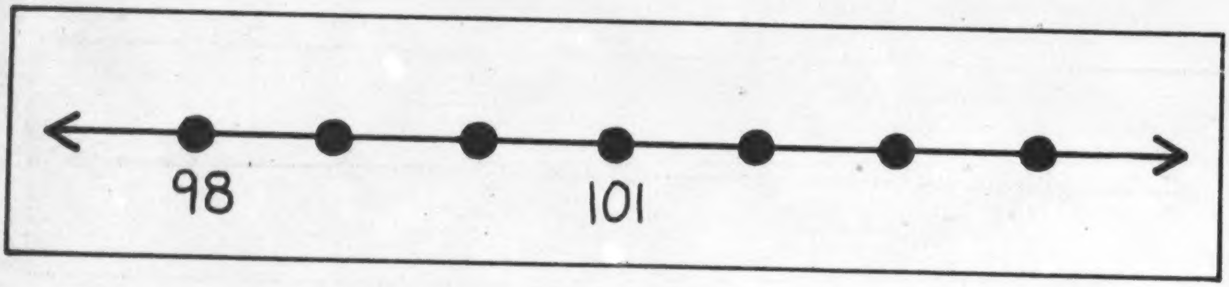
W1 ***

Label the dots on these number lines.





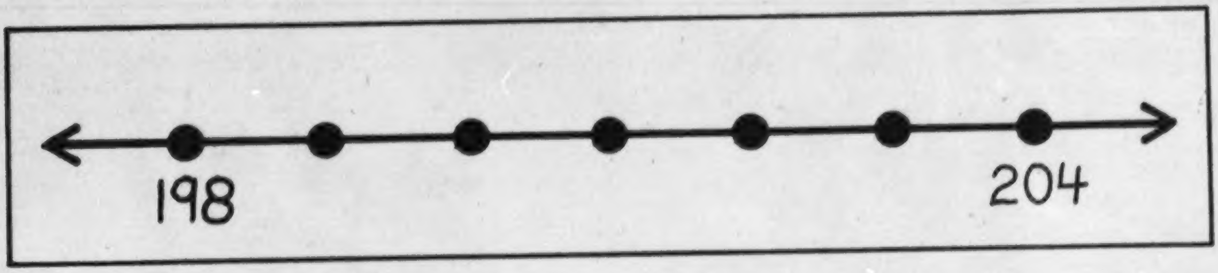


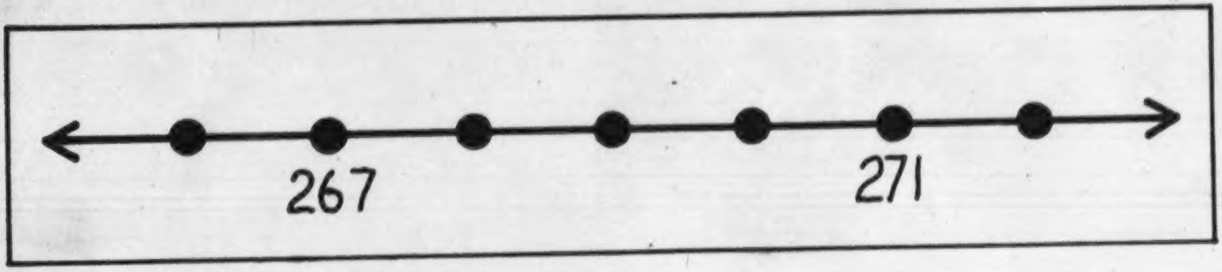


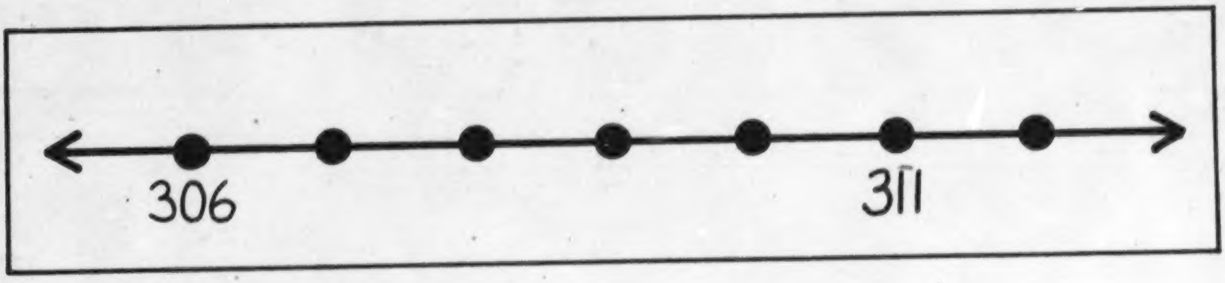
W1 ****

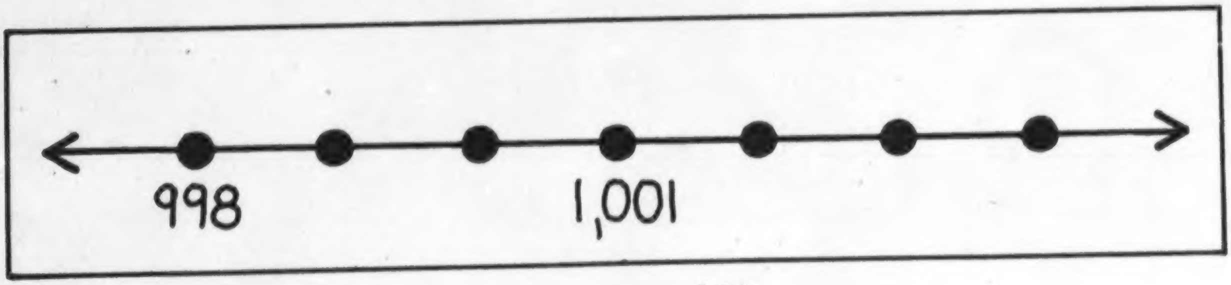
Name ______________________________

Label the dots on these number lines.





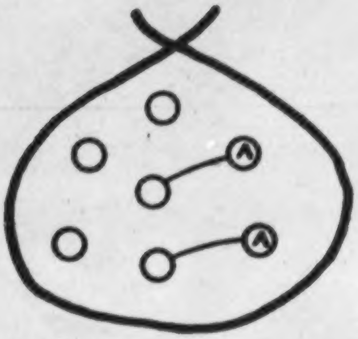




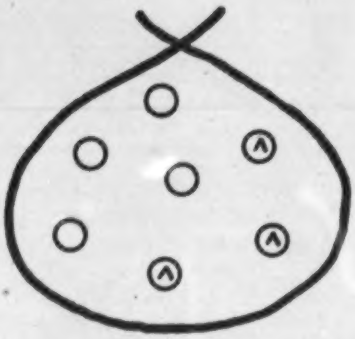
Name ______________________

W 2 *

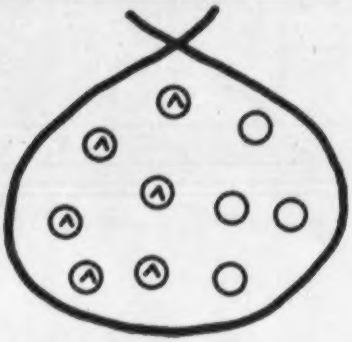
Complete these number stories.



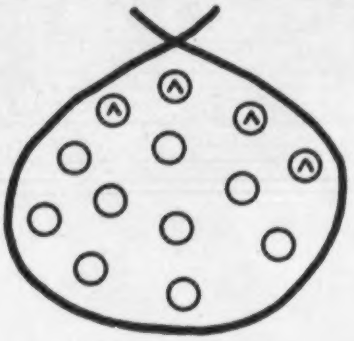
$5 + \hat{2} =$ ______



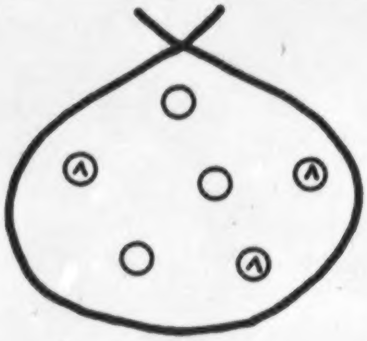
$4 + \hat{3} =$ ______



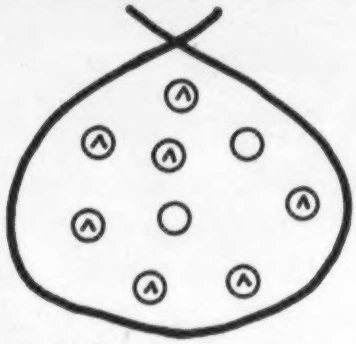
$\hat{6} + 4 =$ ______



$\hat{4} + 9 =$ ______



$3 + \hat{3} =$ ______

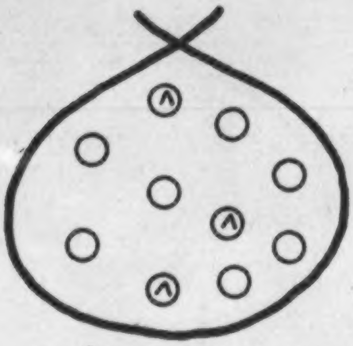


$2 + \hat{7} =$ ______

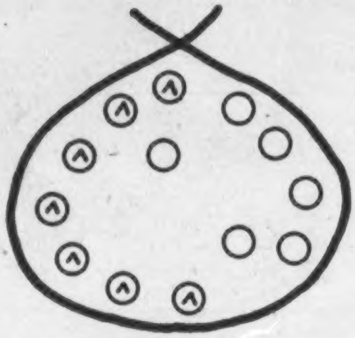
Name ____________________

W 2 **

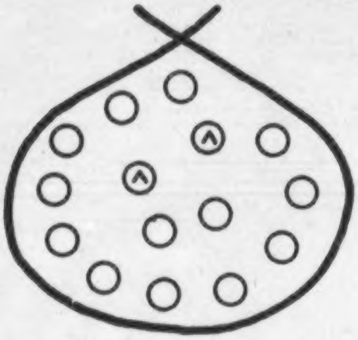
Complete these number stories.



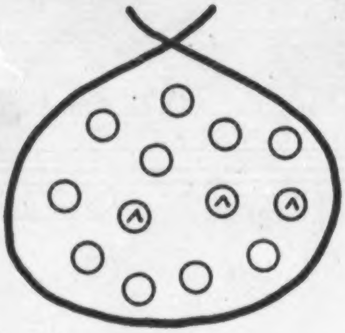
7 + $\hat{3}$ = ______



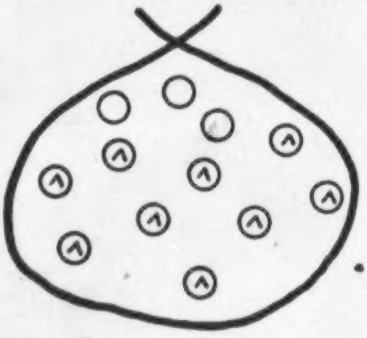
$\hat{7}$ + 6 = ______



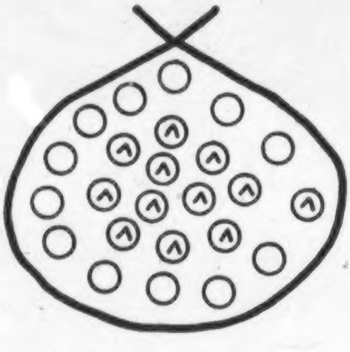
13 + $\hat{2}$ = ______



$\hat{3}$ + 10 = ______



3 + $\hat{9}$ = ______

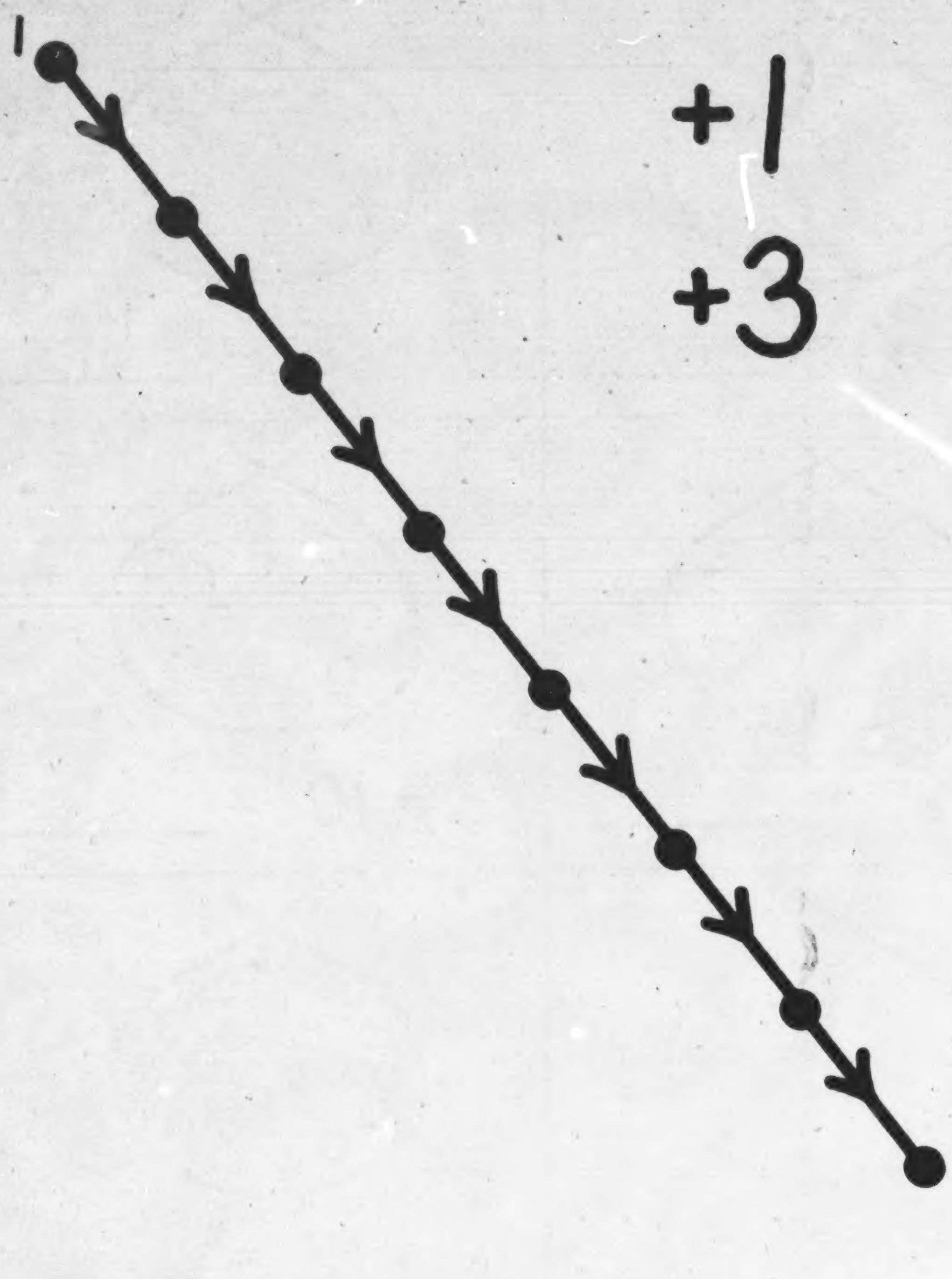


$\widehat{12}$ + 12 = ______

Name ______________________

W 5

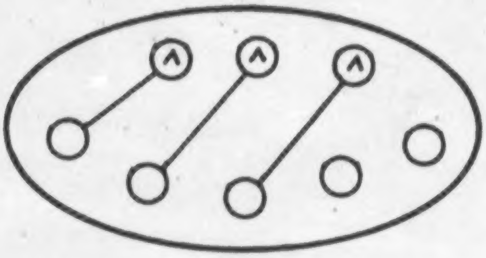
The secret number is in this arrow picture.



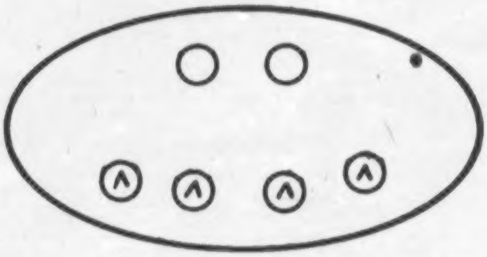
Name ______________________

Finish the number stories.

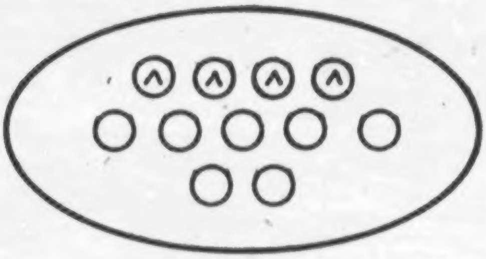
$\hat{3}+5=$ ______



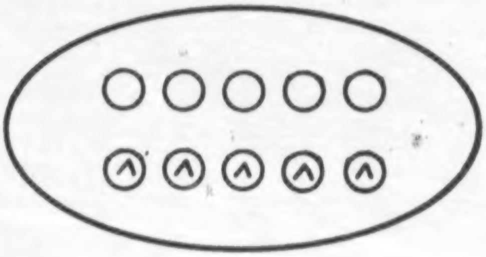
$2+\hat{4}=$ ______



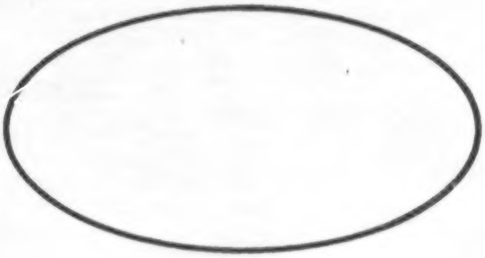
$\hat{4}+7=$ ______



$5+\hat{5}=$ ______



$3+\hat{1}=$ ______



Name ______________________ **W 8 ****

Finish the number stories.

$2 + 4 =$ ________

$5 + \widehat{4} =$ ________

$\widehat{3} + \widehat{3} =$ ________

$4 + \widehat{9} =$ ________

$\widehat{5} + 6 =$ ________

Name ______________________ **W 8 *****

Complete these number stories.

$\hat{2} + 3 =$ ______

$8 + \hat{8} =$ ______

$10 + \hat{1} =$ ______

$4 + 3 =$ ______

$\hat{9} + \hat{3} =$ ______

Name ________________________________

W8 ****

Complete these number stories.

$22 + \widehat{2} =$ ____

$\widehat{18} + 18 =$ ____

$20 + \widehat{10} =$ ____

$5 + \widehat{15} =$ ____

$\widehat{1} + 100 =$ ____

$\widehat{1{,}000} + 1{,}000 =$ ____